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What Drives Inflation? Lessons from Disaggregated Price Data

Elisa Rubbo

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Becker Friedman Institute
FOR ECONOMICS AT UCHICAGO

5757 S. University Ave.
Chicago, IL 60637
Main: 773.702.5599
bfi.uchicago.edu

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Abstract

The Covid pandemic disrupted supply chains and labor markets, with heterogeneous effects on demand and supply across industries. Meanwhile governments responded with unprecedented stimulus packages, and inflation increased to its highest values in 40 years. This paper investigates the contribution of aggregate monetary and fiscal policies to inflation compared to industry-specific disruptions. I argue that, in an economy where multiple industries and primary factors have heterogeneous supply curves, industry-specific shocks to inelastically supplied goods increase aggregate inflation beyond the control of monetary policy. Moreover, industry-specific and aggregate shocks have different effects on relative prices, which allows me to identify their respective contribution to aggregate inflation. For US consumer prices, I find that deflation and subsequent inflation in 2020 were due to industry-specific shocks, while since 2021 inflation is primarily driven by aggregate factors.

1 Introduction

During and after the Covid pandemic, there was widespread belief among policymakers and commentators that disruptions in industry-specific demand and supply played a crucial role in driving aggregate inflation. For instance, as the pandemic began, people refrained from activities like traveling and dining out due to lockdowns and health concerns. Simultaneously, there was an increase in demand for home office upgrades and improvements, shifting expenditure from services to manufacturing. Manufacturing firms, however, often operated below capacity to prevent contagion, while remote-friendly white-collar services remained largely unaffected. These industry-specific

*University of Chicago, Booth School of Business (email: elisarubbo@uchicago.edu). This paper benefited from conversations with David Baqaee, Gadi Barlevy, Larry Christiano, Marco Del Negro, Mishel Ghassibe, Sebnem Kalemli-Ozcan, Guido Lorenzoni, Ekaterina Peneva, Harald Uhlig, Ivan Werning, and seminar participants at the University of Chicago, the New York Fed, the European Central Bank, the Barcelona GSE, and the NBER Impulse and Propagation meetings. Gabriele Romano and Maria Cristina Niero provided outstanding research assistance.

disruptions resulted in significant relative price changes. But why would there be a connection between relative price changes and aggregate inflation?

The theoretical framework most commonly used to study inflation and monetary policy relies on an aggregated view of production, with a single representative industry, and hence it is ill-suited to explain this connection. This paper enhances the New Keynesian framework by introducing a detailed model of production that incorporates multiple industries and primary factors with heterogeneous supply curves. This modification enables us to examine the link between industry-specific shocks, relative price changes, and aggregate inflation. The inclusion of multiple primary factors is a novel aspect compared to earlier New Keynesian models with multiple industries (Carvalho (2006); Nakamura and Steinsson (2010); Pasten et al. (2019); LaO and Tahbaz-Salehi (2019); Rubbo (2023b)). This refinement is essential for modeling supply-chain bottlenecks, a prominent feature of the Covid period.

Within this comprehensive framework, the paper establishes necessary and sufficient conditions for changes in relative supply and demand across industries to impact aggregate inflation. It also elaborates a method to distinguish the component of inflation driven by aggregate shocks (such as aggregate productivity changes, stimulus policies, "greedflation," etc.) from the component driven by industry-specific demand and supply shocks.

To build intuition, in Section 2 I start by illustrating supply bottlenecks through a stylized two-sector example. Bottlenecks are defined as situations where there is a decline in relative productivity or an increase in relative demand for inelastically supplied goods. The supply of certain goods may be inelastic due to the use of inelastic primary factors or due to decreasing returns to scale. Inelastic industries struggle to compensate for high demand or low productivity by using primary factors more intensively. As expenditures shift towards inelastically supplied goods and factors, aggregate supply becomes more inelastic and the efficient aggregate output declines. Moreover, in a sticky-price environment bottlenecks cause an inflation-output tradeoff (manifesting as an endogenous cost-push shock in the aggregate Phillips curve). In fact, with nominal rigidities the demand for inelastically supplied goods and factors increases even more than in a flex-price economy, because their relative price does not increase fast enough. This relative demand distortion makes the aggregate supply even more inelastic. For example, the manufacturing sector is more capital intensive than the service sector. Since capital is more inelastically supplied than labor, manufacturing industries have a steeper supply curve compared to services. Thus a shift in expenditures towards manufacturing and away from services can cause a bottleneck.

