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ABSTRACT

We build a model of optimal fixed-rate mortgage refinancing with fixed costs and inattention and derive a new sufficient statistic that can be used to measure inattention frictions from simple moments of the rate gap distribution. In the model, borrowers pay attention to rates sporadically so they often fail to refinance even when it is profitable. When paying attention, borrowers optimally choose to refinance earlier than under a perfect attention benchmark. Our model can rationalize almost all errors of “omission” (refinancing too slowly) and a large fraction of the errors of “commission” (refinancing too quickly) previously documented in the data.

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1 Introduction

A large body of evidence suggests that inattention leads many borrowers to miss out on mortgage refinancing opportunities that could save them significant amounts of money. Without a benchmark model of optimal refinancing, it is difficult to know how sub-optimal this behavior is. In this paper, we characterize the impact of inattention on fixed-rate mortgage borrowers' optimal refinancing decisions and show that inattention is important for explaining refinancing patterns observed in the data. In our model, borrowers pay attention to rates sporadically, which means they often fail to refinance even when their "rate gap" — the difference between their current rate and the available rate on a new mortgage — is large. However, when they do pay attention, they optimally choose to refinance for smaller rate gaps than in a model with perfect attention. Thus, inattention can explain both errors of "omission" (refinancing too slowly) and errors of "commission" (refinancing too quickly) documented in the data.¹

We begin with an analysis of the borrower's optimal decision when mortgage rates follow a general class of Markov processes. Borrowers have a fixed-rate prepayable mortgage contract that can be refinanced at any time, but their cost-minimization objective is hindered by two different frictions: (i) whenever they refinance, they have to pay a fixed cost, and (ii) they only pay attention to the market sporadically. Borrowers optimally refinance when they pay attention and their rate gap is above a threshold. We show that greater inattention systematically lowers this threshold. While we prove this systematic relationship holds under a very general class of mortgage rate processes, assessing its magnitude requires further assumptions.

For most of the paper, we thus specialize the mortgage rate to be a Brownian motion, which delivers analytical solutions for optimal refinancing. With this assumption, our model is then identical to that of [Agarwal, Driscoll and Laibson \(2013\)](#) (thereafter, "ADL"), except that our borrowers are inattentive and can only make decisions at discrete points in time. In this Brownian motion special case, we analytically solve the borrower's value function and the rate gap threshold at which refinancing is optimal. At the limit where borrowers are infinitely attentive, we recover the ADL threshold formula, but we show that this threshold falls substantially for empirically relevant levels of attention and can thus matter quantitatively for empirical work estimating the frequency and magnitude of mistakes in refinancing decisions.

This key result is robust to two important extensions. First, while our analytical results are obtained when mortgage rates follow a Brownian motion, conclusions are

¹See [Agarwal, Rosen and Yao \(2016\)](#).

quantitatively similar when mortgage rates are instead mean-reverting. Second, we micro-found inattention by studying a perfectly attentive borrower who pays a cost each time they decide to observe the current mortgage market rate. This rational inattention framework delivers an optimal threshold that is decreasing in the observation cost, an ergodic average observation delay that is increasing in the observation cost, and thus a threshold that is increasing in the effective attention rate, like in our baseline model that takes inattention as exogenous.

We then derive our baseline model's implications for the ergodic distribution of rate gaps, and rate gaps upon refinancing. By aggregating individual-level refinancing decisions and characterizing the distribution of refinancing incentives, we derive a sufficient statistic that we use to infer the degree of inattention faced by mortgage borrowers in the US. While the average gap is strictly lower than the optimal threshold, we show that the average gap upon refinancing is exactly equal to the threshold. The difference between these two averages only depends on (i) the borrower's moving rate, (ii) the borrower's attention rate, and (iii) mortgage rate volatility. This observation yields a sufficient statistic approach, which relies on the estimation of these two empirical averages in order to recover the implied attention rate in a borrower population of interest.

Finally, we bring our model to the data to quantify the empirical relevance of inattention for optimal refinancing. Using our framework, we estimate the level of inattention required to rationalize our micro-data on rate gaps and prepayments. With this estimate, we compute borrowers' rate gap thresholds in the presence of inattention and quantify the frequency of errors of commission and omission. We then compare the frequency of these errors to that implied by the ADL framework, which assumes perfect attention. We find that the model with inattention, naturally, removes almost all errors of omission, and reduces the number of errors of commission by 35-46%.

2 Literature

Our paper is most related to [Agarwal, Driscoll and Laibson \(2013\)](#), who derive an analytic expression for the rate gap threshold at which households should be optimally refinancing their long-term fixed rate mortgage. We extend their model by introducing an inattention friction — in the spirit of [Calvo \(1983\)](#) — and by showing that such friction dampens the threshold at which borrowers find it optimal to refinance. We also study more general mortgage rate processes and characterize the effects of inattention on the

distribution of rate gaps.²

Many empirical investigations of mortgage refinancing decisions document some type of mistakes by borrowers (for e.g. [Keys, Pope and Pope \(2016\)](#) or [Andersen et al. \(2020\)](#), amongst many others). [Agarwal, Rosen and Yao \(2016\)](#) separate these mistakes into (i) *errors of commission* (refinancing at the “wrong” rate gap) and (ii) *errors of omissions* (not immediately refinancing once the rate gap threshold is reached). We show in our paper that our framework can rationalize both types of errors.

The study of the optimal prepayment strategy includes numerous articles from a separate literature focused on the valuation of mortgage-backed securities (“MBS”). The paper most related to ours is [Stanton \(1995\)](#) who, using aggregate prepayment data from a set of MBS, estimates a model that features (heterogeneous) fixed costs of refinancing and (homogeneous) inattention. Solving the model via backward induction, the article finds that borrowers (i) face heterogeneous fixed costs and (ii) exhibit substantial amount of time-dependent inaction. Our study, rather than relying on numerical methods, instead derives an analytic formula for borrowers’ optimal refinancing strategy, and it uses mortgage-level (rather than pool-level) micro-data to estimate inattention in the data.

Finally, our paper relates to the role of inattention for the exercise of options beyond mortgage refinancing. For example, [Kadan, Liu and Yang \(2009\)](#) derive the value of executive stock options when the executive is inattentive and show that the optimal exercise barrier is increasing in the attention rate. Thus, they conclude similarly that agents should exercise their option at lower “intrinsic values”, once they get the opportunity to do so.

3 Mortgage refinancing with inattention: general case

We consider a model of mortgage refinancing decisions similar to that in [Berger et al. \(2023\)](#). We study fixed-rate mortgages that can be refinanced at any time — a contract chosen by the majority of borrowers in the US. We specify an exogenous process for the mortgage rate.

²An older version of their paper also included some overlapping conclusions related to inattention. We thank John Driscoll for alerting us to and sharing those results, which do not appear in the published paper. Our results also differ in some important ways from this unpublished material: we analyze more general rate environments as well as the optimal choice of attention, explore distributional implications, and have a more extensive empirical application.

