WORKING PAPER · NO. 2019-33

The Productivity J-Curve: How Intangibles Complement General Purpose Technologies

Erik Brynjolfsson, Daniel Rock, Chad Syverson

SEPTEMBER 2018

Becker Friedman Institute | Industrial Organization Initiative

1126 E. 59th St, Chicago, IL 60637
Main: 773.702.5599
bfi.uchicago.edu

The Productivity J-Curve: How Intangibles Complement General Purpose Technologies*

Erik Brynjolfsson, MIT and NBER
Daniel Rock, MIT
Chad Syverson, University of Chicago and NBER

September 2018

Abstract

General purpose technologies (GPTs) such as AI enable and require significant complementary investments, including business process redesign, co-invention of new products and business models, and investments in human capital. These complementary investments are often intangible and poorly measured in the national accounts, even if they create valuable assets for the firm. We develop a model that shows how this leads to an underestimation of output and productivity in the early years of a new GPT, and how later, when the benefits of intangible investments are harvested, productivity will be overestimated. Our model generates a *Productivity J-Curve* that can explain the productivity slowdowns often accompanying the advent of GPTs, as well as the follow-on increase in productivity later. We use our model to assess how AI-related intangible capital is currently affecting measured total factor productivity (TFP) and output. We also conduct a historical analysis of the roles of intangibles tied to R&D, software, and computer hardware, finding substantial and ongoing effects of software in particular and hardware to a lesser extent.

Acknowledgements: We thank Daron Acemoglu, Seth Benzell, John Fernald, Rebecca Henderson, Austan Goolsbee, Adam Saunders, Larry Summers, Manuel Trajtenberg and numerous seminar participants for helpful comments. The MIT Initiative on the Digital Economy provided valuable funding. We dedicate this paper to the memory of Shinkyu Yang, whose pioneering insights on the role of intangibles inspired us.

* Contact information: Brynjolfsson: erikb@mit.edu MIT Sloan School, E62-414, 100 Main Street, Cambridge, MA 02142; Rock: drock@mit.edu; 100 Main Street, Cambridge, MA 02142; Syverson: chad.syverson@chicagobooth.edu; University of Chicago Booth School of Business, 5807 S. Woodlawn Ave., Chicago, IL 60637.

After I left academe in 2014, I joined the technical organization at iRobot. I quickly learned how challenging it is to build deliberative robotic systems exposed to millions of individual homes. In contrast, the research results presented in papers (including mine) were mostly linked to a handful of environments that served as a proof of concept.

-Alexander Kleiner[1]

I. *Introduction*

In the late 1980s, Robert Solow (1987) famously pointed out that "a technological revolution, a drastic change in our productive lives" had curiously been accompanied by "a slowing-down of productivity growth, not by a step up." His famous productivity paradox, that one "can see the computer age everywhere but in the productivity statistics," named a challenge for economists seeking to reconcile the emergence of exciting technological breakthroughs with tepid productivity growth.

Solow's Paradox was not unique. In this paper, we argue it was one example of a more general phenomenon resulting from the need for intangible investments in early stages of new general purpose technologies. General purpose technologies (GPTs) are "engines for growth." They are pervasive, improve over time, and lead to complementary innovation (Bresnahan and Trajtenberg 1995). These are the defining technologies of their times and can radically change the economic environment. They have great potential from the outset, but realizing that potential requires larger intangible and often unmeasured investments and a fundamental rethinking of the organization of production itself. Thus, the usual measurement of productivity growth as a residual after accounting for input changes in the production function can fall short when the technology changes the production function itself.

[1] Kleiner, Alexander. 2018. "The Low-Cost Evolution of AI in Domestic Floor Cleaning Robots" *AI Magazine*, (Summer).

The extensive investment required to integrate GPTs into an organization is often forgotten. Along with installing more easily measured items like physical equipment and structures capital, firms must create new business processes, develop managerial experience, train workers, patch software, and build other intangibles. The difficulty for productivity measurement arises because intangible investments are not readily tallied on a balance sheet. The invention of a GPT can lead to the creation of entirely new asset classes and the transformation of existing capital varieties. It also presents abundant opportunity for entrepreneurs to discover new ways to deploy existing capital and labor. Moreover, these transformations of the production process do not occur overnight.

Given all of this, it is easy to see how something like Solow's Productivity Paradox can occur. There is a period during which measurable resources are committed (and measurable output forgone) to build new, unmeasured inputs that complement the new GPT. This period can be of considerable length. For example, the technologies driving the British industrial revolution led to "Engels' Pause," a half-century-long period of capital accumulation, industrial innovation, and wage stagnation (Allen 2009; Acemoglu and Robinson 2013). In the later GPT case of electrification, it took a generation as the nature of factory layouts was re-invented (David 1990). Solow was noting a similar phenomenon roughly two decades into the IT era.

We call this phenomenon the *Productivity J-Curve*. As firms adopt a new GPT, total factor productivity growth will initially be underestimated because capital and labor are spent to accumulate unmeasured intangible capital stocks. Later, measured productivity growth overestimates true productivity growth because the capital service flows from those hidden intangible stocks generates measurable output. The error in measured total factor productivity therefore follows a J-curve shape, initially dipping while the investment rate in unmeasured capital is larger than the investment rate in other types of capital, then rising as growing intangible stocks begin to affect measured production. As we will explain later, large capital adjustment costs, correlated intangible investments, and high investment shares of income exacerbate the magnitude of the J-curve effect. In the long run, as investment quantities and capital stocks reach their steady-state, the mismeasurement problem disappears even if the intangible investments do not.

We documented the basic idea of the Productivity J-Curve, along with a discussion of the Productivity Paradox in the context of Artificial Intelligence (AI), in Brynjolfsson, Rock, and Syverson (2017), building on earlier work by Yang and Brynjolfsson (2001). The current paper formalizes and expands on these ideas, offering a set of quantitative methods designed to measure the value and productivity effects of intangible investments. Namely, we propose using a set of forward-looking measures derived from stock market valuations as a means of assessing the magnitude of intangible investment value in the style of the traditional growth accounting framework. The basic idea is that these hidden intangibles are still captured by market valuations. As noted by Yang and Brynjolfsson (2001), a combination model of the Q-Theory of investment (Hayashi 1982; Wildasin 1984; Hayashi and Inoue 1991) and neoclassical growth accounting (Solow 1956; Solow 1957; Barro 1998; Corrado, Hulten, and Sichel 2009; Oliner and Sichel 2000; Oliner, Sichel, and Stiroh 2008) can deal simultaneously with the magnitudes of the intangible component of GPT-related investment and lags in implementation. We extend the model in Yang and Brynjolfsson (2001) to adjust the traditional growth accounting methods to include unmeasured intangible capital investments, showing that the J-curve is a consequence of the growth of associated intangible investments. We use market value regressions (following Hall 2001 and 2004, and Hall 2006) to inform estimates of the currently installed value of intangible capital stocks. We show how one can use these estimates to infer more accurate measures of total factor productivity growth on a contemporaneous basis.

II. Technology, Investment Theory, and Productivity Growth

Economic historians have emphasized the transformative effects of GPTs in history. We mentioned the work of David (1990), Allen (2009), and Acemoglu and Robinson (2013) above. Rosenberg and Trajtenberg (2004) identify the Corliss steam engine as an "icon of the Industrial Revolution," shifting population centers from rural to urban areas as water power was abandoned in favor of steam. Crafts (2004) explores the contribution of steam power to growth in greater detail for the British economy during the Industrial Revolution. Lipsey, Carlaw, and Bekar (2006) offer a list of possible GPTs (including electrification, mass production, and the factory system), also relating those inventions to the presence of the Productivity Paradox. Bresnahan (2010) conducts a wide review of the GPT concept, making the point that the modern era's information and communication technologies (ICT) broadly constitute a GPT with transformative effects on the economy. Particularly relevant to our analysis work Helpman and Trajtenberg (1994) which notes how general purpose technologies can generate alternating periods of investment and harvesting.

We focus in this paper on the most recent potential GPT: artificial intelligence. AI, and in particular the subfield of AI called machine learning (ML), is potentially pervasive, improves over time, and can spawn complementary innovation, meeting the Bresnahan and Trajtenberg (1995) criteria for a GPT. We would therefore expect, after an implementation lag period, for AI to significantly impact economic growth as other GPTs have (Brynjolfsson, Rock, and Syverson 2017; Cockburn, Henderson, and Stern 2018; Aghion, Jones, and Jones 2017; Agrawal, McHale, and Oettl 2018; Trajtenberg 2018). Nevertheless, the formal arguments presented here are applicable to other technologies and intangible capital accumulation more generally. The GPT context is useful because this is where we expect firm investment in unmeasured intangible capital goods to be large. Incremental innovations that do not transform productive activity are likely well captured by standard models. The complementary innovations necessitated by GPTs motivate our approach. If it were not necessary to transform existing business processes via complementary intangible investments, new GPTs would immediately boost output in straightforward and measurable ways. Creating complementary innovation both introduces implementation lags and predisposes the new intangible capital accumulation dynamics to mismeasurement.

Part of the productivity growth slowdown of the past decade may be due to these dynamics.[2] We argued in earlier work that implementation and restructuring lags are a possible explanation for the juxtaposition of optimism about AI's potential and currently low productivity growth (Brynjolfsson, Rock, and Syverson 2017). An alternative explanation is that the current technological promise is unfounded (Gordon 2015) and we are in a period of secular stagnation (Summers 2015). A third story is that mismeasurement of productivity growth can arise from changes in measures of output quality, consumer surplus, or price indices, particularly for digital goods (Brynjolfsson, Eggers, and Gannamaneni 2018a, 2018b; Goolsbee and Klenow 2018). Syverson (2017) shows that that these types of mismeasurement, while important, are likely to be insufficient to explain the productivity growth decline. We focus instead on a different type of mismeasurement: the forgone output due to investment in unmeasured *capital* goods. Identifying these hidden asset values makes it possible to better measure true productivity growth.

Intangible assets are an increasingly important component of economic activity, especially IT-related intangibles (Brynjolfsson and Hitt, 2000; Hall 2000; Hall 2001; Brynjolfsson, Hitt, and Yang 2002; Tambe, Hitt, and Brynjolfsson 2012, Saunders and Brynjolfsson 2016). This has led to numerous updates to the standard growth accounting frameworks and an emphasis in recent productivity studies on IT's role in productivity dynamics (Jorgenson and Stiroh 2000; Marrano, Haskel, and Wallis 2009; Corrado, Hulten, and Sichel 2009; Byrne, Oliner, and Sichel 2013; Byrne, Fernald, and Reinsdorf 2016), and specifically in the ICT-as-GPT case in (Basu, Fernald, Oulton,

[2] Of course, there are many other possible explanations. For instance, Acemoglu (2002) argues that "an acceleration in skill bias could cause a TFP slowdown because it creates an imbalance in the composition of R&D."

and Srinivasan 2003). Haskel and Westlake (2017) argue that intangible capital tends to have high fixed costs, low marginal costs, spillovers, and complementarities with other assets.[3] Further, the existence of significant intangible assets might explain the relatively poor historical performance of Tobin's Q (the ratio of a firm's market-to-book value) in explaining capital investment (Crouzet and Eberly 2018). Accounting for organizational investments, human capital, and business processes can strengthen the link between observed investment and asset prices (Eisfeldt and Papanikolaou 2013, 2014; Peters and Taylor 2017; Kogan et al. 2017; Andrei, Mann, and Moyen 2018).