The following Sections 3 and 4 formalize the role of heterogeneous supply curves in a general environment, by linking micro-level supply and demand elasticities and price adjustment frequencies with industry-level Phillips curves. The Phillips curves isolate two factors contributing to inflation: the output gap[1] and relative demand or supply shocks affecting prices even when output is at potential. A sufficient statistic for these industry-specific shocks are the wedges $\boldsymbol{\chi}$ between lagged sectoral relative prices and efficient relative prices. The key insight is that relative price wedges affect aggregate inflation, beyond the aggregate output gap. That is, when inelastic industries face a negative price wedge – meaning that they would like to increase their price – the demand for inelastically

[1]The output gap is defined as the difference between realized aggregate output and the potential aggregate output that would prevail in a hypothetical flex-price economy exposed to the same shocks.

supplied factors is inefficiently high and there is positive inflation even under a zero output gap.

In practice it is difficult to identify the components of aggregate inflation driven by the output gap vs relative demand and supply shocks, because we do not observe the aggregate output gap nor the relative price wedges $\boldsymbol{\chi}$. To address the issue, in Section 5 I show that the aggregate output gap and industry-specific demand and supply shocks have different effects on relative prices. Levering up on this observation, I construct a weighted-average inflation rate – the divine coincidence index – that does not depend on relative price wedges, thereby providing an inflation proxy for the aggregate output gap. This result extends Rubbo (2023b) to an economy with multiple primary factors, highlighting the need to discount inflation in inelastically supplied industries as well as in flex-price industries.

The component of any inflation index attributable to the output gap can be obtained by rescaling the divine coincidence index according to the relevant Phillips curve slope. The residual is driven by relative price wedges. With only price data, one cannot further distinguish between the underlying drivers of the two inflation components. For example, the output gap could be driven by pure monetary shocks or by TFP shocks that are not accommodated by monetary policy. Likewise, price wedges reflect a combination of various industry-specific demand, preference, technology and markup shocks. Teasing those apart would require high-frequency disaggregated output or expenditure data, beyond the scope of this paper.

Section 6 provides a decomposition of US consumer prices between January 2020 and December 2022. In 2020 consumer price inflation was primarily driven by relative demand and supply shocks. The same is true for measures of core inflation, such as the core PCE. This period saw inflation concentrated in flex-priced, capital-intensive "bottleneck" sectors, while wage inflation remained subdued, suggesting that aggregate demand was not the primary driver. Conversely, from 2021 onwards inflation became broad-based and wage inflation started to increase, and as a result the aggregate component closely aligns with the core PCE.

1.1 Related literature

Ball and Mankiw (1995) first suggested that large relative price movements can be associated with high aggregate inflation, showing a positive empirical correlation between aggregate inflation and the right skewness of the distribution of sectoral price adjustments. Ruge-Murcia et al. (2022) confirmed that a similar empirical relationship remains valid in recent years. They also argue that both low inflation between 2012-2019 and high inflation in the first months of the Covid pandemic were driven by a small number of sectors, while inflation has become broad-based since the second half of 2021. In comparison to these papers, I provide a formal theory of when and why relative price movements matter for aggregate inflation.

In other related work, Shapiro (2020) and Shapiro (2022) classifies PCE categories as demand- versus supply-driven depending on whether quantities and prices change in the same or opposite directions, finding that demand and supply effects are equally important to explain the recent inflation episode. Ball et al. (2021) show that core inflation

– as measured by the weighted median inflation rate – was initially low, and the increase in prices until 2021 was driven by inflation in non-core sectors. Only later inflation propagated to other sectors. I model the propagation of real and monetary shocks in a multi-sector economy, and hence I can properly account for the fact that demand-like shocks in one sector (i.e. prices and quantities change in the same direction) could actually originate from a supply shock in some customer sector (and vice versa). Moreover, my theory provides a rationale for why aggregate inflation indices should exclude sectors which experience disproportionately large price changes.