3.1 Setup

Time t is continuous. We consider a risk-neutral, long-lived borrower with subjective discount rate ρ . The borrower has financed the purchase of a house with a long-term fixed-rate prepayable mortgage with coupon c_t and constant unit balance. Let m_t be the prevailing mortgage rate, i.e., the rate that can be locked in when refinancing at time t . Two separate frictions limit the borrower's refinancing ability. First, the borrower is inattentive and makes decisions only at discrete times, modeled as i.i.d. Poisson events occurring with intensity λ — the *attention rate*. Second, the borrower bears closing costs ψ when refinancing. Last, the borrower moves from one house to another at intensity ν ; when doing so, the mortgage coupon gets reset to the prevailing mortgage rate.³

We assume that the mortgage rate is a smooth function $m(\cdot)$ of a latent state vector x_t , a possibly multidimensional, time-homogeneous Itô process with drift $\mu(x)$, diffusion $\sigma(x)$ and infinitesimal generator \mathcal{L} .⁴

3.2 The role of inattention

Denote $V(x, c)$ the valuation of all future mortgage liabilities for a borrower paying a coupon c , when the latent state is x . The borrower solves

$$\begin{aligned} V(x, c) &:= \inf_{a \in \mathcal{A}} \mathbb{E}_{x, c} \left[\int_0^{+\infty} e^{-\rho t} \left(c_t^{(a)} dt + a_t \psi dN_t^{(\lambda)} \right) \right], \\ \text{s.t.} \quad dc_t^{(a)} &= \left(m(x_t) - c_{t-}^{(a)} \right) \left(a_t dN_t^{(\lambda)} + dN_t^{(\nu)} \right), \end{aligned} \quad (1)$$

where \mathcal{A} is a set of progressively measurable binary actions $a = \{a_t\}_{t \geq 0}$ such that $a_t \in \{0, 1\}$ at all times, $N_t^{(\lambda)}$ and $N_t^{(\nu)}$ are counting processes with jump intensity λ and ν , $c_t^{(a)}$ is the coupon rate on the mortgage for a borrower following strategy a , and the subscript on the expectation indicates that it is conditional on the information available at time t . At the random points in time when the borrower can pay attention, the choice $a_t = 1$ represents a decision to refinance, while $a_t = 0$ means that the borrower chooses to keep their existing mortgage.

A straightforward application of stochastic control theory, in connection with technical conditions related to the generator \mathcal{L} ⁵, yields a valuation V that solves

$$(\rho + \nu + \lambda) V(x, c) = c + \nu V(x, m(x)) + \lambda \min \{ V(x, c), V(x, m(x)) + \psi \} + \mathcal{L}V(x, c), \quad (2)$$

³Moving-related fixed costs could be added to the model without changing any of our conclusions.

⁴ \mathcal{L} is defined over functions f of class \mathcal{C}^2 via $\mathcal{L}f(x) = \mu(x) \cdot \partial_x f(x) + \frac{1}{2} \text{trace}(\sigma'(x) \partial_{xx'} f(x) \sigma(x))$.

⁵See [Strulovici and Szydlowski \(2015\)](#) or [Fleming and Soner \(2006\)](#) for example.

with an optimal refinancing choice that satisfies $a^*(x, c) = \mathbb{1}_{\{c - m(x) \geq \theta(x)\}}$, where the threshold $\theta(x)$ satisfies the indifference condition

$$V(x, m(x)) + \psi = V(x, m(x) + \theta(x)). \quad (3)$$

According to (3), whenever the current mortgage coupon is $\theta(x)$ above the current mortgage rate $m(x)$, the borrower is indifferent between (i) refinancing the mortgage, paying the fixed cost ψ and resetting the coupon to the current rate $m(x)$, and (ii) staying put. In our first proposition (proven in [Appendix A.1](#)), we study the extent to which the threshold $\theta(x)$ varies with the degree of borrower inattention.

Proposition 1 *Assume that the valuation V is differentiable in the parameter λ . The optimal rate gap threshold $\theta(x)$ is an increasing function of λ .*

Lower attention reduces the threshold, so borrowers optimally choose to refinance for smaller rate gaps when paying attention, since they only pay attention to rates sporadically. Intuitively, inattentive borrowers “pull the trigger” earlier when given the opportunity to refinance, since they might not have the opportunity to refinance again for some time.

4 Mortgage rates as a random walk

[Proposition 1](#) establishes a general result that relies only on weak assumptions on the statistical properties of the mortgage rate. In this section, we characterize analytically the threshold $\theta(x)$ in the special case where mortgage rates follow a random walk.

Proposition 2 *Assume that m_t is a Brownian motion with volatility σ . Introduce the constants η_0, η_λ and ϵ_λ , equal to:*

$$\eta_0 := \frac{\sqrt{2(\rho + \nu)}}{\sigma} \quad \eta_\lambda := \frac{\sqrt{2(\rho + \nu + \lambda)}}{\sigma} \quad \epsilon_\lambda := \frac{(\rho + \nu)(\eta_0 + \eta_\lambda)}{\lambda}.$$

The borrower valuation satisfies $V(m, c) = \frac{c}{\rho} + v(z)$, with $z := c - m$ and

$$v(z) = \begin{cases} k_- e^{\eta_0(z-\theta)} + \frac{\nu}{\rho+\nu} \left[v(0) - \frac{z}{\rho} \right] & \text{if } z \leq \theta \\ k_+ e^{-\eta_\lambda(z-\theta)} + \frac{\nu}{\rho+\nu+\lambda} \left[v(0) - \frac{z}{\rho} \right] + \frac{\lambda}{\rho+\nu+\lambda} \left[v(0) + \psi - \frac{z}{\rho} \right] & \text{if } z \geq \theta. \end{cases} \quad (4)$$

The constants of integration k_-, k_+ are given in [Appendix A.2](#), and v is a strictly decreasing and concave function of z . The (state-independent) rate gap threshold $\theta > 0$ is given by

$$\theta = (\rho + \nu) \psi + \frac{1}{\eta_0 + \epsilon_\lambda} + \frac{1}{\eta_0} W \left(\frac{-\eta_0}{\eta_0 + \epsilon_\lambda} \exp \left\{ \frac{-\eta_0}{\eta_0 + \epsilon_\lambda} [1 + (\rho + \nu) (\eta_0 + \epsilon_\lambda) \psi] \right\} \right), \quad (5)$$

where W is the Lambert-W function. θ increases with the attention rate λ , and asymptotically:

$$\lim_{\lambda \rightarrow 0} \theta = (\rho + \nu) \psi \quad (6)$$

$$\lim_{\lambda \rightarrow +\infty} \theta = (\rho + \nu) \psi + \frac{1}{\eta_0} + \frac{1}{\eta_0} W(-\exp\{-[1 + (\rho + \nu)\eta_0\psi]\}). \quad (7)$$

A Taylor expansion of the implicit equation underlying (5) around $\theta = 0$ yields an approximation $\hat{\theta}$ of the threshold θ with formula:

$$\hat{\theta} = \sqrt{\frac{2}{\eta_0} \left(1 + \frac{\epsilon_\lambda}{\eta_0}\right) (\rho + \nu) \psi + \left(\frac{\epsilon_\lambda}{\eta_0^2}\right)^2} - \frac{\epsilon_\lambda}{\eta_0^2}. \quad (8)$$

Our proof in [Appendix A.2](#) relies on the observation that the value function can be decomposed into (i) the present value of all future interest payments c/ρ (based on the current mortgage coupon) plus (ii) the value of a refinancing option which, given the unit root behavior of mortgage rates, only depends on the rate gap z .