Our approach applies the Q-theory of investment to recover the portion of productivity growth attributable to unmeasured intangible capital outputs. To adjust aggregate productivity estimates from a growth accounting framework for intangible output, we need to know 1) the growth rates of tangible investment and capital, and 2) the quantity of intangible investment per unit of correlated tangible investment. We empirically pin down this second component from the estimation of market value regressions. Part of Q is treated as intangible capital instead of excess valuation over asset replacement costs. When we observe a firm's market value rise by an amount greater than observed investment, we infer the difference as reflecting the value of intangible capital investments that were correlated with the tangible investment. We call these correlated intangible investments *intangible correlates*. Our framework also handles the case in which intangible capital is used to produce more intangible capital.

The Productivity J-Curve that we describe in this paper is related to, but distinct from, the trade balance J-curve discussed in Rose and Yellen (1989) and Magee (1973).[4] Their J-curve describes how trade balances react over time to changes in real exchange rates.[5] The similarity between the two J-curves is that there is a change in the sign of derivatives of focal quantities with

[3] They refer to the "4 S's" of intangible capital: sunk costs, scalability, spillovers, and synergies.
[4] We thank Larry Summers for suggesting how the dynamics we model are similar to the trade J-curve..
[5] Assuming export prices between countries are sticky, one country depreciating its currency makes sticky-priced imports (exports) more (less) attractive, while later prices adjust and foreign import demand increases.

respect to time as time passes (trade balances in the earlier case, productivity in this one), reflecting the adjustment of production processes in response to an external shock. In Rose and Yellen, the shock comes from a large change in exchange rates. In our paper, it is from a large technological innovation.

III. Growth Accounting in the Presence of Unmeasured Intangible Investment

Our setup builds on the approach of Yang and Brynjolfsson (2001) as follows. Suppose a competitive firm produces output with a general constant returns to scale production function. Then

$$Y = pF(K, N, A) \quad (1)$$

where *Y* is the final goods output of the firm, *p* is the price of final goods output (stable over time), *K* is the vector of capital goods, *N* is the vector of variable inputs (e.g., labor), and *A* represents the level of total factor productivity at time *t*. With flexible capital and input prices (*r, w*), we have the following, with *g* denoting a growth rate:

$$g_Y = \frac{\dot{Y}}{Y} = \frac{p\left(F_K\dot{K} + F_N\dot{N} + F_A\dot{A}\right)}{Y} = \left(\frac{rK}{Y}\right)g_K + \left(\frac{wN}{Y}\right)g_N + g_A \quad (2)$$

$$g_K = \frac{\dot{K}}{K};\ g_N = \frac{\dot{N}}{N};\ g_A = \frac{\dot{A}}{A}$$

Values with an upper dot represent the total derivative with respect to time.

In words, one can decompose the growth in output over time into the growth in capital stock multiplied by capital's share of output plus the growth in flexible input quantity multiplied by the expenditure share of flexible inputs and a final total factor productivity growth term. This last term is the familiar Solow Residual. It represents an improvement in productive efficiency, or more modestly a kind of "measure of our ignorance" in how a firm converts inputs to outputs.[6] Equation (2) is the basis for traditional growth accounting, which we revisit in equation (9) with an adjustment for unmeasured intangible investments.

To incorporate adjustment costs, we modify (1) following Lucas (1967):

$$Y = pF(K, N, I, A) \quad (3)$$

Now the production function incorporates an investment term I with market price z such that the total cost of investment in one unit of capital goods is $(z - pF_I)$. F is assumed non-increasing and convex in I to represent the idea that adjustment costs grow increasingly costly for larger I. In other words, the firm must forgo an increasing amount of output as its rate of capital investment increases. This helps explain why firms cannot, for example, instantaneously replicate the capital stocks of their competitors without incurring larger costs.

We can relate firm investment behavior to market value using this production function.[7] For the price-taking firm, market value equals the sum of the capitalized adjustment costs. The firm must solve:

$$\max_{I,N} \left[\int_0^\infty \pi(t) u(t) dt \right] = V(0)$$

$$\text{where} \quad \pi(t) = pF(K, N, I, A) - w'N - z'I$$

$$\text{and} \quad \frac{dK_j}{dt} = I_j - \delta_j K_j \; \forall j = 1, 2, \ldots, J. \quad (4)$$

That is, K_j is the capital stock of type j (indexes capital variety), N is a vector of flexible inputs, $u(t)$ denotes the compound discount rate at time t, and δ_j is the depreciation rate of capital of type j. As in Yang and Brynjolfsson (2001), F is assumed non-decreasing and concave in K and N. With

6 Abramovitz, Moses. "Resource and output trends in the United States since 1870." In *Resource and output trends in the United States since 1870*, pp. 1-23. NBER, 1956.

7 See, for example, Hayashi (1982), Wildasin (1984), and Hayashi and Inoue (1991).

homogeneity of degree one for *F*, we get the solution to the maximization of the Hamiltonian in (5) at time 0:

$$H(K,N,I,A) = (pF(K,N,I,A) - w'N - z'I)u(t) + \sum_{j=1}^{J} \lambda_j (I_j - \delta_j K_j) \quad (5)$$

with constraints:

$$\frac{\partial H}{\partial \lambda_j} = \dot{K}_j = I_j - \delta_j K_j \quad \forall j \in \{1,2,\dots,J\}, \forall t \in [0,\infty]$$

$$\frac{\partial H}{\partial K_j} = -\dot{\lambda}_j = pF_{K_j} u - \lambda_j \delta_j \quad \forall j, \forall t$$

$$\frac{\partial H}{\partial I_j} = 0 = \left(pF_{I_j} - z_j\right) u + \lambda_j \quad \forall j, \forall t$$

$$\frac{\partial H}{\partial N_i} = 0 = \left(pF_{N_i} - w_i\right) u \quad \forall i \in \{1,2,\dots,L\}, \forall t$$

$$\lim_{t \to \infty} \boldsymbol{\lambda}(t) \boldsymbol{K}(t) = 0$$

leading to an equation for the value of the firm:

$$V(0) = \sum_{j=1}^{J} \lambda_j(0) K_j(0) \quad (6)$$

Equation (6) shows that the value of the firm at $t = 0$ is the sum over all varieties of the capital stock quantities multiplied by the "shadow price" of investment of the respective varieties. In our context, this shadow price reflects adjustment costs.[8]

---

[8] Following equation (6) in Hall (2000), if λ_j represents the marginal q value (incremental market value created divided by asset replacement cost), then the marginal adjustment cost for the firm (set by the firms' competitors) at its chosen capital investment rate is set by:

$$c'\left(\frac{k_t - (1-\delta)k_{t-1}}{k_{t-1}}\right) = q_t - 1 = \lambda_t - 1$$

Where c'(x) is the marginal adjustment cost function and δ is the depreciation rate of capital. In this case, there are no unmeasured intangible correlates, only adjustment costs of investment. Our framework below allows for both adjustment costs and unmeasured intangibles. In that case, the sum of these two elements is our λ value. (One interpretation of this summation is that capitalized convex adjustment costs are, in effect, a nonlinear component of correlated intangible investments.)

Assuming all asset stocks are measured correctly and market prices correctly represent the value of claims on publicly traded firms, equation (6) suggests that a regression of firm value on dollar quantities of asset varieties will yield a coefficient vector that represents the average present value of one unit of each type of capital. In a frictionless efficient market, that vector would contain values of one for all assets. In the presence of adjustment costs, however, the coefficient will be greater than one.

We can extend this logic to unmeasured intangible GPT investments that are complementary to tangible assets. Suppose a firm adopting a new GPT must invest proportionately in two assets: computer equipment and firm-specific GPT specialist training (e.g., training AI engineers). For a firm with a measurable quantity of tangible computer equipment, the estimated shadow price coefficient for the computer equipment investment will exceed the "true" computer equipment coefficient by the amount necessary to represent the value of the complementary training as well. The specialist training is not capitalized on the firm's formal balance sheet, yet the financial market must also value the future service flows from training if no arbitrage conditions are to hold. The market value premium over book value implies a Tobin's Q above unity; the value of the firm is higher than the simple replacement cost of its *observed* assets.

When we examine data from financial markets, we find that technology firms have, on average, considerably higher values of Q. This suggests that they have either higher levels of adjustment costs, intangible correlated investments relative to booked assets, or both. This is consistent with the idea that implementing a new GPT requires complementary intangible investment to reorganize production.

In a growth accounting framework, the value of final goods in any given year is divisible into the value of consumption goods and the value of capital goods as follows:

$$p_c C + zI = Y = p_y F(K, N, I) = p_y F_n N + p_y F_k K + p_y F_I I = wN + rK + (z - \lambda)I \quad (7)$$

alternatively, the total payments to capital (including intangible stocks) and labor are:

$$p_c C + zI + (\lambda - z)I = wN + rK$$

This is the growth accounting identity. The value of consumption goods plus the value of capital investment is equal to total output Y. This, in turn, is equal to the total income of flexible inputs, capital rental costs, and investment (both measured and unmeasured).[9]

If the $(\lambda - z)I$ value of capital goods production goes unmeasured, then part of the expenditure on capital goods is missing when the growth decomposition is performed. In the context of a GPT, this means that much of the training, the investment in implementing new decision processes, the reorganization costs, and the incentive designs necessary to generate productive service flows from GPT capital are left out.

If the economy is accumulating the new GPT-related capital faster than it accumulates measurable capital, TFP will be underestimated. To see why, we can update the growth decomposition equation as follows:

$$g_Y = \frac{\dot{Y}}{Y} = \frac{p\left(F_K \dot{K} + F_N \dot{N} + F_I \dot{I} + F_A \dot{A}\right)}{Y} \quad (8)$$

Following the first order conditions for the Hamiltonian above, we have

$$\lambda_j(0) = (z_j - pF_{I_j}) \qquad \text{and}$$

$$g_Y = \left(\frac{pF_K K}{Y}\right)\left(\frac{\dot{K}}{K}\right) + \left(\frac{pF_N N}{Y}\right)\left(\frac{\dot{N}}{N}\right) + \left(1 - \frac{\lambda}{z}\right)\left(\frac{zI}{Y}\right)\left(\frac{\dot{I}}{I}\right) + \left(\frac{F_A A}{Y}\right)\left(\frac{\dot{A}}{A}\right) \quad (9)$$

There are several differences between this expression and the typical growth accounting equation (2). Because expressions like (2) are the standard method of computing TFP growth, deviations between the true growth decomposition (9) and the decomposition as implemented (2) reflect the sources of TFP mismeasurement.

[9] This framework assumes that the firm's Hamiltonian can be aggregated to the economy level, as does the standard growth accounting framework. This is not always the case (Houthakker 1955; Basu and Fernald 1997).