Closer to my work, Baqaee and Farhi (2020a) also provide a decomposition between supply- and demand-driven inflation, based on a model with multiple industries and primary factors. In Baqaee and Farhi (2020a) primary factors have vertical supply curves and wages are downward rigid. This can cause labor rationing in some sectors, which in turn creates shortages in the customer sectors, thereby increasing inflation. They identify demand and supply shocks by matching changes in employment and changes in nominal spending by sector. di Giovanni et al. (2022) extend the framework in Baqaee and Farhi (2020a) to an open economy and a longer time period.

Relatedly, Comin et al. (2023) consider a quantitative two-sector environment where industries move along a non-linear supply curve. Shocks that trigger binding capacity constraint push firms toward the steep part of the supply curve and generate inflation. They use changes in prices and quantities to identify binding capacity constraints. I instead study an environment where supply curves are linear but have different slopes across industries. I show that shocks which increase the demand for industries with steeper supply curves are inflationary, and my identification strategy is entirely based on relative price movements.

Importantly, Baqaee and Farhi (2020a), di Giovanni et al. (2022) and Comin et al. (2023) define aggregate demand shocks as changes in interest rates or in the discount factor which cause aggregate nominal GDP to deviate from the (pre-pandemic) steady-state. In contrast, I define aggregate demand shocks relative to an economy with flexible prices, which faces the same pandemic shock as the actual sticky-price economy. Accordingly, I estimate a much smaller role for aggregate demand in the early phases of the pandemic.

While in this paper I focus on the contemporaneous response of inflation to industry-specific demand and supply shocks, Werning and Lorenzoni (2023) study the dynamic response of real wages to aggregate demand shocks in an economy with decreasing returns to scale. Their setting is isomorphic to a constant-returns production function with an inelastic primary factor, as in the bottleneck model of Section 2 below.

2 A simple model of bottlenecks

In this Section I demonstrate the impact of bottlenecks using a simplified model with two industries and two primary factors. I model supply bottlenecks as instances of declining relative productivity or rising relative demand for inelastically supplied goods. From a short-run perspective, supply is inelastic for industries that rely on inelastic primary factors (such as machinery or land) or face decreasing returns to scale. As it is well known, one can model decreasing returns by adding industry-specific fixed factors into a constant-returns-to-scale production function. Therefore I assume constant returns to scale going forward.

Essentially, inelastic sectors are unable to compensate for increased demand or reduced productivity by using primary factors more intensively. This leads to two distinct negative effects. First — even with constant aggregate demand, productivity, and factor supply — the aggregate potential output decreases when inelastic sectors face higher relative demand, lower relative productivity, or lower relative factor supply. Second, these same shocks create an inflation-output tradeoff, where inflation increases even if monetary policy maintains output at potential.

Beyond its implications for aggregate inflation, the model has important implications for the dynamics of real wages. Assuming that labor is more elastically supplied than capital, either "bottleneck shocks" or increases in aggregate demand will lower wages relative to the price of capital. Consequently, real wages decline even when firms' desired markups remain constant.[2]

Simplified environment Consider an economy with two sectors (manufacturing and services) and two primary factors (labor and capital). There is a representative household, with per-period preferences over consumption (C) and labor supply (L) given by

$$U_t = C - \frac{L^{1+\varphi_L}}{1+\varphi_L}$$

The consumption bundle has equal expenditure shares on manufacturing (M) and services (S), with substitution elasticity θ:

$$C = \left(C_M^{\frac{\theta-1}{\theta}} + C_S^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$$

Capital is a fixed endowment, with variable utilization. The utilization cost is in units of the consumption bundle (see Section 3.1.2 for a fully-specified model of capital utilization).