[Proposition 2](#) generalizes the results of [Agarwal, Driscoll and Laibson \(2013\)](#) to the case where borrowers are inattentive. Formula (6) suggests that a completely inattentive borrower should only refinance when its rate gap is above the flow value of the fixed cost $(\rho + \nu)\psi$. At the other extreme, if the borrower is infinitely attentive, the threshold reduces to the ADL formula (7). Importantly, a decrease in λ reduces the threshold — a special case of the more general result of [Proposition 1](#). [Figure 1a](#) illustrates how the threshold θ given in (5) varies with attention λ , and also the degree of precision of our approximation formula (8). Given our estimated borrower attention (see [Section 7](#)) and parameter values consistent with [Figure 1a](#), the rate gap threshold with inattention is 54-65% smaller than that in the perfect attention benchmark, depending on the assumed mortgage rate volatility.⁶

This analysis sheds new light on empirical studies focusing on mistakes made by bor-

⁶Using our preferred value $\sigma = 0.70\%$, [Section 7](#) estimates an attention rate $\lambda = 23.4\%$ p.a., which leads to a threshold $\theta = 0.46\%$, which is 54% smaller than the relevant ADL threshold; assuming instead $\sigma = 1.00\%$ (used by [Agarwal, Driscoll and Laibson \(2013\)](#) and others), [Section 7](#) estimates an attention rate $\lambda = 11.8\%$ p.a., which leads to a threshold $\theta = 0.41\%$, which is 65% smaller than the relevant ADL threshold.

rowers in connection with their refinancing decisions. Agarwal, Rosen and Yao (2016) and Fuster et al. (2019), for instance, conclude that borrowers refinance at rate gaps that are on average too small relative to the ADL threshold — what the former article refers to as *errors of commission*.⁷ Once we take into account the fact that borrowers exhibit inattention, their rate gap thresholds are reduced significantly. Rather than making mistakes (by refinancing at excessively low rate gaps, as these empirical studies suggest), borrowers may act rationally, refinancing aggressively when they have the chance, subject to their attention friction. Similarly, both articles find that borrowers wait too long before refinancing, once their rate gap reaches the theoretically optimal threshold — sometimes referred to as *errors of omission*. With inattention, borrowers naturally wait a random amount of time before “pulling the trigger” to refinance, and that random delay depends on the degree of attention of the borrower. We investigate these two types of errors systematically in Section 7.

5 Extensions

In this section, we investigate two extensions of our benchmark model. In the first, we show that our results are quantitatively insensitive to our assumption that mortgage rates follow a random walk. In the second, we micro-found inattention by introducing (pecuniary) observation costs, and show that the key insight of Section 4 remains.

5.1 Mean-reverting mortgage rates

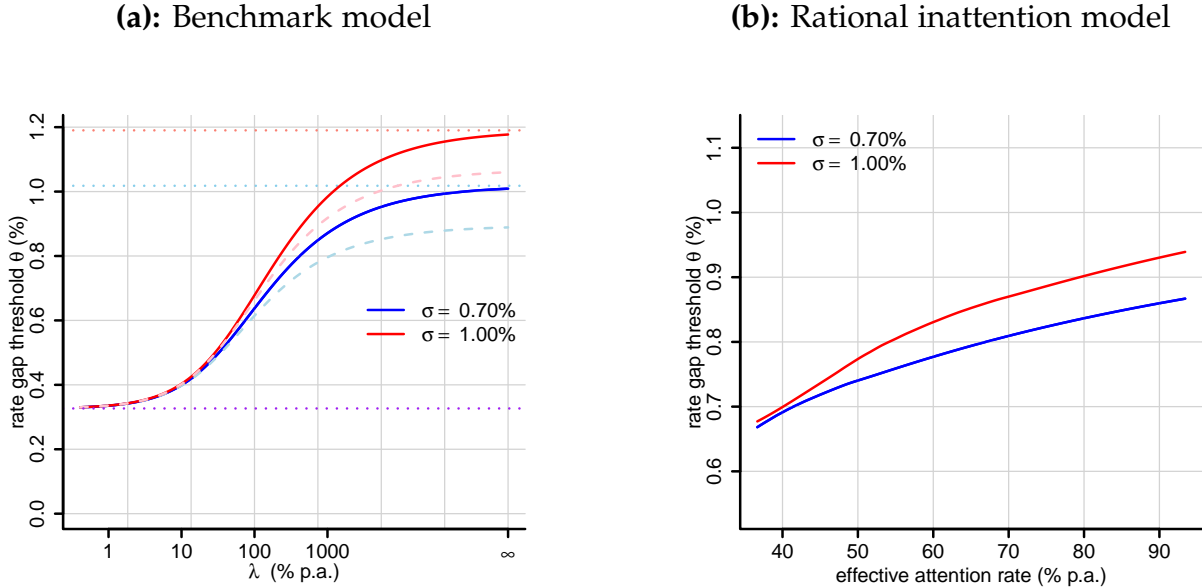
Section 4 relies on the assumption that the mortgage rate m_t is a random walk — a strongly debated empirical question.⁸ When mortgage rates are mean-reverting, refinancing decisions are necessarily state dependent, i.e., $\theta = \theta(x)$. To what extent does mean reversion alter a borrower’s threshold? To address this question we analyze mortgage rates that follow an Ornstein-Uhlenbeck process:

$$dm_t = -\chi(m_t - \bar{m})dt + \sigma dB_t \tag{9}$$

⁷Agarwal, Rosen and Yao (2016) find that borrowers refinance at rate gaps with an average of 121 bps, relative to the average ADL threshold of 158 bps. Similarly, Fuster et al. (2019) find that among refinancing borrowers, more than half of the refinancings are executed at rates that are too small when assessed against the ADL threshold.

⁸Stock and Watson (1988) conclude that various short term interest rates appear to contain a unit root; Perron (1989) cannot reject the unit root hypothesis at a 10% level, but can reject at lower levels; more recently, Bierens (1997) conducts various tests, some of them rejecting, and others failing to reject the unit root hypothesis, and concludes that the short term rate is a nonlinear trend stationary process.

Figure 1: Rate gap threshold θ



Left-hand side: solid (dashed) lines show the threshold θ (approximation $\hat{\theta}$) as a function of attention λ in the benchmark model of Section 4. The horizontal dotted purple line shows the limit of θ when $\lambda \rightarrow 0$ and the dotted red and blue lines show the limit as $\lambda \rightarrow +\infty$. Right-hand side: threshold θ as a function of the effective attention rate, defined as $1/\mathbb{E}[\mathbb{T}(z)]$, with $\mathbb{T}(z)$ the observation delay when the rate gap is z , when varying the observation costs ϕ , in the rational inattention model of Section 5.2. Figures computed assuming $\rho = 5\%$, $\nu = 11.33\%$, $\psi = 2\%$ and for two different mortgage rate volatilities $\sigma = 0.70\%$ and $\sigma = 1.00\%$.

This specification nests the special case of the Brownian motion studied in Section 4, by setting $\chi = 0$.

We solve our model numerically and find that the average threshold $\mathbb{E}[\theta(m_t)]$ is barely affected by the persistence of mortgage rates.⁹ Using our baseline parameters, the average threshold is at most 2 bps away from the benchmark case studied in Section 4, even as we vary the half-life from 1 to 20 years.¹⁰ Thus, our conclusions are very similar when using stationary mortgage rates instead of assuming a random walk like in our baseline model. We maintain the simpler random walk assumption for the remainder of the paper.