The first difference is in the first term on the right side of (9). The capital services' share of income needs to be adjusted to account for the capital services from intangibles. Given homogeneity of degree one in the production function, the adjustment is:

$$\frac{pF_K K}{Y} = 1 - \left(\frac{wN}{Y}\right) + \sum_{j=0}^{J}\left(\frac{\lambda_j}{z_j} - 1\right)\left(\frac{z_j I_j}{Y}\right) \quad (10)$$

Where j indexes different capital varieties. In (2), the summation term was omitted from the value of capital's output share and so in a standard growth decomposition would be rolled into measured productivity growth. The difference between the capital share of income without unmeasured intangibles and the adjusted capital share is equal to the summation term in (10). Labor's share of income remains the same under either setup. This is why the left side of (10) is stated precisely as the residual income share without labor added to the summation term.

The true growth decomposition (9) also clearly shows in its second-to-last term the investment component missing from (2). The contribution to output growth of this term would also be subsumed into productivity growth when applying decomposition (2).

Denote with g^*_{TFP} the productivity growth measure derived from the true decomposition (9)—i.e., the final term in that expression. Denote the productivity growth measure derived from a standard decomposition (2) as g_{TFP}. Taking the difference gives us an expression for the measurement error in productivity growth:

$$g^*_{TFP} - g_{TFP} = \sum_{j=0}^{J}\left(\frac{\lambda_j}{z_j} - 1\right)\left(\frac{z_j I_j}{Y}\right)(g_{I_j} - g_k) \qquad (11)$$

where j indexes the capital variety type. This equation states that the measurement error in TFP growth is the sum of the differences between investment growth rates in a given capital variety and the overall growth rate of capital, multiplied by the investment share of observable income and the per-observable–investment-unit value of intangible correlates and adjustment costs. With a fixed multiplier and investment share of income, the growth rate differential between investment and overall capital drive the dynamics of measurement error.

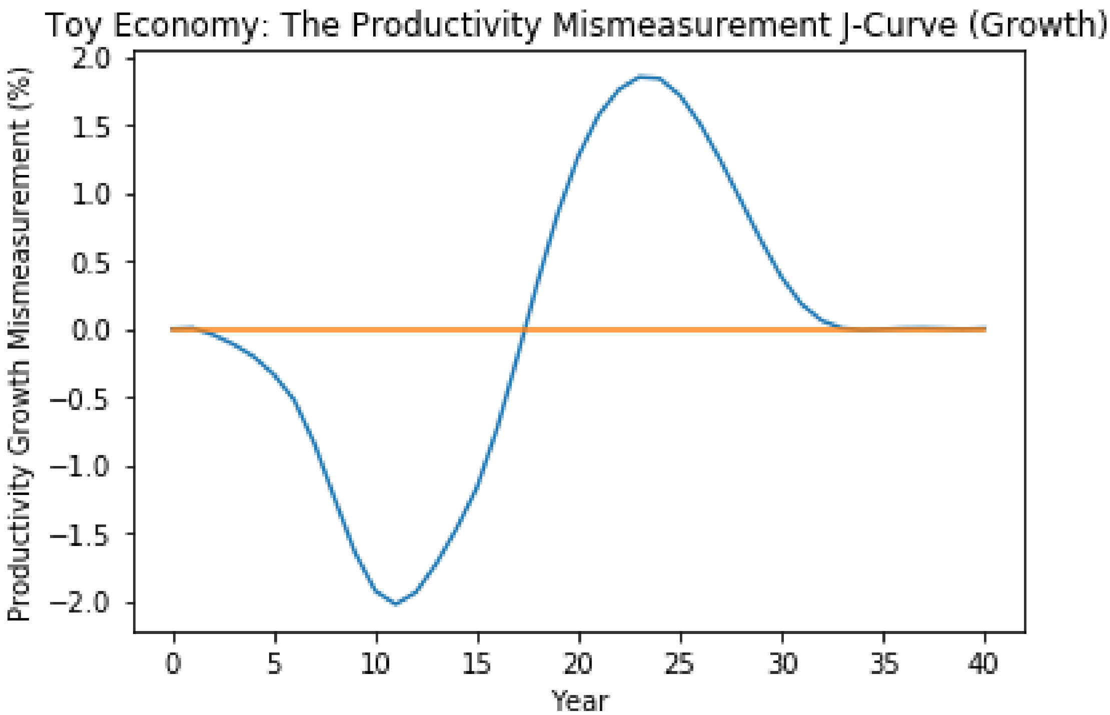
If the shadow price of technological investment is simply the market price of the investment good, so that $\lambda/z = 1$, there is no missing growth in output. However, for GPTs, it is likely that there will be a need for extensive unmeasured investments that correlate with tangible capital goods production, making λ/z greater than one. Because investment share of measurable output (zI/Y) is positive, the sign of productivity mismeasurement will depend on the difference between the growth rates of *investment* in GPT capital and the *installed stock* of GPT capital. Therefore, if GPT diffusion leads to a period of differential growth rates of intangible investments and intangible stocks—and we offer intuition below for why this is likely to be the case—then measured and true productivity growth will diverge during that period. In the long run, if the economy reaches steady state, the two growth rates will converge and productivity will no longer be mismeasured, even in the presence of unmeasured intangibles.

IV. The Productivity J-Curve

The most straightforward way to understand the Productivity J-Curve is to consider foregone output used to produce unmeasured capital goods. Suppose a company wants to become more "data-driven" and reorganize its production processes to take advantage of new machine learning prediction technologies (Brynjolfsson and McElheran 2016). This firm might want, for example, to change its labor mix to build more software and to teach its customers to order products online instead of in person. While the company develops online product ordering applications and business processes for that purpose, it will not be able to use those investment resources to produce more final goods inventory. At the same time, though, the capital assets the firm *is* building—institutional software knowledge in the company, hiring practices, organization building, and customer retraining to use digital systems—are left unmeasured on the balance sheet.

On the margin, the (present-discounted and risk-adjusted) value of these unmeasured assets equals the costs incurred to produce them. But during the period in which that output is foregone, the firm's (traditionally measured) productivity will suffer because it will seem as though the company produces proportionately less output relative to its inputs. Later, when those hidden intangible investments start to generate a yield as inputs, it will seem as though the measured capital stock and employed workers have become much more productive. Therefore, in early investment periods productivity is understated, whereas the opposite is true later when investment levels taper off.

The mismeasurement in this example regards a J-curve in productivity *levels*, and we derive a general expression describing its evolution below. That said, a similar J-curve exists for productivity *growth rates*. The math behind this growth rate J-curve is precisely that from our analysis above and is summarized in equation (11). The underlying intuition is very much like that for the J-curve in the productivity level. Early in the GPT diffusion process, intangible investment growth is likely to be larger than intangible capital stock growth. With missed output growth dominating missed input growth, measured TFP growth is lower than true TFP growth. Later in the GPT diffusion process, investment growth slows below the growth rate of the installed intangible stock. Measured productivity growth then exceeds its true level. Eventually the growth rates equalize in steady state, and productivity mismeasurement disappears.



The mathematics behind the J-curve in level terms follow from equation (7). Differentiating our earlier production function (assuming numeraire output price for simplicity) yields:

$$dY = F_K dK + F_N dN + F_I dI + F_A dA = rdK + wdN + (z - \lambda)dI + F_A dA \quad (12)$$

If we have measured capital service flows, labor service flows, and we know investment prices and their installed shadow values, we can back out the component of output driven by productivity improvements *dA*. In efficiency units and log terms (for an ordinary Cobb-Douglas production function):

$$\ln(Y + \lambda I) = \ln(A) + \alpha \ln(K) + (1 - \alpha) \ln(N)$$

$$\ln(Y + zI) = \ln\left(\frac{Y + zI}{Y + \lambda I}\right) + \ln(A) + \alpha \ln(K) + (1 - \alpha) \ln(N) \quad (13)$$

That is, with factor shares of α and 1-α for capital (including investment) and labor (respectively), we now see that the productivity level decomposition in (13) has an additional term of the ratio of total measured output to total output including intangible correlates and adjustment costs. Fixing a multiplier λ, this ratio is always going to be (weakly) less than one, implying a drag on the measured productivity level. But recall that the measure of capital stock *K* includes the intangible investment stock as well. We modify (13) to explicitly include the unmeasured intangible capital stock:

$$\ln(Y + zI) = \ln\left(\frac{Y + zI}{Y + \lambda I}\right) + \ln(A) + \boldsymbol{\alpha}\ln(\boldsymbol{K}) + (1 - \alpha)\ln(N) \quad (14)$$

K denotes the vector of ordinary capital stocks and unmeasured intangible capital stock. If we have the marginal condition that the present value of investment returns in unmeasured intangibles equals the cost of the investment, in the long run the positive and negative effects of these additional terms on the level of productivity must net out. This is because the total risk-adjusted net present value of capital service flows from unmeasured intangible capital investments must equal the costs to the firm of making those investments.

To see the Productivity J-Curve, consider the case that the stock of intangible capital is zero. In this case, we only have the first negative term creating mismeasurement of productivity. Later, if net investment is zero instead, we only have the intangible component of the positive adjustment term $\boldsymbol{\alpha}\ln(\boldsymbol{K})$, causing an overestimate of the productivity level $\ln(A)$.

Recall that the J-curve in productivity *growth* can be seen in the right-hand side of (11). When the growth rate of unmeasured intangible investments exceeds (is lower than) the growth rate of the total capital stock, the true productivity growth rate will exceed (be lower than) the measured productivity growth rate. This effect in either direction is amplified by 1) a large installed-to-purchase price ratio λ/z of investment (or large quantities of unmeasured intangible investment required per unit of measured investment) and 2) a large measured investment share of measured output. This latter effect is part of the explanation for the Productivity Paradox; when the economy is in the early stages of accumulating GPT-related capital, measured investment's share of measured output will be relatively low. The figure below shows the total factor productivity growth J-curve.

Because the values are in growth terms, the sum of overestimates and underestimates need not be zero over time. The *level* impact must necessarily be zero in expectation, however, if assets are efficiently priced.

There is a similar description in equation (5) of Brynjolfsson, Rock, and Syverson (2017), where the mismeasured components of investment and capital stock work against each other to generate the difference between measured and actual productivity growth. In this case, *r* and *z*, respectively, represent the total prices of capital and investment (adjusted to account for intangible quantities).

$$g_{TFP_{measured}} - g_{TFP_{corrected}} = \left(\frac{r_{intan}K_{intan}}{Y}\right) g_{K_{intan}} - \left(\frac{z_{intan}I_{intan}}{Y}\right) g_{I_{intan}} \quad (15)$$

Note the similarity with equation (11) above. We can extend (15) with (11) to situate this difference in terms of all intangible correlates associated with different capital varieties while separating capital and investment effects. Doing so gives:

$$g_{TFP_{measured}} - g_{TFP_{corrected}} = \left(\sum_{j=0}^{J}\left(\frac{z_j I_j}{Y}\right)\left(\frac{\lambda_j}{z_j} - 1\right) g_{K_j}\right) - \left(\sum_{j=0}^{J}\left(\frac{z_j I_j}{Y}\right)\left(\frac{\lambda_j}{z_j} - 1\right) g_{I_j}\right) \quad (16)$$

The difference in measurement between the standard approach and ours is just the investment share and intangible value-weighted difference in growth rates between capital stocks and investments.