The service sector is more labor intensive. For simplicity, I assume that the service sector uses only labor in production, while the good sector uses only capital, with constant returns to scale. Labor is more elastically supplied than capital, with inverse Frisch elasticities $\varphi_L < \varphi_K$.

Sectors face productivity shocks $\log A_M$ and $\log A_S$, consumers face relative preference shocks $d\log b_M = -d\log b_S$ (where $d\log \mathbf{b}$ denotes the change in expenditure shares at constant prices), and primary factors face supply shocks $\mathcal{E}_L^{\varphi_L}$ and $\mathcal{E}_K^{\varphi_K}$.

I define a bottleneck as a shock that inefficiently shifts expenditures towards the manufacturing sector. Below I show that bottlenecks can be caused by a decline in capital supply relative to labor ($\log(\mathcal{E}_K) < \log(\mathcal{E}_L)$), or by an increase in the relative demand for manufacturing ($d\log b_S > d\log b_M$), or by a decline in the productivity of manufacturing relative to services ($\log A_M < \log A_S$).

[2] Werning and Lorenzoni (2023) make a similar point.

Before illustrating the implications of bottlenecks for the inflation-output tradeoff, I discuss their effect on potential output.

Potential output The potential output y^{nat} is defined as the aggregate output that would prevail in an economy with flexible prices. I start by expressing potential output in our example economy a function of primitive shocks $\log \mathbf{A}$, $d \log \mathbf{b}$ and $\boldsymbol{\mathcal{E}}$.

To compute y^{nat} let's derive the equilibrium equations of this simplified model, expressed in log-changes with respect to the steady-state.[3]

For any given vector $\mathbf{x}$, denote by $\bar{x}$ the equally weighted average of its components: $\bar{x} \equiv \frac{1}{2}(x_1 + x_2)$. Log-changes in factor supply and factor prices are related through the supply equations

$$
\begin{aligned}
w_L &= \varphi_L (\ell_L - \epsilon_L) + \bar{p} \\
w_K &= \varphi_K (\ell_K - \epsilon_K) + \bar{p}
\end{aligned}
\tag{1}
$$

where ℓ_L and ℓ_K are log-deviations in the employment of labor and capital from steady-state, w_L and w_K are log-deviations of factor prices, $\bar{p}$ is the equally weighted average deviation of good and service prices, and $\epsilon_h \equiv \log(\mathcal{E}_h)$ for $h = K, L$. Importantly, the supply curve for capital in (1) has a steeper slope ($\varphi_K > \varphi_L$).

Changes in relative factor demand and relative factor prices are related through the demand equations

$$
\ell_L - \ell_K = -\theta (p_S - p_M) + [(d \log b_S - \log A_S) - (d \log b_M - \log A_M)] \tag{2}
$$

Assuming constant unit markups, prices must change proportionately to marginal costs:

$$
\begin{pmatrix} p_S \\ p_M \end{pmatrix} = \begin{pmatrix} w_L \\ w_K \end{pmatrix} - \begin{pmatrix} \log A_S \\ \log A_M \end{pmatrix} \tag{3}
$$

Combining the supply, demand and pricing equations allows us to relate changes in the natural aggregate output with changes in productivity, relative demand, and factor supply:

$$
y^{nat} = \bar{\ell}^{nat} = \frac{1}{\bar{\varphi}\left[1 - \frac{1}{4}\left(\frac{\varphi_L - \varphi_K}{\bar{\varphi}}\right)^2 \frac{\theta\bar{\varphi}}{1+\theta\bar{\varphi}}\right]} \left[\bar{\epsilon} + \bar{A} + \frac{Cov(\varphi, (1-\theta)\log \mathbf{A} + \theta\epsilon - d\log b)}{1 + \theta\bar{\varphi}}\right] \tag{4}
$$

In equation (4) the natural level of output is lower when the manufacturing sector faces higher relative demand ($Cov(\varphi, d \log b) > 0$), or when the supply of capital exogenously declines ($Cov(\varphi, \epsilon) < 0$). Moreover, the natural level of output is also lower when manufacturing productivity declines and manufacturing and services are comple-

[3] These equations are different from (19)-(21) below, which are expressed in gaps with respect to the natural allocation that we are computing in this paragraph. Rubbo (2024) derives equations (1)-(3) in a general setting.

mentary ($Cov(\boldsymbol{\varphi}, \log \mathbf{A}) < 0$ and $\theta < 1$). Hence equation (4) illustrates how shocks that shift expenditure towards inelastic sectors reduce the potential output, because these sectors cannot compensate for higher demand or lower productivity by utilizing their primary factors more intensively.