⁹This conclusion is robust to a broad range of realistic attention rates and mortgage volatility parameters — not only those we estimate empirically.

¹⁰This half-life is equal to $\ln(2)/\chi$.

5.2 Rational inattention

Next, consider an environment in which the borrower makes decisions continuously, but must pay a cost ϕ to observe the current mortgage rate, and a cost ψ to refinance. Each time the borrower observes the market, they decide (i) whether or not to refinance, and (ii) the length of time T until their next market observation. The borrower’s refinancing option value remains a function of the rate gap z , and the optimal refinancing strategy remains a cutoff — i.e. refinance at an observation date whenever the rate gap satisfies $z \geq \theta$, for an optimally chosen threshold θ . Different from our benchmark model, the borrower now optimally picks their next observation date $T(z)$ as a function of the current rate gap, balancing (i) the cost ϕ of paying attention with (ii) the future potential benefit of refinancing, if the rate gap at such time is above the threshold. This extended version of our model delivers rational inattention, as in [Abel, Eberly and Panageas \(2007\)](#); given the observation costs, the borrower chooses an optimal observation delay $\mathbb{T}(z)$ (and thus an implicit attention time).

[Appendix A.4](#) develops the mathematical notation for this extension. We solve our model numerically and compute, for a range of observation costs ϕ , (i) the optimal rate gap θ at which borrowers refinance and (ii) the ergodic average attention delay $\mathbb{E}[\mathbb{T}(z)]$. We plot in [Figure 1b](#) the threshold θ vs. the effective attention rate $1/\mathbb{E}[\mathbb{T}(z)]$ for two different mortgage rate volatilities. We find that (a) the optimal rate gap at which to refinance θ is a decreasing function of the observation cost, (b) the ergodic average attention delay $\mathbb{E}[\mathbb{T}(z)]$ is an increasing function of the observation cost, resulting in an upward sloping relationship between the optimal rate gap θ and the effective attention rate — echoing our insight from previous sections.

6 Distributional implications

Now that we have explore implications for individual decisions, we next discuss the resulting implications of the benchmark model of [Section 4](#) for the distribution of rate gaps, and the distribution of rate gaps upon refinancing. These distributions are not only easily measurable in the data, but they have also been the focus of a component of the household finance literature measuring the magnitude and frequency of mistakes made by households in their refinancing decisions. Under the assumptions of [Proposition 2](#), while the mortgage rate is non-stationary, the rate gap z_t admits an ergodic density $f(z)$:

$$f(z)dz := \lim_{t \rightarrow +\infty} \frac{1}{t} \int_0^t \mathbb{1}_{\{z_t \in [z, z+dz]\}} dt. \quad (10)$$

The density f can be characterized analytically, as the next proposition shows.

Proposition 3 Assume that m_t is a Brownian motion with volatility σ . Introduce the constants χ_0 and χ_λ equal to:

$$\chi_0 := \frac{\sqrt{2\nu}}{\sigma} \quad \chi_\lambda := \frac{\sqrt{2(\nu + \lambda)}}{\sigma}.$$

The rate gap z_t admits an ergodic density f that is an asymmetric Laplace distribution:

$$f(z) = \frac{1}{\frac{1}{\chi_0} + \frac{1}{\chi_\lambda}} \begin{cases} \exp(\chi_0(z - \theta)) & \text{if } z \leq \theta \\ \exp(-\chi_\lambda(z - \theta)) & \text{if } z \geq \theta \end{cases} \quad (11)$$

Instead, the rate gap upon prepayment admits an ergodic density \hat{f} equal to

$$\hat{f}(z) = \frac{1}{\left(\frac{1}{\chi_0} + \frac{1}{\chi_\lambda}\right) \nu + \frac{\lambda}{\chi_\lambda}} \begin{cases} \nu \exp(\chi_0(z - \theta)) & \text{if } z \leq \theta \\ (\nu + \lambda) \exp(-\chi_\lambda(z - \theta)) & \text{if } z \geq \theta \end{cases} \quad (12)$$

The following ergodic statistics admit analytic expressions

$$\text{average prepayment rate} = \lim_{t \rightarrow +\infty} \frac{1}{t} \left(N_t^{(\nu)} + \int_0^t a_s dN_s^{(\lambda)} \right) = \nu + \left(\frac{\chi_0}{\chi_0 + \chi_\lambda} \right) \lambda \quad (13)$$

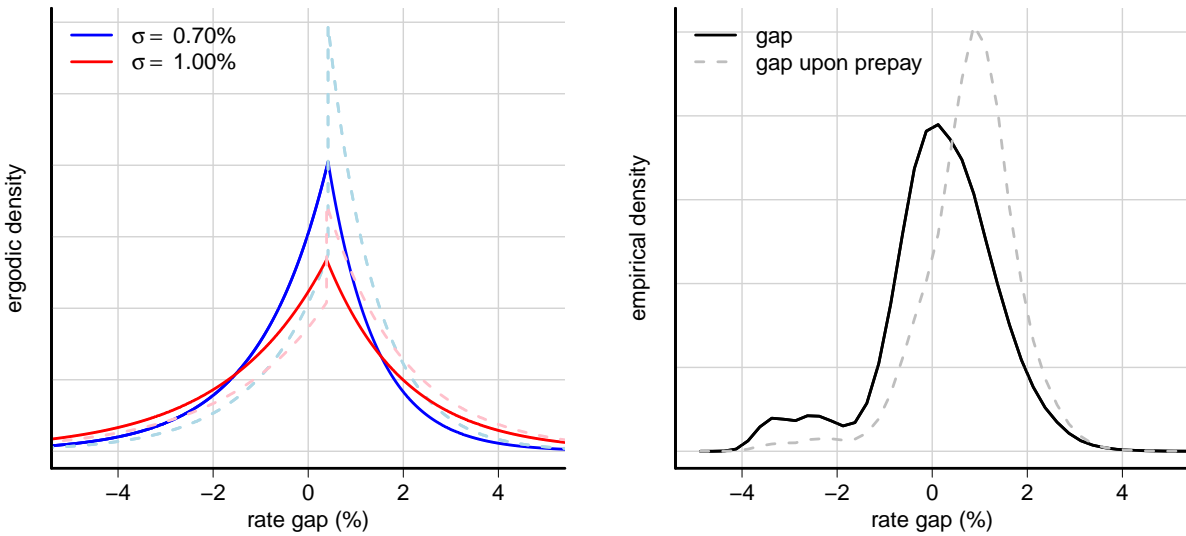
$$\text{average rate gap} = \lim_{t \rightarrow +\infty} \frac{1}{t} \int_0^t z_s ds = \theta + \frac{1}{\chi_\lambda} - \frac{1}{\chi_0} \quad (14)$$

$$\text{average rate gap upon prepay} = \lim_{t \rightarrow +\infty} \frac{\int_0^t z_{s-} \left(dN_s^{(\nu)} + a_s dN_s^{(\lambda)} \right)}{N_t^{(\nu)} + \int_0^t a_s dN_s^{(\lambda)}} = \theta \quad (15)$$