Note that (16) can be rearranged to form a regression specification, in which the intercept is defined by the corrected measure and the coefficient estimates are defined by the investment shares and lambda values:

$$\boldsymbol{g_{TFP_{measured}}} = \boldsymbol{g_{TFP_{corrected}}} + (\boldsymbol{g_K} - \boldsymbol{g_I})'\boldsymbol{\beta} + \boldsymbol{\epsilon} \quad (17)$$

In steady state, the growth rates of capital and net investment converge, mitigating the mismeasurement problem. In the short run, the deployment of resources of different types to produce outputs of measured and unmeasured varieties can influence the degree to which productivity growth is mismeasured. These unmeasured intangible capital stocks might be used to produce even more unmeasured intangible assets, in which case the hidden output and hidden input effects can offset each other somewhat. In the case that the rate of intangible capital production accelerates and uses measured capital and labor services in increasingly greater quantities, the J-curve effects are more pronounced.[10] This will also occur if the quantity of intangible correlates (including adjustment costs) per unit of tangible investment increases.

V. Is Hidden AI Capital Investment Already Causing a Productivity Shortfall?

Gross Domestic Product in the U.S. in 2017 was $19.5 trillion and in real terms grew at an average annual rate of 2.17% over 2010 to 2017, down from 2.72% per year from 2000 to 2007 (the eight years prior to the Great Recession).[11] This implies that unmeasured intangible capital investment over 2010 to 2017 would need to average $107 billion per year (= 19.5 trillion * [2.72% - 2.17%]) in 2017 dollars to explain the entire slowdown in in GDP growth. How much of this slowdown could a Productivity J-Curve for investment in AI and related intangibles explain?

The economy is early in the AI adoption cycle, but recent growth has been impressive. There has been a rapid increase in the use of AI and robotics technology over the past decade (Furman and Seamans 2018). Startup funding for AI has increased from $500 million in 2010 to $4.2 billion by 2016 (growing by 40% between 2013 and 2016) (Himel and Seamans 2017). Though concentrated heavily in the information technology sector, overall measurable corporate investment in AI in 2016 was $26-39 billion, marking 300% growth since 2013 (Bughin et al. 2017). Similarly, international industrial robot shipments since 2004 have nearly doubled overall and almost quadrupled in the consumer electronics industry (Furman and Seamans 2018).

---

[10] There is also a degenerate scenario in which firms shift toward focusing on intangible output production using intangible assets. In this case, the productivity measurement apparatus starts to lose its value.
[11] From the Bureau of Economic Analysis GDP statistics.

For AI to account for the 0.55% of "lost" output in 2017 GDP, the quantity of correlated intangible investments per unit of tangible investment must be between roughly 2.7 and 4.1 times the observable investment values (using the Bughin et al. (2017) estimate).[12] This is not implausible. Brynjolfsson, Hitt, and Yang (2002) find that the total market value of measured computer capital investments is as much as $11.8 per $1 in measured expenditure, with a standard error of $4.025.[13] None of the intangible "shadow" value will show up in the productivity statistics. Because the foregone output cannot be explained by growth in labor or observable capital inputs alone, the output shortfall will be attributed to slower productivity growth. Further, this investment (discounted and risk-adjusted) will later generate a capital service flow that produces measurable output.

Of course, these numbers are just for 2017, when measured AI investment was several multiples of what it was only a few years prior. Thus analogous pre-2017 values would be notably smaller, and it is unlikely that much of the GDP slowdown gaps in those earlier years would be attributable to AI-related intangibles. However, given that AI investments are likely to continue growing quickly, and the fact that where it exists, AI capital has a high market valuation and as such a considerable shadow value for intangible correlates, we could well be likely entering the period in which AI-as-GPT could have noticeable impacts on estimates of output and productivity growth.

VI. Deploying the Framework Using R&D, Software, and Computer Hardware Investment

While the results in the previous section imply AI-related intangibles *per se* have only very recently been large enough to noticeably affect measured output and productivity, other technology-

[12] The required forgone output in 2017 was $107 billion. Dividing by the low observed investment figure of $26 billion implies a required intangible investment that was 107/26 = 4.1 times the observed investment. Using the larger $39 billion figure implies intangibles that were 107/39 = 2.7 times observed investment.

[13] This uses a series of regression specifications motivated by a version of equation (6) in the previous section.

related investments may have had more substantial effects over greater horizons, creating their own J-curve dynamics as a result. We explore this possibility in this section.

Specifically, we estimate the per unit magnitudes of intangible capital investment that coincides with observable R&D, software, and computer hardware capital. We then use those values to adjust total factor productivity estimates and explore if substantial J-curve effects exist for those capital types. To estimate the magnitude of intangible investments, we use the approach described above for obtaining intangible capital shadow values by comparing firms' observable investments to their market capitalization.[14] We obtain shadow values for R&D, software, and hardware (each) since 1961 and use these to build up time series estimates of intangible stocks from 1961-2017.

Our productivity baselines, net capital stock, and investment by capital variety estimates all come from Fernald (2014), extended through 2017.[15] We take estimates of the total stock of research and development (R&D) capital and the total stock of capitalized selling, general, and administrative (SG&A) expense from the Peters and Taylor (2017) measures available in Wharton Research Data Services (WRDS) (we extend these measures through 2017 as well, following the guidelines in their paper). These estimates are joined to Compustat to construct a panel from 1961-2017 of firm market value, total asset book value, total R&D capital, total "organizational" capital (the capitalized SG&A expenditure), and other firm identifiers of all publicly-held companies in the U.S. In our regressions, we define industry by four-digit NAICS code.

R&D capital provides a useful context for understanding Productivity J-curve dynamics for a few reasons. Corporate research leads to the development of new technologies that diffuse over time, and there has been a steady flow of investment into R&D for decades. Further, the link between R&D investment and market value is well established (Hall 1993, 2006). Because investment in R&D has persisted over the long term, we are more likely to find investment in R&D

[14] Recall as discussed above that adjustment costs can be thought of as a nonlinear component of intangible investment.
[15] Capital stock estimates for this series are also available from the U.S. Bureau of Economic Analysis (BEA).

at nearly steady-state levels. This implies that the intangible-related challenges for productivity estimation coming from R&D are likely to be minimal at present. (Remember from our analysis above that as the growth rates of intangible investment and stocks converge, productivity mismeasurement falls to zero.) Nevertheless, the exercise presented here for R&D is applicable to other capital varieties.

In contrast, heavy software and computer hardware capital investment is a more recent phenomenon in which firm behavior might (still) not have entirely matured, so J-curve dynamics may still be present. We find evidence of this further below after first demonstrating our approach with analysis of R&D-related intangibles.

The first step in estimating the productivity mismeasurement effect of intangible correlates is estimating how many units of intangible investment correspond to observable investment quantities. We begin with R&D and capitalized SG&A stock measures from Peters and Taylor (2017). For this, we run a market value regression of the style in Hall (1993) and Brynjolfsson, Hitt, and Yang (2002). The specification for firm *i* in industry *j* at time *t* is:

$$Market\ Value_{ijt} = \beta_0 + \beta_1 TotalAssets_{it} + \beta_2 R\&D_{it} + \eta_{jt} + \epsilon_{it} \quad (18)$$

The coefficient on R&D picks up the ratio of dollars of market value created per unit of R&D stock at the firm in a given year. This, which we refer to as the *intangible multiplier*, is the ratio (λ/z) from our analysis above. We estimate specifications both including and excluding capitalized SG&A and industry-year fixed effects. The results are in Table 1 below.

Table 1: Market Value Regressions on R&D and SG&A Stocks

Market Value Regressions	(1) Basic R&D	(2) Basic R&D and SG&A	(3) Industry-Year Fixed Effects: R&D	(4) Industry-Year Fixed Effects: R&D and SG&A	(5) Firm and Year Fixed Effects: R&D	(6) Firm and Year Fixed Effects: R&D and SG&A

	(1)	(2)	(3)	(4)	(5)	(6)
Total Assets	1.019	1.002	1.018	1.002	1.019	1.004
	(0.00125)	(0.00163)	(0.00992)	(0.0119)	(0.00740)	(0.0102)
R&D Stock	2.889	2.045	2.980	2.195	2.362	1.852
	(0.0805)	(0.0788)	(0.510)	(0.418)	(0.332)	(0.306)
SG&A Stock		1.493		1.424		1.100
		(0.0700)		(0.251)		(0.216)
Constant	625.7	434.2				
	(9.699)	(12.35)				
Observations	665,536	665,536	665,515	665,515	665,408	665,408
R-squared	0.985	0.986	0.986	0.987	0.990	0.991
Industry-Year FE	No	No	Yes	Yes	No	No
Firm and Year FE	No	No	No	No	Yes	Yes

Robust standard errors in parentheses

Total Assets are the total assets on the firm's balance sheet, Industry is the four-digit NAICS code. Market Value is the sum of the book value of debt, preferred stock, and the end-of-year equity share price multiplied by common shares outstanding. Specifications (5) and (6) include firm and year fixed effects, but not firm-year fixed effects. Standard errors in parentheses (robust for (1) and (2), clustered by industry in (3) and (4), clustered by firm in (5) and (6)).

The coefficients on Total Assets is very close to 1 as expected, whereas estimates for R&D are 2.89, 2.98, and 2.36 for specifications without capitalized SG&A and with (respectively) no fixed effects, industry-year fixed effects, and firm and year fixed effects). Including capitalized SG&A, the estimates decrease somewhat to 2.04, 2.20, and 1.85 for the respective specifications, with the coefficients on capitalized SG&A picking up much of the difference. Thus, these models suggest that the market value of capitalized research and development expenses is between about two and three dollars per net dollar of investment.

We also estimate a year-by-year regression of market value on total book assets and capitalized R&D with industry fixed effects. Figure 3 presents the time series of R&D coefficient estimates for that specification.[16] The year-by-year regressions reveal substantial variation in the shadow value of R&D-related intangible assets, consistent with overall valuation dynamics. It is with

[16] The full table of coefficients is available in the Online Appendix. Available: http://drock.mit.edu/Research

this set of values that we proceed to adjust productivity growth measurement. Figure 4 shows the same coefficient estimates for Total Assets, which are considerably lower in comparison (note the vertical scale is an order of magnitude smaller than Figure 3).