See proof in [Appendix A.3](#). [Figure 2](#) illustrates these model-implied densities, and relates them to their empirical counterpart, computed using a sample of 30-year fixed-rate mortgages from Fannie Mae's Single-Family Loan Performance ("SFLP") data. Formula (13) tells us that the long-term average prepayment rate is between ν (the non-strategic prepayment rate) and $\nu + \lambda$ whereas formula (14) suggests that the average rate gap in this model is below the threshold θ — both intuitive results. Formula (15) is however more surprising; indeed, given the threshold θ and borrowers' optimal strategy to wait for the rate gap to be above this threshold before refinancing (at a point in time where they are attentive), one might have expected the ergodic average rate gap upon prepayment to be greater than θ . Instead, this ergodic average is exactly equal to θ due to a second, countervailing force: whenever the rate gap is below θ , borrowers prepay at a low intensity ν , allowing the rate gap to reach values materially lower than θ , thereby pushing the ergodic average gap upon prepayment downwards. This latter result is

important in another way. So far, [Section 3](#) and [Section 4](#) have shown that higher inattention makes borrowers want to refinance at lower rate gaps (relative to the perfect attention benchmark) *when they have the opportunity to do so*. This does not necessarily mean that *on average*, inattentive borrowers refinance their mortgage at lower rate gaps, since inattention “slows down” borrowers. The result in this section that the average rate gap upon prepayment is exactly equal to θ means that the average gap upon prepayment is also an increasing function of the attention rate.

Figure 2: Rate gap density: theory vs data.



Theoretical (left-hand side) and empirical (right-hand side) rate gap distribution $f(z)$ (solid lines) and rate gap upon prepayment distribution $\hat{f}(z)$ (in dash lines) for two mortgage rate volatilities. The left-hand side assumes that m_t is a Brownian motion, and is computed using parameters $\rho = 5\%$, $\nu = 11.33\%$, $\psi = 2\%$ and attention rates estimated in [Section 7](#).

Moreover, (14) and (15) are measurable in the data and deliver a sufficient statistic to measure the effective attention rate of a population of borrowers:

$$(\text{avg rate gap upon prepay}) - (\text{avg rate gap}) = \frac{\sigma}{\sqrt{2}} \left(\frac{1}{\sqrt{\nu}} - \frac{1}{\sqrt{\nu + \lambda}} \right) \quad (16)$$

Formula (16) is useful since, conditional on two moments measurable in the data and assumptions on only (i) the moving rate ν and (ii) the mortgage rate volatility σ , we can recover the effective attention of a population of interest, irrespective of the values of the subjective discount rate ρ or the closing costs ψ . We use this approach in [Section 7](#) to estimate the average attention rate in the population of agency mortgage borrowers

using SFLP data.¹¹

7 Empirical study

Many papers use the ADL formula to study the degree and magnitude of refinancing mistakes made by borrowers (Keys, Pope and Pope, 2016; Agarwal, Rosen and Yao, 2016; Fuster et al., 2019). Given parameter values ψ, ν, σ and ρ , these studies compute the threshold at which a borrower should be optimally refinancing. Borrowers observed with rate gaps above the ADL threshold are labeled as making “errors of omission”, while borrowers refinancing at rate gaps below the ADL threshold are labeled as making “errors of comission”. Our model can rationalize both type of errors: inattentive borrowers make decisions at discrete *points* in time, which means that they will frequently be observed at rate gaps above the threshold θ ; and inattention lowers their optimal threshold, thereby reducing the number of errors of omission that an econometrician would measure in the data.

To illustrate our point, we use a 2% random sample of 30-year fixed rate mortgages originated between January 1999 and September 2023, for which we have origination and performance data from the SFLP dataset. For each mortgage-month observation, we compute the mortgage rate available to borrowers refinancing at such time.¹² This allows us to estimate rate gaps for each mortgage over its life, controlling for observables that drive mortgage rates at origination. The right-hand side of Figure 2 shows the empirical distribution of rate gaps and rate gaps upon prepayment in our data. We then use the summary statistic (16) to determine the attention rate in our borrower population, assuming a mortgage rate volatility of either $\sigma = 0.70\%$ (the relevant empirical value for our sample period) or $\sigma = 1.00\%$ (the value used by Agarwal, Driscoll and Laibson (2013)), and a moving/amortization rate $\nu = 11.33\%$.¹³ Given an average rate gap of 0.15% and average rate gap upon prepayment of 0.78%, inverting (16) leads to an

¹¹It is important to emphasize that the distributions f and \hat{f} are *long run* rate gap distributions, which would be obtained if we follow one borrower for an arbitrarily large amount of time. In other words, f does *not* characterize the cross-sectional distribution of rate gaps at a particular point in time, but rather the *average* cross-sectional distribution of rate gaps over an arbitrarily long time-horizon.

¹²With our origination data, we regress mortgage coupons onto (i) original LTV, (ii) original FICO, (iii) original principal balance, (iv) dummy for loan purpose, and (v) origination-month fixed effects. For each mortgage-month, we then use our linear model to estimate the rate at which the borrower could refinance, taking into account the then-current mortgage balance and LTV, computed using HPI at the 3-digit zip obtained from the FHFA.

¹³Focusing on the prepayment rate for households with negative rate gaps allows us to estimate an average moving rate of 8% p.a., to which we add the amortization rate of 1/30, leading to our parameter choice $\nu = 11.3\%$.

attention rate of $\lambda = 23.4\%$ (when $\sigma = 0.70\%$) or $\lambda = 11.8\%$ (when $\sigma = 1.00\%$).¹⁴

For each mortgage-month observation (i, t) and for both mortgage rate volatilities considered, we then compute the optimal rate gap threshold $\theta_{\infty, it}$ under the perfect attention benchmark, and the corresponding threshold θ_{it} under our estimated attention rates.¹⁵ These calculations allow us to quantify the frequency and magnitude of errors of omission and of commission imputed by both models, which we show in [Table 1](#).

Table 1: Errors of omission and of commission

	Perfect Attention	With Inattention	Perfect Attention	With Inattention
mortgage rate (annual) volatility σ	0.70%	0.70%	1.00%	1.00%
average threshold θ_{it}	1.09%	0.75%	1.27%	0.71%
errors of omission				
mortgage-month obs (M) with $z_{it} > \theta_{it}$	7.07	10.86	5.43	11.47
nb of prepayments (M) when $z_{it} > \theta_{it}$	0.21	0.3	0.16	0.31
empirical monthly prepay rate when $z_{it} > \theta_{it}$	2.97	2.77	3.01	2.72
theoretical monthly prepay rate when $z_{it} > \theta_{it}$	100	2.85	100	1.91
errors of commission				
mortgage-month obs (M) with $z_{it} < \theta_{it}$	28.35	24.56	29.99	23.95
nb of prepayments (M) when $z_{it} < \theta_{it}$	0.31	0.21	0.35	0.2
empirical monthly prepay rate when $z_{it} < \theta_{it}$	1.08	0.87	1.17	0.85
theoretical monthly prepay rate when $z_{it} < \theta_{it}$	0.94	0.94	0.94	0.94
mortgage-month obs (M) with $0 < z_{it} < \theta_{it}$	13.67	9.88	15.31	9.27
nb of prepayments (M) when $0 < z_{it} < \theta_{it}$	0.21	0.12	0.26	0.11
empirical monthly prepay rate when $0 < z_{it} < \theta_{it}$	1.57	1.24	1.7	1.2
mortgage-month obs (M) with $z_{it} < 0$	14.68	14.68	14.68	14.68
nb of prepayments (M) when $z_{it} < 0$	0.09	0.09	0.09	0.09
empirical monthly prepay rate when $z_{it} < 0$	0.62	0.62	0.62	0.62

Random sample of 780k mortgages issued between January 1999 and September 2023 (mortgage rate volatility over that sample period is estimated to be equal to 0.70% per year). Panel consists of 35.42M mortgage-month observations. Prepayment rates are expressed in % per month.

Comparing the model with inattention to the model with perfect attention in [Table 1](#),

¹⁴These relatively low estimates for the attention rate are in part due to the trend decline in mortgage rates from the early 80s to 2022, causing the empirical average rate gap to be biased upwards relative to the theoretical ergodic value, thereby potentially biasing downwards our attention rate estimate.

¹⁵To compute the optimal rate gaps $\theta_{\infty, it}$ and θ_{it} , we use an approach consistent with [Agarwal, Driscoll and Laibson \(2013\)](#): we assume refinancing costs of \$2,000 plus 1% of outstanding loan balance, a tax rate of 28% — since mortgage interest is assumed to be tax deductible — and a subjective discount rate $\rho = 5\%$. We keep $\nu = 11.33\%$ and in order to compute θ_{it} , we use the estimated attention rate consisted with our mortgage volatility assumption. The i, t dependence of the threshold stems from the time-varying balance of the mortgage and the assumed fixed costs that do not scale with loan balance.

several takeaways emerge. For brevity, we focus on the results with $\sigma = 0.70\%$, but conclusions are similar for the case $\sigma = 1.00\%$. First, as expected, the average optimal threshold θ is significantly lower in the calibrations with inattention than those with perfect attention. Second, 2.97% of mortgage-month observations with a rate gap above the ADL threshold θ_∞ end up prepaying, supporting the analysis from previous academic work suggesting that borrowers make frequent errors of omission, *under the assumption that they are perfectly attentive*. Once we factor in their inattention, 2.77% of mortgage-month observations with a rate gap above the threshold θ end up prepaying, vs. a theoretical prepayment rate of 2.85%; in other words, inattention rationalizes almost all errors of “omission”.¹⁶ Third, about 310k prepayments occur at rate gaps that are *too low* relative to the ADL threshold θ_∞ — what the literature labels errors of commissions. Taking into account borrower inattention reduces the optimal threshold, so that 32% of these errors of commission in the perfect attention model can instead be rationalized by the model with inattention.¹⁷ Thus, our model of mortgage refinancing with inattention considerably reduces the two types of mistakes identified in the previous literature, at the same time emphasizing the significant role played by inattention frictions in the data.

8 Conclusion

Fixed-rate mortgage refinancing decisions are hindered by various frictions — both behavioral, as well as financial. These frictions interact in non-trivial ways, such that more inattention leads borrowers to refinance earlier, when they have the opportunity to do so. The inattention friction can help rationalize why borrowers both (i) refinance too “early” (compared to a full attention benchmark) and (ii) wait too long to refinance, when it is profitable for them to do so. Our model of borrower behavior also has consequences for the distribution of rate gaps, which can be informative of the degree of state- and time-dependent frictions faced by borrowers.

¹⁶The theoretical prepayment rate for these observations is $1 - \exp(-(\lambda + \nu)dt)$, with $dt = 1/12$ years.

¹⁷It is worth noting that the SFLP data does not differentiate prepayments occurring due to refinancing, vs. moves. If one were to assume that all prepayments occurring at $z_{it} < 0$ are actually moves, and if one were to label as errors of omissions only those prepayments occurring when $0 < z_{it} < \theta$, then 210k prepayments occur at rate gaps that are *too low* relative to the ADL threshold, amongst which 43% could be rationalized with our borrower inattention model.

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A Appendix

A.1 Effect of inattention on threshold

Proof of Proposition 1. Consider equation (2) satisfied by V , and differentiate w.r.t. λ :

$$\begin{aligned} (\rho + \nu + \lambda) \partial_\lambda V(x, c) &= \min(V(x, m(x)) + \psi - V(x, c), 0) + \nu \partial_\lambda V(x, m(x)) \\ &\quad + \lambda \left[\mathbb{1}_{\{c - m(x) \geq \theta(x)\}} \partial_\lambda V(x, m(x)) + \mathbb{1}_{\{c - m(x) < \theta(x)\}} \partial_\lambda V(x, c) \right] + \mathcal{L} \partial_\lambda V(x, c) \end{aligned}$$

Noting τ a Poisson time with arrival rate $\nu + \lambda \mathbb{1}_{\{c - m(x_t) \geq \theta(x_t)\}}$, then

$$\begin{aligned} \partial_\lambda V(x, c) &= \mathbb{E}_x \left[\int_0^\tau e^{-\rho t} \min(V(x_t, m(x_t)) + \psi - V(x_t, c), 0) dt \right. \\ &\quad \left. + e^{-\rho \tau} \partial_\lambda V(x_\tau, m(x_\tau)) \right] \quad (\text{A.1}) \end{aligned}$$

$\partial_\lambda V(x, c) < 0$ since a borrower with a higher attention rate must have a lower present value of future mortgage costs. The source term $\min(V(x_t, m(x_t)) + \psi - V(x_t, c), 0)$ in (A.1) is decreasing in c , which means that $\partial_\lambda V(x, c)$ must also be decreasing in c . Consider then (3) and differentiate w.r.t. λ :

$$\partial_\lambda V(x, m(x)) = \partial_\lambda V(x, m(x) + \theta(x)) + \partial_\lambda \theta(x) \partial_c V(x, m(x) + \theta(x))$$

Thus, we have

$$\partial_\lambda \theta(x) = \frac{\partial_\lambda V(x, m(x)) - \partial_\lambda V(x, m(x) + \theta(x))}{\partial_c V(x, m(x) + \theta(x))} \quad (\text{A.2})$$

V is increasing in c , which means that the denominator in (A.2) is positive. Similarly, we have established above that $\partial_\lambda V(x, c)$ is decreasing in c , meaning that we must have $\partial_\lambda V(x, m(x)) > \partial_\lambda V(x, m(x) + \theta(x))$. Thus $\theta(x)$ is increasing in λ . ■

A.2 Special case: m_t as a Brownian motion

Proof of Proposition 2. Assume that $m_t = \sigma B_t + m_0$, with B_t a Brownian motion. V , solution of (1), satisfies $V(m, c) = \frac{c}{\rho} + v(z)$, where $z = c - m$ is the rate gap and

$$v(z) := \inf_{a \in \mathcal{A}} \mathbb{E}_z \left[\int_0^{+\infty} e^{-\rho t} \left[\left(\psi - \frac{z_{t-}^{(a)}}{\rho} \right) a_t dN_t^{(\lambda)} - \frac{z_{t-}^{(a)}}{\rho} dN_t^{(v)} \right] \right] \quad (\text{A.3})$$

$$dz_t^{(a)} = -\sigma dB_t - z_{t-}^{(a)} \left(a_t dN_t^{(\lambda)} + dN_t^{(v)} \right). \quad (\text{A.4})$$