Figure 3: R&D Market Value Year-by-Year Regression Coefficients 1962-2017

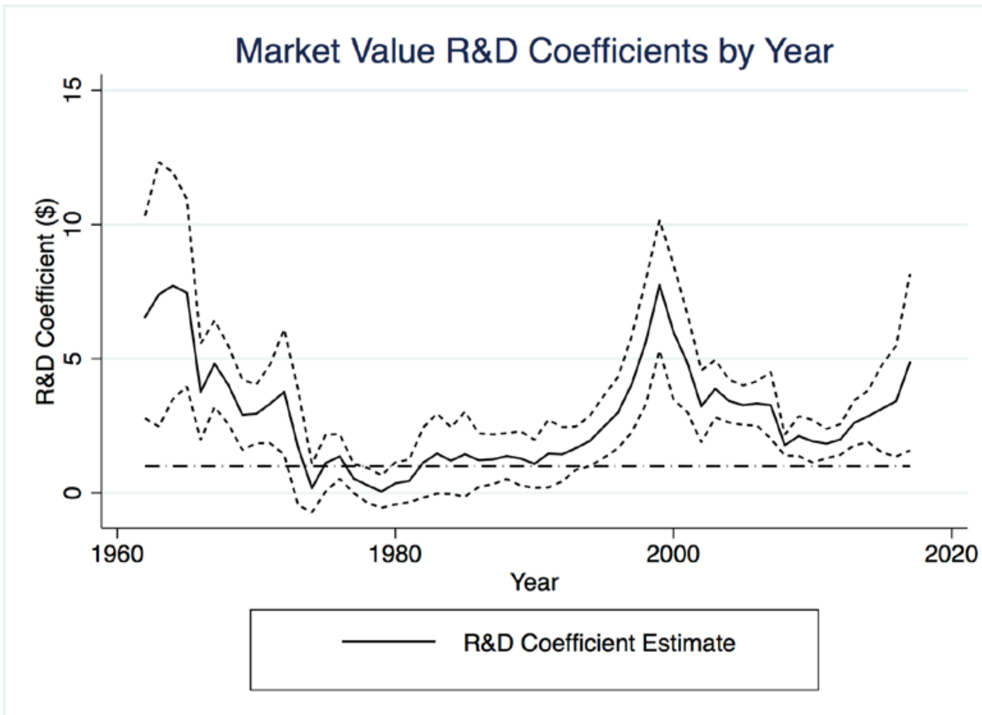
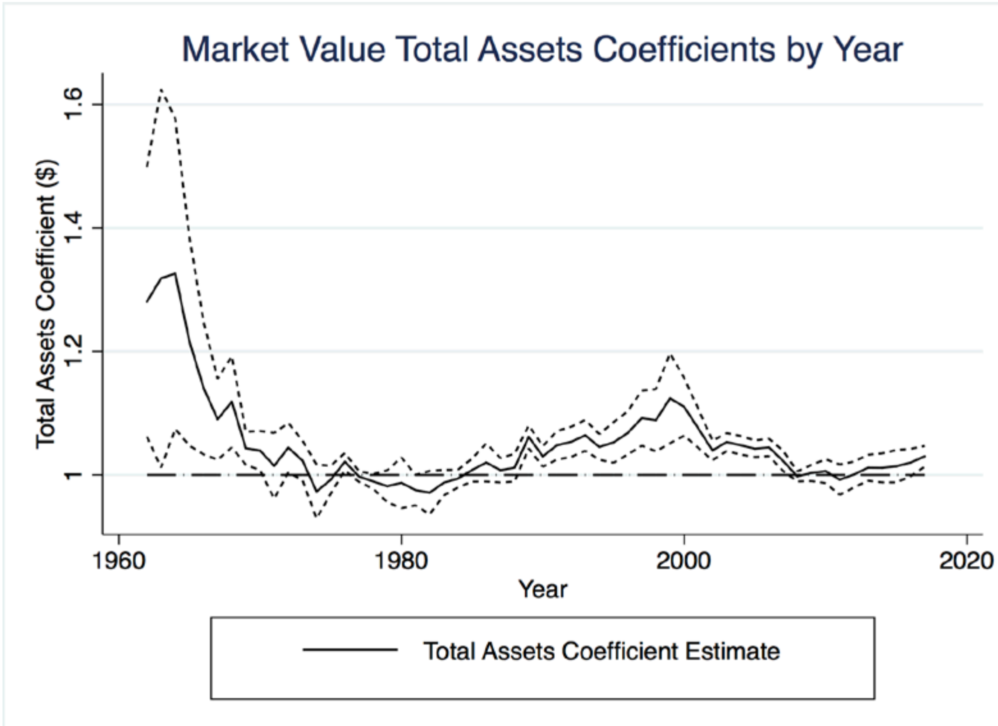
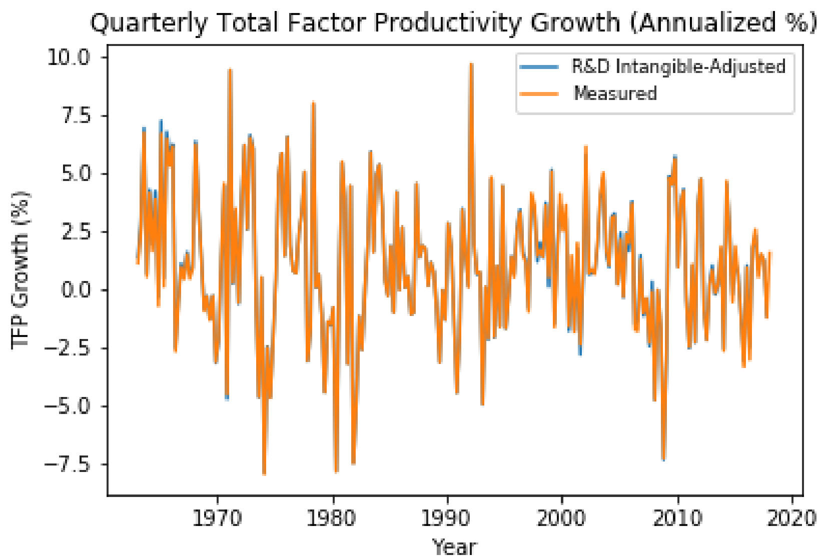


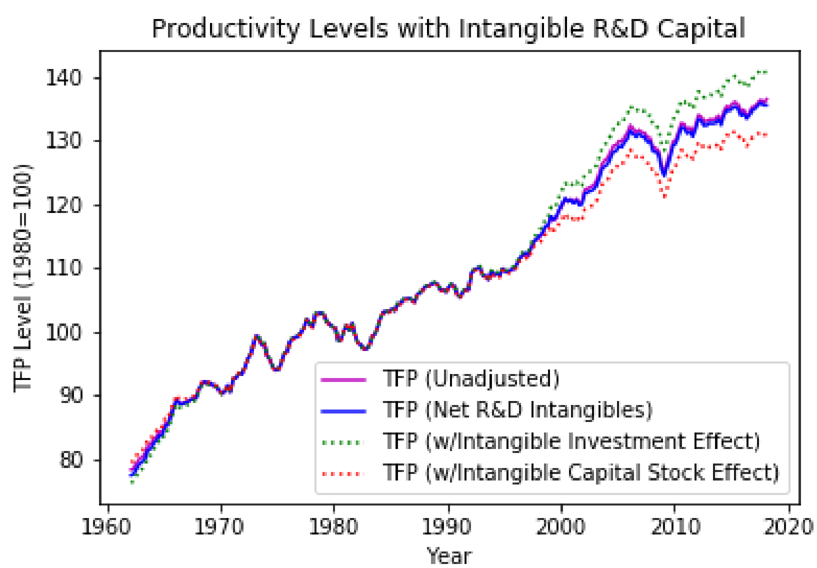
Figure 4: Total Asset Market Value Regression Coefficients 1962-2017



Given an estimate of the total amount of intangible correlates per unit of investment, we proceed to adjust the productivity level and growth estimates to include the missing intangible outputs and inputs using equation (16). Figure 5 shows the time series of TFP growth, both as measured in Fernald's data and adjusting for unmeasured R&D-related intangibles. Figure 6 shows the effects in level terms, obtained by integrating the growth rates.

The unadjusted series differs very little from the net adjusted series. The reason is that R&D capital investment rates have been steady over the observation period, roughly canceling out the countervailing influences of intangible outputs and intangible inputs. This is made clear by the dotted green and red lines in Figure 6, which isolate the influence of the two terms in equation (16). The red dotted line shows the downward adjustment to measured productivity due to the failure to measure intangible capital input service flows. The green dotted line reflects the nearly equal-sized upward adjustment to productivity due to uncounted outputs tied to intangible investment.



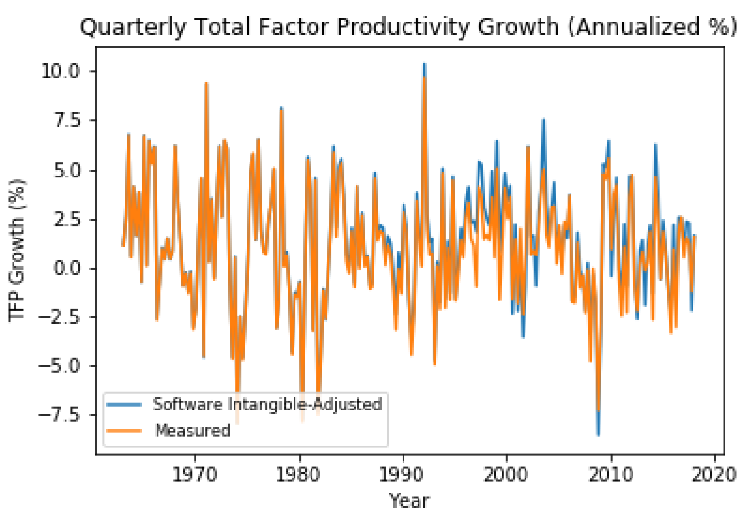


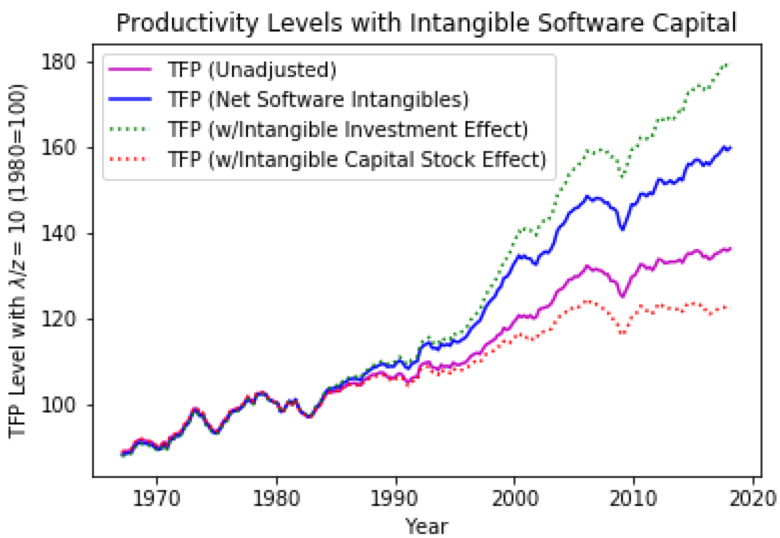
Figures 5 (top) and 6 (bottom): R&D-related Intangible Capital-adjusted Total Factor Productivity

Although the net measurement effects of R&D-related intangibles are negligible, the same is not true for software and computer investments. We do not have similar firm-level data on IT capital stocks and investment to run the market value regression in equation (18) for IT, so we apply the productivity adjustment analysis under a series of plausible values for the intangible multiplier λ/z, guided by the Brynjolfsson, Hitt, and Yang (2002) estimate that each unit of observable software and computer hardware is associated with roughly \$12 (standard error \$4.02) of firm market value.

In contrast to the adjustment for R&D-related intangibles, the Productivity J-curves for both software and computer hardware capital (we separately analyze each) have yet to reach positive territory in terms of levels.