Consider $z_1 < z_2$, an arbitrary feasible policy $a \in \mathcal{A}$, and denote by $v(z; a)$ the option value under policy a . Denote $z_{i,t}^{(a)}$ for $i \in \{1, 2\}$ the rate gap under policy a with starting

point z_i , then

$$v(z_2; a) - v(z_1; a) = \mathbb{E} \left[e^{-\rho\tau} \left(\frac{z_{1,\tau-}^{(a)} - z_{2,\tau-}^{(a)}}{\rho} \right) \right],$$

where τ is the first refinancing time under policy a . Since $z_1 < z_2$, then $z_{1,\tau-}^{(a)} < z_{2,\tau-}^{(a)}$ a.s., which means that $v(z_2; a) < v(z_1; a)$. Since a was chosen arbitrarily, v is strictly decreasing in z . Similarly, take $z_1 < z_2$ and $\lambda \in (0, 1)$, and let $a_\lambda^* \in \mathcal{A}$ be the optimal strategy under starting point $z_\lambda := \lambda z_1 + (1 - \lambda)z_2$. Let τ_λ^* be the first refinancing time under a_λ^* , and notice that for $t \leq \tau_\lambda^*$, $z_{\lambda,t}^{(a_\lambda^*)} = \lambda z_{1,t}^{(a_\lambda^*)} + (1 - \lambda)z_{2,t}^{(a_\lambda^*)}$. This means that

$$\begin{aligned} v(z_\lambda) &= \mathbb{E}_{z_\lambda} \left[\int_0^{\tau_\lambda^*} e^{-\rho t} \left[\left(\psi - \frac{z_{\lambda,t}^{(a_\lambda^*)}}{\rho} \right) a_{\lambda,t}^* dN_t^{(\lambda)} - \frac{z_{\lambda,t}^{(a_\lambda^*)}}{\rho} dN_t^{(v)} \right] + e^{\tau_\lambda^*} v(0) \right] \\ &= \lambda \mathbb{E}_{z_1} \left[\int_0^{\tau_\lambda^*} e^{-\rho t} \left[\left(\psi - \frac{z_{1,t}^{(a_\lambda^*)}}{\rho} \right) a_{\lambda,t}^* dN_t^{(\lambda)} - \frac{z_{1,t}^{(a_\lambda^*)}}{\rho} dN_t^{(v)} \right] + e^{\tau_\lambda^*} v(0) \right] \\ &\quad + (1 - \lambda) \mathbb{E}_{z_2} \left[\int_0^{\tau_\lambda^*} e^{-\rho t} \left[\left(\psi - \frac{z_{2,t}^{(a_\lambda^*)}}{\rho} \right) a_{\lambda,t}^* dN_t^{(\lambda)} - \frac{z_{2,t}^{(a_\lambda^*)}}{\rho} dN_t^{(v)} \right] + e^{\tau_\lambda^*} v(0) \right] \end{aligned}$$

Since policy a_λ^* is feasible when starting at z_1 , the first expectation on the right hand side above is equal to $v(z_1; a_\lambda^*)$, which is greater than $v(z_1)$; for the same reason, the second expectation on the right hand side above is to $v(z_2; a_\lambda^*)$, which is greater than $v(z_2)$. This means that $v(z_\lambda) \geq \lambda v(z_1) + (1 - \lambda)v(z_2)$, i.e. v is concave (and thus continuously differentiable almost everywhere). The option value v satisfies

$$(\rho + v + \lambda)v(z) = \frac{\sigma^2}{2}v''(z) + \lambda \min \left(v(z), v(0) + \psi - \frac{z}{\rho} \right) + v \left(v(0) - \frac{z}{\rho} \right) \quad (\text{A.5})$$

Noting θ the rate gap above which the borrower finds it optimal to refinance when given the opportunity to do so, we must have

$$(\rho + v)v(z) = \frac{\sigma^2}{2}v''(z) + v \left(v(0) - \frac{z}{\rho} \right) \quad z \leq \theta \quad (\text{A.6})$$

$$(\rho + v + \lambda)v(z) = \frac{\sigma^2}{2}v''(z) + v \left(v(0) - \frac{z}{\rho} \right) + \lambda \left(v(0) + \psi - \frac{z}{\rho} \right) \quad z \geq \theta \quad (\text{A.7})$$

Since $v(z) = O(z)$ as $z \rightarrow +\infty$ and as $z \rightarrow -\infty$, the value function v must satisfy (4). The constants k_-, k_+ must be such that v is continuously differentiable at $z = \theta$. Moreover, since $\theta > 0$, it must be the case that

$$v(0) = \frac{\rho + v}{\rho} k_- e^{-\eta_0 \theta}$$

The requirement that v be continuously differentiable at $z = \theta$ yields a system of 2 linear equations in the 2 unknown k_-, k_+ , with solution (using our formula for $v(0)$):

$$k_- = \frac{\lambda}{(\rho + \nu + \lambda)(\eta_0 + \eta_\lambda) - \lambda\eta_\lambda e^{-\eta_0\theta}} \left[\eta_\lambda \left(\psi - \frac{\theta}{\rho + \nu} \right) - \frac{1}{\rho + \nu} \right]$$

$$k_+ = \frac{-\lambda}{(\rho + \nu + \lambda)(\eta_0 + \eta_\lambda) - \lambda\eta_\lambda e^{-\eta_0\theta}} \left[\left(1 - \frac{\lambda e^{-\eta_0\theta}}{\rho + \lambda + \nu} \right) \left(\frac{1}{\rho + \nu} \right) + \eta_0 \left(\psi - \frac{\theta}{\rho + \nu} \right) \right]$$

At $z = \theta$, the borrower is indifferent between (a) continuing with the current mortgage, or (b) paying the fixed cost and refinancing. This means that

$$v(\theta) = v(0) + \psi - \frac{\theta}{\rho} \Rightarrow k_- = \frac{\rho}{\rho + \nu} v(0) + \psi - \frac{\theta}{\rho + \nu}$$

But since we know $v(0)$ as a function of k_- , this yields

$$k_- = k_- e^{-\eta_0\theta} + \psi - \frac{\theta}{\rho + \nu}$$

Using our formula for k_- , this yields, after some algebra, the implicit equation

$$e^{-\eta_0\theta} + (\eta_0 + \epsilon_\lambda) \theta = 1 + (\rho + \nu) \psi (\eta_0 + \epsilon_\lambda), \quad (\text{A.8})$$

where ϵ_λ is defined in [Proposition 2](#). (A.8) is solved by

$$\theta = (\rho + \nu) \psi + \frac{1}{\eta_0 + \epsilon_\lambda} + \frac{1}{\eta_0} W \left(\frac{-\eta_0}{\eta_0 + \epsilon_\lambda} \exp \left\{ \frac{-\eta_0}{\eta_0 + \epsilon_\lambda} [1 + (\rho + \nu) (\eta_0 + \epsilon_\lambda) \psi] \right\} \right),$$

with $W(\cdot)$ the primary branch of the Lambert-W function. Notice that ϵ_λ is a positive and decreasing function of λ , converging to zero as $\lambda \rightarrow +\infty$. Differentiate (A.8) w.r.t. λ to obtain