Of the three capital varieties we investigate in this study, software's J-curve is in the least mature stage. Software investment has been and continues to be growing faster than overall capital investment, and its level is sufficiently large to suggest that part of the productivity slowdown might be explained by a compositional shift of investment toward digital assets. Figures 7 and 8, analogously to Figures 5 and 6 for R&D-related intangibles, show the annualized quarterly growth rates and levels of measured TFP and software-intangible-adjusted TFP. The differences between measured and corrected estimates are starkly larger than those arising from R&D.





Figures 7 (top) and 8 (bottom): Software-related Intangible Capital-adjusted TFP

The J-curve dynamics of software investment began in the 1990s and have not waned since. If we assume an intangible multiplier of $10, roughly the level but somewhat lower than Brynjolfsson, Hitt, and Yang (2002), the net adjusted TFP level (163.9) is 18.2% higher than measured TFP (138.6) as of the beginning of 2017. Figure 9 shows the productivity level adjustments for more conservative intangible multipliers. Even for lower levels of the multiplier, the productivity level differences are notable and growing.

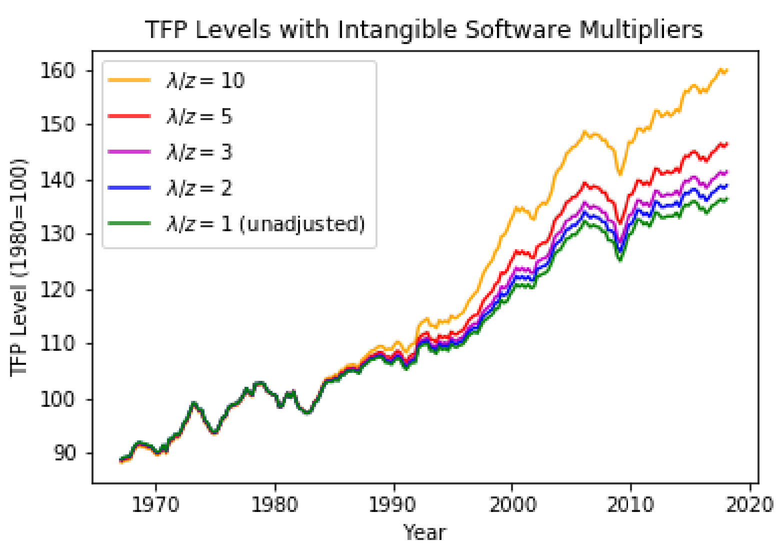
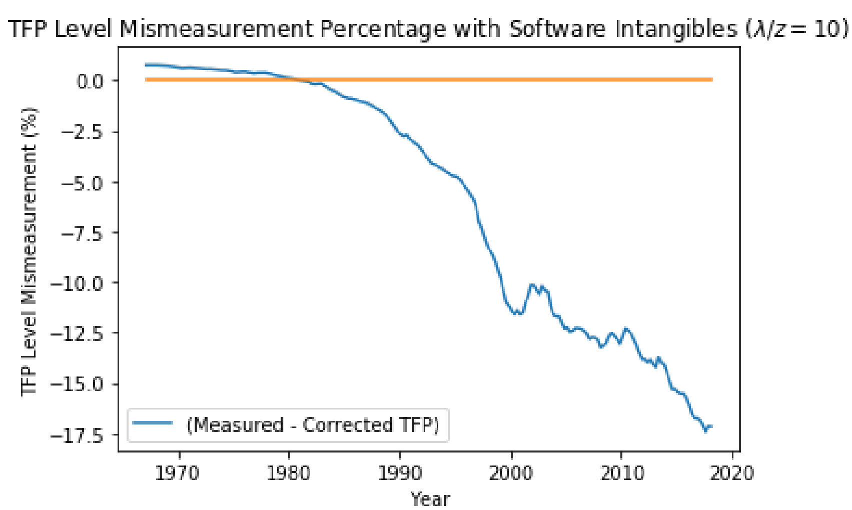
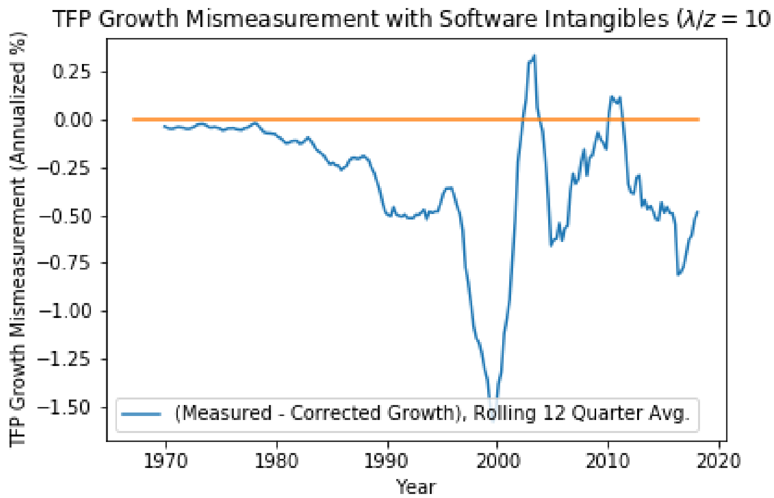


Figure 9: Total Factor Productivity Levels Corrected for Different Software Intangible Multipliers

The reason behind the growing understatement of productivity due to software-related intangibles is the growing rate of software investment. Aside from brief periods following the dot-com bust and the financial crisis, investment in software has grown significantly. As a result, software-related intangible investment rates are not yet in steady state. As the analysis above shows, when the investment growth rate exceeds the growth rate of the intangible stock, productivity growth is understated. Since 2010, when the productivity growth mismeasurement effect was very nearly zero, annualized quarterly productivity growth underestimation increased to 0.86% by the end of 2016. The implied understatement was even larger at the end of the 1990s, where measured productivity was 1.6% lower than software-adjusted productivity. Figures 10 and 11 show the respective mismeasurements of TFP levels and growth for software-related intangible capital outputs (i.e., the vertical distances between the adjusted and measured series in Figures 7 and 8) since 1967. At least in level terms, we are still in the capital accumulation phase of a deep Productivity J-curve. Tables in the Online Appendix show the productivity growth adjustments for R&D, computer software, and computer hardware from 1967-2017.[17]

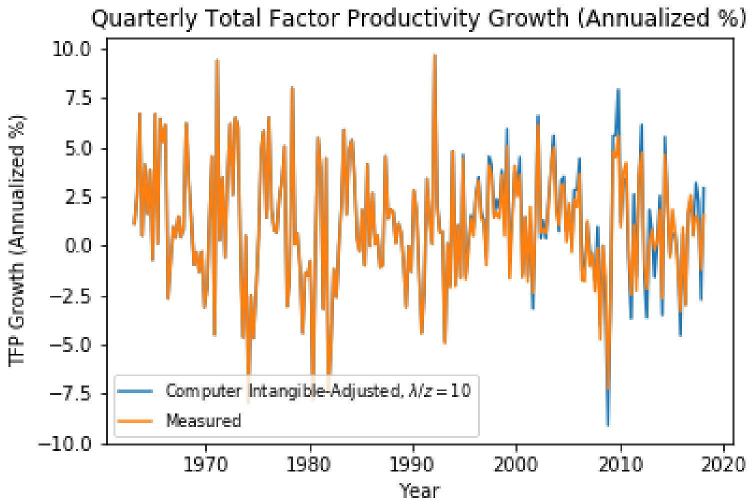


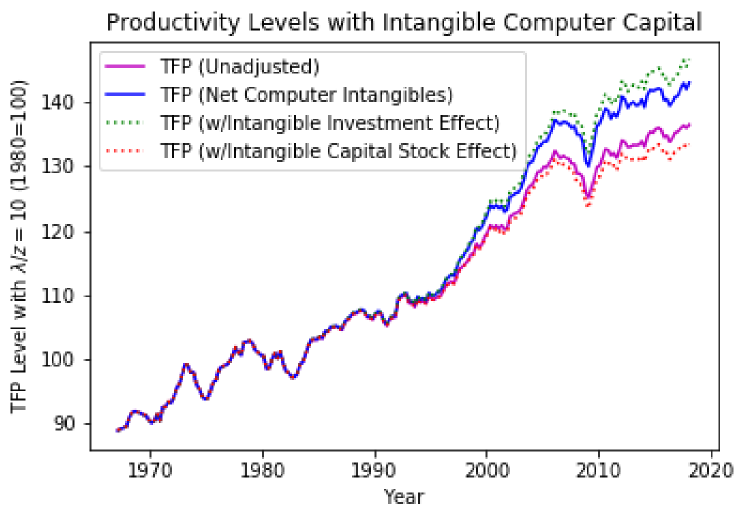
[17] Available at http://drock.mit.edu/Research



Figures 10 (top) and 11 (bottom): Computer Software-related TFP Mismeasurement in Levels and Growth Terms (respectively)

We extend our analysis to computer *hardware*-related intangible investment. Figures 12 and 13 show adjusted and measured TFP growth and levels, again assuming an intangible multiplier of \$10 for each dollar of hardware investment. Again, the divergence between measured and corrected TFP begins to become noticeable in the 1990s, after Solow's famous quip. Figure 13 also shows where the TFP level would be without adjustment (purple), the net intangible-adjusted series (blue), isolating only the missing intangible inputs effect (dotted red), and isolating only the missing intangible outputs effect (dotted green). Figure 14 compares the adjusted series for an intangibles value of \$10, \$5, \$3, \$2, and \$1 (unadjusted).





Figures 12 (top) and 13 (bottom): Computer-Hardware-Related Intangible Capital-adjusted TFP