$$\frac{\partial \theta}{\partial \lambda} = \frac{((\rho + \nu) \psi - \theta) \frac{\partial \epsilon_\lambda}{\partial \lambda}}{\epsilon_\lambda + \eta_0 (1 - e^{-\eta_0\theta})} > 0,$$

with the last inequality following from $\frac{\partial \epsilon_\lambda}{\partial \lambda} < 0$ and $(\rho + \nu) \psi - \theta = \frac{e^{-\eta_0\theta} - 1}{\eta_0 + \epsilon_\lambda} < 0$. Thus θ increases with λ . Since $\lim_{\lambda \rightarrow \infty} \epsilon_\lambda = 0$,

$$\theta_\infty := \lim_{\lambda \rightarrow \infty} \theta = (\rho + \nu) \psi + \frac{1}{\eta_0} + \frac{1}{\eta_0} W(-\exp\{-[1 + (\rho + \nu) \eta_0 \psi]\}),$$

which is the ADL formula. Lastly, since $\lim_{\lambda \rightarrow 0} \epsilon_\lambda = \infty$ and $W(0) = 0$, we have

$$\theta_0 := \lim_{\lambda \rightarrow 0} \theta = (\rho + \nu) \psi.$$

We can then perform a Taylor expansion of (A.8) around $\theta = 0$, which allows us to obtain an approximation $\hat{\theta}$ of the value θ :

$$\frac{\eta_0^2}{2}\hat{\theta}^2 + \epsilon_\lambda\hat{\theta} - (\rho + \nu)(\eta_0 + \epsilon_\lambda)\psi = 0$$

This allows us to conclude that the approximation $\hat{\theta}$ is equal to

$$\hat{\theta} = \sqrt{\frac{2}{\eta_0} \left(1 + \frac{\epsilon_\lambda}{\eta_0}\right) (\rho + \nu) \psi + \left(\frac{\epsilon_\lambda}{\eta_0^2}\right)^2} - \frac{\epsilon_\lambda}{\eta_0^2}$$

Finally, it is straightforward (but tedious) to verify that the optimal rate gap threshold θ is identical to that derived above if one were to assume a fixed cost upon moving. The intuition behind this result is straightforward: since there is equal probability that the borrower moves when the mortgage is in- or out-of-the-money (given the fact that the mortgage rate is a Brownian motion), the borrower's refinancing strategy does not change in the presence of fixed moving costs. ■

A.3 Rate gap ergodic density when m_t is a Brownian motion

Proof of Proposition 3. The rate gap follows dynamics described by (A.4). This stochastic process admits an ergodic density f , which satisfies the Kolmogorov-Forward equation, for $z \neq \theta$:

$$0 = \frac{\sigma^2}{2}f''(z) - (\nu + \lambda\mathbb{1}_{z \geq \theta})f(z) \quad (\text{A.9})$$

Moreover, f is continuous at $z = \theta$, it vanishes at $z \rightarrow \pm\infty$, and it integrates to 1. Using the constants χ_0, χ_λ defined in Proposition 3, we can integrate (A.9) with the above boundary conditions to derive (11). The ergodic average rate gap $\mathbb{E}[z_t]$ is then equal to

$$\mathbb{E}[z_t] = \int_{-\infty}^{+\infty} zf(z)dz = \theta + \frac{\chi_0 - \chi_\lambda}{\chi_0\chi_\lambda}$$

Similarly, the ergodic average refinancing rate equals

$$\lim_{t \rightarrow +\infty} \frac{1}{t} \left(N_t^{(\nu)} + \int_0^t a_t dN_t^{(\lambda)} \right) = \int_{-\infty}^{+\infty} (\nu + \lambda\mathbb{1}_{z \geq \theta}) f(z) dz = \nu + \left(\frac{\chi_0}{\chi_0 + \chi_\lambda} \right) \lambda$$

Lastly, the ergodic average rate gap observed at the time of refinancing equals

$$\lim_{t \rightarrow +\infty} \frac{\int_0^t z_{t-} \left(dN_t^{(\nu)} + a_t dN_t^{(\lambda)} \right)}{N_t^{(\nu)} + \int_0^t a_t dN_t^{(\lambda)}} = \frac{\int_{-\infty}^{+\infty} z (\nu + \lambda\mathbb{1}_{z \geq \theta}) f(z) dz}{\int_{-\infty}^{+\infty} (\nu + \lambda\mathbb{1}_{z \geq \theta}) f(z) dz} = \theta$$

■

A.4 Rational inattention

We continue to assume that the mortgage rate satisfies $m_t = m_0 + \sigma B_t$, with B_t a standard Brownian motion. Let V be the valuation for a borrower at the time she observes the current market rate and is not refinancing:

$$V(m, c) = \phi + \min_{T \geq 0} \mathbb{E}_m \left[\int_0^{\tau \wedge T} e^{-\rho t} c dt + e^{-\rho \tau} \mathbb{1}_{\{\tau \leq T\}} V(m_\tau, m_\tau) + e^{-\rho T} \mathbb{1}_{\{\tau > T\}} \min(V(m_T, c); V(m_T, m_T) + \psi) \right],$$

where τ is an exponentially distributed moving time (with parameter ν), at which the borrower is forced to refinance. We guess the valuation function can be written $V(m, c) = \phi + \frac{c}{\rho} + v(z)$. After some algebra, the option value $v(z)$ solves

$$v(z) = \min_{T \geq 0} \frac{\nu}{\rho + \nu} \left(1 - e^{-(\rho + \nu)T} \right) \left(v(0) + \phi - \frac{z}{\rho} \right) + e^{-(\rho + \nu)T} \left[\phi + \mathbb{E} \left[\min \left(v(z - \sigma B_T); v(0) + \psi - \frac{z - \sigma B_T}{\rho} \right) \right] \right]$$

One can also express v in sequence form to show, using a method identical to that used in [Appendix A.2](#), that v is decreasing and concave in z . The optimal rate gap threshold θ satisfies

$$\frac{\theta}{\rho} + v(\theta) = v(0) + \psi.$$

The optimal wait time $\mathbb{T}(z)$ satisfies the first order condition

$$(\rho + \nu)v(z) - \nu \left(v(0) + \phi - \frac{z}{\rho} \right) = e^{-(\rho + \nu)\mathbb{T}(z)} \frac{d\mathbb{E} \left[\min \left(v(z - \sigma B_T); v(0) + \psi - \frac{z - \sigma B_T}{\rho} \right) \right]}{dT} \Big|_{T=\mathbb{T}(z)},$$

in other words the flow cost of waiting (the left-hand side above) has to equal the marginal increase in the present value of expected option payoff from waiting an incremental unit of time. The state of a given borrower is (T, z) , with T the time until next observation and z the rate gap. (T, z) is a Markov process that satisfies

$$\begin{aligned} dT_t &= -dt + \mathbb{T}(z_t) \mathbb{1}_{\{T_{t-}=0\}} \\ dz_t &= -\sigma dB_t - z_{t-} \left(dN_t^{(\nu)} + \mathbb{1}_{\{T_{t-}=0\}} \mathbb{1}_{\{z_{t-} \geq \theta\}} \right) \end{aligned}$$

(T, z) admits an ergodic density that can be computed via simulation, allowing us to derive various moments — in particular, $\mathbb{E}[\mathbb{T}(z_t)]$, the ergodic average attention delay.