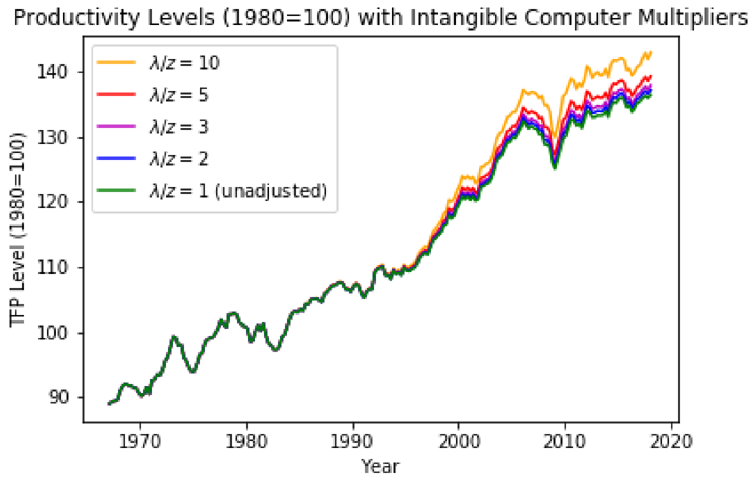
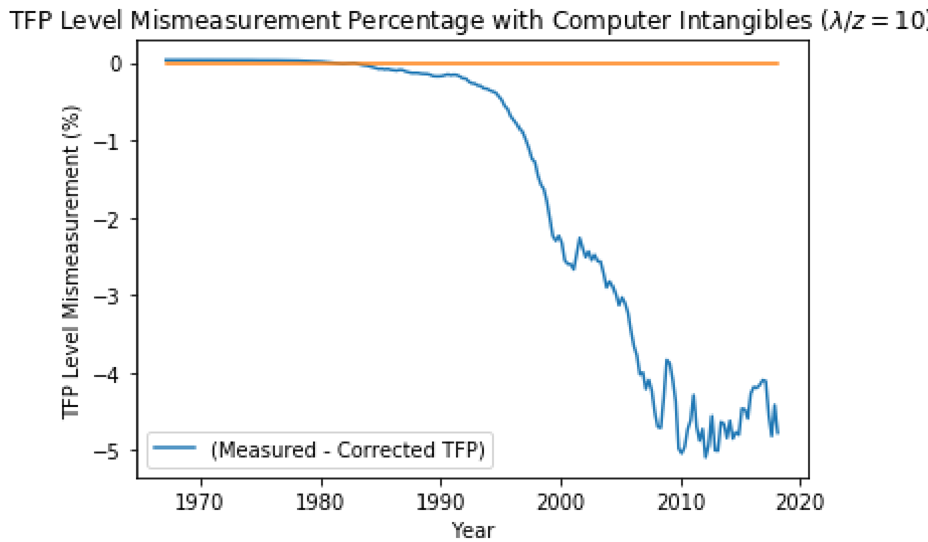
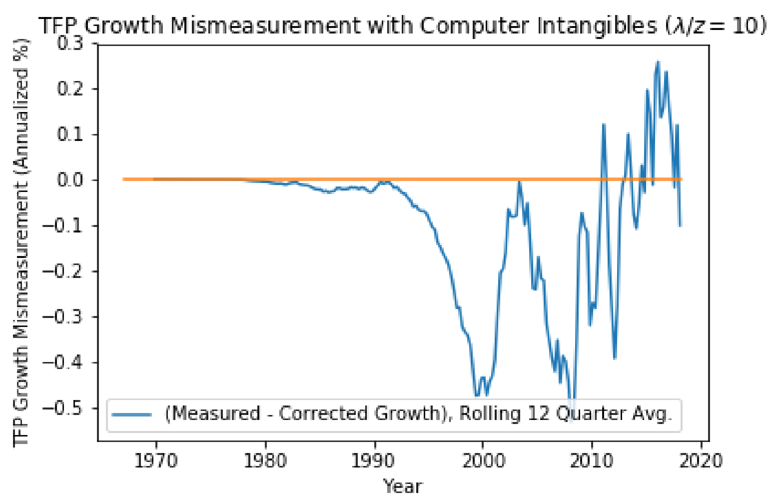


Figure 14: Total Factor Productivity Levels Corrected for Different Computer Intangible Multipliers

The quantitative patterns for hardware are different than what we found for software. First, the accumulated mismeasurement due to hardware-correlated intangibles is much more modest. Adjusted TFP at the end of 2016 is 4.4% higher than the measured series—a considerably smaller gap than that associated with software-related intangibles. Second, and interestingly, the recent slowdown in the rate of hardware investment has actually caused a small *overstatement* of productivity growth, and as a result, has started to bring the level difference back toward measured TFP. The reversal started following the dot-com bust, reverted as computer hardware investment rebounded in the following years, and then reversed again at the start of the Great Recession. Figures 15 and 16 show the magnitudes of the deviations in TFP growth and levels between the measured and corrected series. In growth terms, the latter (overstatement) part of the Productivity J-curve (at least that component tied to hardware investment) appears to have begun, and in level terms productivity understatement has stabilized.[18] Assuming an intangible multiplier on hardware capital of 10, the growth overestimate was about 0.2% in 2016.

[18] Figure 15 is a trailing three-year average of quarterly annualized total factor productivity growth estimates.





Figures 15 (top) and 16 (bottom): Computer Hardware-related TFP Mismeasurement in Levels and Growth Terms (respectively)

VII. Can Intangible Capital Outputs Explain the Productivity Slowdown?

We now take the above estimates of the TFP adjustments due to intangible capital related to R&D, software, and computer hardware to ask if the measured productivity slowdown after 2004 (see, e.g., Gordon 2015; Summers 2015; Syverson 2017) can be accounted for by such intangibles. Some role seems plausible; while our calculations above imply intangibles related to software and (to a lesser extent) hardware started having a noticeable influence on true TFP in the 1990s, they also made contributions in more recent periods. If these are larger than their earlier influence, they would in part explain the measured productivity slowdown.

The slowdown in measured annual TFP growth from 1995-2004 to 2005-2017 was approximately 1.23% per year.[19] Had measured TFP grown since 2005 at the same rate it did from 1995-2004, and holding labor and tangible capital inputs fixed at their observed levels, U.S. GDP at the end of 2017 would have been $3.46 trillion higher than it was.[20]

To see if intangible capital accumulation tied to R&D, software, and computer hardware investments can account for any of this shortfall, we use our calculated TFP growth adjustments above to construct an intangible-adjusted TFP series. As discussed above, this series is substantially higher than the measured values in the post-slowdown period. Adjusted annual TFP growth over 2005-2017 was 0.85%, up from the measured value of 0.40%. However, the adjusted series was also larger before the productivity slowdown, averaging 2.53% per year from 1995-2004, higher than the measured value of 1.63%. Thus the productivity slowdown also exists in the adjusted series. Indeed, at 1.68% per year it is larger than the measured slowdown of 1.23%.[21] Of course, this analysis assumes that the multiplier for intangibles—the amount of intangibles associated with each dollar of tangible investments—is constant throughout the period. If it is higher in recent periods, mismeasurement would be greater in recent periods, and vice-versa.

Note that the fact that intangibles, at least in the simplest formulation with a constant multiplier, do not explain the productivity slowdown (and actually somewhat deepen it) does not

[19] We calculate this as the difference between the average quarterly TFP growth values for 1995-2004 and 2005-2017, respectively. We then annualize this average difference.

[20] At the end of 2017, counterfactual TFP would be 1.235 (= 1.00407^52) times its level at the end of 2004, where 0.407% was average quarterly TFP growth over 1995-2004. Measured TFP was instead 1.052 times larger in 2017. Thus, assuming observed labor and capital inputs remain as observed, counterfactual GDP at the end of 2017 would be 1.174 (= 1.235/1.052) times larger than the observed value of $19.83 trillion. The difference, $3.46 trillion, is 17.4% of $19.83 trillion.

[21] For the adjusted series, counterfactual TFP is 1.388 (=1.00632^52) times its level at the end of 2004, where 0.632% was average quarterly adjusted TFP growth over 1995-2004. Measured TFP was 1.15 times larger in 2017 in adjusted terms.

imply that intangibles' influence on productivity and GDP is small. Adjusted TFP (again holding observed labor and tangible capital constant) is 15.9% higher than observed at the end of 2004, and 22% higher than observed at the end of 2017. To put it in other words, in addition to all the measured assets, including housing, property plant and equipment, and so on that the U.S. economy produced over the past several decades, it also produced trillions of dollars' worth of unmeasured intangible capital. It is just that the long-lived nature of intangibles' effects itself causes these upward adjustments to be differenced out when seeking to explain the slowdown.[22]

VIII. Conclusion

Our approach has shown how accounting for intangible investments correlated with measurable ones can meaningfully change estimates of productivity growth and dynamics. Both capital inputs and capital outputs are affected by intangibles. Productivity is underestimated in cases where the growth rate of investment (which contributes to output) exceeds the growth rate of capital services (inputs), and overestimated when the investment growth rate it lower. The first of these effects tends to dominate early in the capital accumulation cycle, when firms and organizations devote resources to building unmeasured intangible capital. The second effect dominates later, when these unmeasured assets generate capital services that increase measured output. Finally, when the capital accumulation reaches steady state, there is no longer any mismeasurement. These dynamics generate what we call the Productivity J-curve.

Because technological improvement often leads to the creation of new capital varieties and necessitates investment in intangible complements, the introduction of a new GPT often causes such a J-curve to occur. We show how this has been the case for IT-related capital in recent decades,

[22] Our empirical framework can capture firm-specific intangible investments. However, if there are intangibles built at the industry- or economy-wide levels (perhaps by governments or other organizations that can solve free-riding problems), our empirics will miss them even if they create a large J-curve..

for which our calculations suggest that trillions of dollars of intangibles output has been produced but not counted in the national income accounts. There is also some evidence that the phenomenon appears to have begun again, very recently, in AI-related intangible investments.

The mere presence of intangible correlate investment is not a guarantee of the existence of the Productivity J-curve. Although R&D investments are large and are associated with large intangibles, we find that mismeasurement related to R&D investments has a negligible effect on the estimation of productivity growth.[23] On the other hand, computer hardware, and to a greater extent software, have a large effect. The difference reflects the interaction of three quantities: the investment share of output of the asset type, the intangible correlate quantity and adjustment costs per unit of observable investment, and the difference between the growth rate of investment in the asset and the growth rate of capital services. In the case of R&D, the investment share is large and the intangible multiplier is historically larger than two. But, as a mature asset type, the difference between the growth rate of R&D investment and the growth rate of capital is not very large. Software, in contrast, has a meaningfully large investment share of output, has intangible multipliers close to 10, and the investment growth rate in software has often exceeded the growth of capital services overall.

By integrating aspects of Q-theory of investment and traditional growth accounting methods, we offer a means of adjusting the productivity statistics such that new, seemingly omnipresent GPTs might show up in the productivity statistics. Assuming that capital markets price corporate securities efficiently, then market value regressions can estimate the value of intangible correlates and adjustment costs per unit of observable capital. The forward-looking nature of market valuation means that lags in capital services would rationally be considered correctly in expectation. Of course, these multipliers reflect a risk-adjusted discounted expected value of the accumulated

[23] With a minor deviation present in the late 1990s and early 2000s.

asset stock which might to come to bear. The mismeasurement issues might accordingly be sensitive to differences in the timing of expected returns. Lower interest rates, for example, could encourage longer duration investments and therefore prolong the effects of the J-Curve. This investment timing component of productivity mismeasurement is left to future research.

The J-Curve method also suggests an indicator of whether or not a new technology is indeed a *general-purpose technology*. If measures of the investment in a given new technology fail to generate economically significant intangibles, that particular technology at that moment in time would not qualify as general-purpose. Equation (17) offers a framework to use firm-level estimates of technological capital (e.g. AI, IT, or robotics capital) to determine if productivity growth estimation is need adjustment in the presence of that new capital type. This framework also might inform whether or not intangible capital accounts for the wide differences between frontier and median productivity firms (Andrews, Criscuolo, and Gal 2015).

The Productivity J-curve explains why a productivity paradox can be both a recurrent and expected phenomenon when important new technologies are diffusing throughout the economy. Adjusting productive processes to take advantage of new types of capital requires the kind of investments the statistics miss. In future, after making appropriate adjustments accounting for the Productivity J-curve, we can see new technologies everywhere *including* the productivity statistics.

References

Acemoglu, Daron, and James A. Robinson. 2013. Why Nations Fail: The Origins of Power, Prosperity, and Poverty. Broadway Business.

Acemoglu, Daron (2002). Technical change, inequality, and the labor market. *Journal of economic literature*, 40(1), 7-72.

Aghion, Philippe, Benjamin F Jones, and C Jones. 2017. "Artificial Intelligence and Economic Growth." *National Bureau of Economic Research*, no. w23928.

Agrawal, Ajay, John McHale, and Alex Oettl. 2018. "Finding Needles in Haystacks: Artificial Intelligence and Recombinant Growth." In *Economics of Artificial Intelligence*. University of Chicago Press. http://www.nber.org/papers/w24541.

Allen, Robert C. 2009. "Engels' Pause: Technical Change, Capital Accumulation, and Inequality in the British Industrial Revolution." *Explorations in Economic History* 46 (4):418–35.

Andrei, Daniel, William Mann, and Nathalie Moyen. 2018. "Why Did the Q Theory of Investment Start Working?" *Working Paper*.

Andrews, Dan, Chiara Criscuolo, and Peter N Gal. 2015. "Frontier Firms, Technology Diffusion and Public Policy: Micro Evidence from OECD Countries." *Working Paper*, 39.

Barro, Robert. 1998. "Notes on Growth Accounting." *Journal of Economic Growth* 137 (June):119–37.

Basu, Susanto, and John G. Fernald. 1997. Returns to Scale in US Production: Estimates and Implications." *Journal of Political Economy* 105 (2): 249-283.

Basu, Susanto, John G. Fernald, Nicholas Oulton, and Sylaja Srinivasan. 2003. "The case of the missing productivity growth, or does information technology explain why productivity accelerated in the United States but not in the United Kingdom?." NBER *macroeconomics annual* 18: 9-63.

Bresnahan, Timothy. 2010. "General Purpose Technologies." *Handbook of the Economics of Innovation* 2 (1):761–91.

Bresnahan, Timothy F., and M. Trajtenberg. 1995. "General Purpose Technologies 'Engines of Growth'?" *Journal of Econometrics* 65 (1):83–108.

Bresnahan, Timothy F, Shane Greenstein, David Brownstone, and Ken Flamm. 1996. "Technical Progress and Co-Invention in Computing and in the Uses of Computers." *Brookings Papers on Economic Activity: Microeconomics*, 1–83.

Brynjolfsson, Erik, Felix Eggers, and Avinash Gannamaneni. 2018a. "Measuring Welfare with Massive Online Choice Experiments: A Brief Introduction." *American Economic Association: Papers and Proceedings* 108:473–76.

———. 2018b. "Using Massive Online Choice Experiments to Measure Changes in Well-Being." *NBER Working Paper Series*.

Brynjolfsson, Erik and Kristina McElheran. 2016. "The Rapid Adoption of Data-Driven Decision-Making." *American Economic Review: Papers and Proceedings* 106(5): 133-39.

Brynjolfsson, Erik and Lorin M. Hitt. 2000. "Beyond Computation: Information Technology, Organizational Transformation and Business Performance." *Journal of Economic Perspectives* (4):23-48.

Brynjolfsson, Erik, Lorin M. Hitt, and Shinkyu Yang. 2002. "Intangible Assets: Computers and Organizational Capital." *Brookings Papers on Economic Activity* (1):137–98.

Brynjolfsson, Erik, Daniel Rock, and Chad Syverson. 2017. "Artificial Intelligence and the Modern Productivity Paradox: A Clash of Expectations and Statistics." In *Economics of Artificial Intelligence*. University of Chicago Press.

Bughin, Jacques, Eric Hazan, Sree Ramaswamy, Michael Chui, Tera Allas, Peter Dahlstrom, Nicolas Henke, and Monica Trench. 2017. "Artificial Intelligence – The Next Digital Frontier?"

Artificial Intelligence, 80.
Byrne, David M., John G. Fernald, and Marshall B. Reinsdorf. 2016. "Does the United States Have a Productivity Slowdown or a Measurement Problem." *Finance and Economics Discussion Series* 2016 (017):1–74.
Byrne, David M., Stephen D. Oliner, and Daniel E. Sichel. 2013. "Is the Information Technology Revolution Over?" *SSRN Electronic Journal*.
Cockburn, Iain M, Rebecca Henderson, and Scott Stern. 2018. "The Impact of Artificial Intelligence on Innovation: An Exploratory Analysis." *National Bureau of Economic Research* No. w24449 (September).
Corrado, Carol, Charles Hulten, and Daniel Sichel. 2009. "Intangible Capital and U.S. Economic Growth." *Review of Income and Wealth*.
Crafts, Nicholas. 2004. "Steam as a General Purpose Technology: A Growth Accounting Perspective." *Economic Journal* 114 (495):338–51.
Crouzet, Nicolas and Janice Eberly. 2018. "Understanding Weak Capital Investment: the Role of Market Concentration and Intangibles." Working Paper.
David, Paul A. 1990. "The Dynamo And The Computer: An Historical Perspective On T. *American Economic Review* 80 (2):355.
Eichengreen, Barry. 2015. "Secular Stagnation: The Long View." *American Economic Review* 105 (5):66–70.
Eisfeldt, Andrea L., and Dimitris Papanikolaou. 2013. "Organization Capital and the Cross-Section of Expected Returns." *Journal of Finance* 68 (4):1365–1406.
———. 2014. "The Value and Ownership of Intangible Capital." *American Economic Review* 104 (5):189–94.
Fernald, John. 2014. "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity." *Federal Reserve Bank of San Francisco*. http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.310.1766&rep=rep1&type=pdf.
Furman, Jason, and Robert Seamans. 2018. "AI and the Economy," 1–33. https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3186591.
Goolsbee, Austan D, and Peter J Klenow. 2018. "Internet Rising , Prices Falling: Measuring Inflation in a World of E-Commerce." *AEA Papers and Proceedings* 108:488–92.
Gordon, Robert J. 2015. "Secular Stagnation: A Supply-Side View." *American Economic Review* 105 (5):54–59.
Hall, Bronwyn H. 1993. "The Stock Market's Valuation of R&D Investment during the 1980's." *The American Economic Review* 83 (2):259–64.
———. 2006. "R&D, Productivity, and Market Value." *IFS Working Papers, Institute for Fiscal Studies (IFS) 06/23*.
Hall, R. E. 2004. "Measuring Factor Adjustment Costs." *The Quarterly Journal of Economics* 119 (3):899–927.
Hall, Robert E. 2001. "The Stock Market and Capital Accumulation." *The American Economic Review* 91 (5):1185–1202. http://www.jstor.org/stable/2677921%5Cnhttp://www.jstor.org/stable/pdfplus/2677921.pdf?acceptTC=true.
Hall, Robert E. 2000. "E-Capital: The Link between the Stock Market and the Labor Market in the 1990s." *Brookings Papers on Economic Activity* 2:73–118.
Haskel, Jonathan, and Stian Westlake. 2017. *Capitalism without Capital: The Rise of the Intangible Economy*. Princeton University Press.
Hayashi, Fumio. 1982. "Tobin's Marginal q and Average q: A Neoclassical Interpretation." *Econometrica* 50 (1):213–24.

Hayashi, Fumio, and Tohru Inoue. 1991. "The Relation Between Firm Growth and Q with Multiple Capital Goods: Theory and Evidence from Panel Data on Japanese Firms." *Econometrica* 59 (3):731–53.

Helpman, Elhanan & Trajtenberg, Manuel (1994). A time to sow and a time to reap: Growth based on general purpose technologies (No. w4854). National Bureau of Economic Research.

Henderson, Rebecca. 1993. "Underinvestment and Incompetence as Responses to Radical Innovation: Evidence from the Photolithographic Alignment Equipment Industry." *The RAND Journal of Economics* 24 (2):248–70.

Himel, Samuel, and Robert Seamans. 2017. "Artificial Intelligence, Incentives to Innovate, and Competition Policy." *Antitrust Chronicle* 1 (3).

Houthakker, Hendrik S. "The Pareto distribution and the Cobb-Douglas production function in activity analysis." *The Review of Economic Studies* 23, no. 1 (1955): 27-31.

Jorgenson, Dale Weldeau, and Kevin J. Stiroh. 2000. "Raising the Speed Limit: U.S. Economic Growth in the Information Age." *Brookings Papers on Economic Activity* 2000 (1):125–210.

Kogan, Leonid, Dimitris Papanikolaou, Amit Seru, and Noah Stoffman. 2017. "Technological Innovation, Resource Allocation, and Growth." *Quarterly Journal of Economics* 132 (2):665–712.

Lipsey, Richard, Kenneth Carlaw, and Clifford Bekar. 2006. "Economic Transformations: General Purpose Technologies and Long-Term Economic Growth." *The Economic History Review* 59 (4):881–82.

Lucas, Robert E. 1967. "Adjustment Costs and the Theory of Supply." *Journal of Political Economy* 75 (4, Part 1):321–34.

Marrano, Mauro Giorgio, Jonathan Haskel, and Gavin Wallis. 2009. "What Happened to the Knowledge Economy? ICT, Intangible Investment, and Britain's Productivity Record Revisited." *Review of Income and Wealth*.

Magee, Stephen P. 1973. "Currency Contracts, Pass Through and Devaluation," *Brookings Papers on Economic Activity*, Vol. 1, pp. 303-325.

Oliner, Stephen D., Daniel E. Sichel, and Kevin J. Stiroh. 2008. "Explaining a Productive Decade." *Journal of Policy Modeling* 30 (4):633–73.

Oliner, Stephen D, and Daniel E Sichel. 2000. "The Resurgence of Growth in the Late 1990s: Is Information Technology the Story?" *Journal of Economic Perspectives.*

Peters, Ryan H., and Lucian A. Taylor. 2017. "Intangible Capital and the Investment-q Relation." *Journal of Financial Economics* 123 (2):251–72.

Rose, Andrew K., and Janet L. Yellen. 1989. "Is There a J-Curve?" *Journal of Monetary Economics* 24 (1):53–68.

Rosenberg, Nathan, and Manuel Trajtenberg. 2004. "A General-Purpose Technology at Work: The Corliss Steam Engine in the Late-Nineteenth-Century United States." *The Journal of Economic History* 64 (1):61–99.

Saunders, Adam, and Erik Brynjolfsson. "Valuing Information Technology Related Intangible Assets." *Mis Quarterly* 40, no. 1 (2016).

Solow, R.M. 1957. "Technical Change and the Aggregate Production Function." *The Review of Economics and Statistics* 39 (3):312–20.

Solow, Robert. 1987. "We'd Better Watch Out." *New York Times Book Review*, (July 12).

Solow, Robert M. 1956. "A Contribution to the Theory of Economic Growth." *The Quarterly Journal of Economics* 70 (1):65.

Summers, Lawrence H. 2015. "Demand Side Secular Stagnation." In *American Economic Review*, 105:60–65.

Syverson, Chad. 2017. "Challenges To Mismeasurement Explanations for the U.S. Productivity Slowdown." *Journal of Economic Perspectives* 31 (2):165–86.

Tambe, Prasanna, Lorin M. Hitt, and Erik Brynjolfsson. 2012. "The Extroverted Firm: How External Information Practices Affect Innovation and Productivity." *Management Science* 58 (5):843–59.
Trajtenberg, Manuel. 2018. "AI as the Next GPT: A Political-Economy Perspective." 24245. Working Paper Series.
Wildasin, David E. 1984. "The q Theory of Investment with Many Capital Goods." *American Economic Review*.
Yang, S, and E Brynjolfsson. 2001. "Intangible Assets and Growth Accounting: Evidence from Computer Investments." *Unpublished Paper. MIT* 85 (61):28.