The next paragraph shows that these same shocks also create an inflation-output tradeoff. To set the stage, let's solve for the efficient relative price changes:

$$p_S^{nat} - p_M^{nat} = -\frac{1+\frac{1}{\varphi}}{\theta+\frac{1}{\varphi}}(\log A_S - \log A_M) + \frac{d\log b_S - d\log b_M}{\theta+\frac{1}{\varphi}} - \frac{\epsilon_L - \epsilon_K}{\theta+\frac{1}{\varphi}} - \frac{\left(\frac{1}{\varphi_L} - \frac{1}{\varphi_K}\right)}{\theta+\frac{1}{\varphi}}\bar{A} \tag{5}$$

As expected, relative prices increase with relative demand and decrease with relative productivity or relative factor supply. Note that the natural relative prices also respond to aggregate productivity, not just to relative productivity. As aggregate productivity increases, the relative supply of labor (the more elastic factor) increases, while the relative price of capital (the less elastic factor) increases. Consequently, the price of manufacturing becomes relatively higher compared to services.

Inflation-output tradeoff Suppose that output prices are flexible, while wages and capital prices are sticky, with the same adjustment probability δ. In section 4 I show that the average inflation rate $\bar{\pi}$ is given by

$$\bar{\pi} = \kappa^Y \tilde{y} - \frac{\theta\delta\bar{\varphi}}{1+\theta\delta\bar{\varphi}} Cov\left(\frac{\varphi}{\bar{\varphi}}, \log A + \epsilon - d\log b\right) \tag{6}$$

where $\kappa^Y = \frac{\delta}{1-\delta}\bar{\varphi}\left[1 - \frac{1}{4}\frac{\theta\delta\bar{\varphi}}{1+\delta\theta\bar{\varphi}}\left(\frac{\varphi_L - \varphi_K}{\bar{\varphi}}\right)^2\right]$ is the slope of the aggregate Phillips curve, and the output gap $\tilde{y} \equiv y - y^{nat} = \bar{\ell} - y^{nat}$ is the difference between aggregate output in the sticky price and in the flex-price economy.

Relative prices instead evolve according to

$$\pi_S - \pi_M = -\frac{\delta}{1-\delta}\frac{\varphi_K - \varphi_L}{1+\theta\delta\bar{\varphi}}\tilde{y} - \delta\frac{1+\theta\bar{\varphi}}{1+\theta\delta\bar{\varphi}}\left[(\log A_S + \epsilon_S - \log b_S) - (\log A_M + \epsilon_M - \log b_M)\right] \tag{7}$$

If labor and capital had the same supply elasticity (so that $\varphi_L - \varphi_K = 0$ and $Cov\left(\frac{\varphi}{\bar{\varphi}}, \log A + \epsilon - d\log b\right) = 0$), aggregate inflation $\bar{\pi}$ would only depend on the aggregate output gap, while relative prices would only depend on relative demand and supply shocks. If instead capital and labor have different supply elasticities, aggregate inflation depends on both the aggregate output gap and industry-specific shocks.

Specifically, there is a worse inflation-output tradeoff whenever the relative supply of capital declines, the relative productivity of the manufacturing sector declines, or the demand for manufacturing increases. Note that all these shocks lead to an inefficiently high demand for capital relative to labor. To see this, let's solve for the relative employment gaps of labor and capital. I refer to these as "bottleneck shocks".

When $\tilde{y} = 0$, employment gaps are given by

$$\tilde{\ell}_K = -\tilde{\ell}_L = \frac{1}{2}\frac{\theta\bar{\varphi}}{1+\theta\bar{\varphi}\delta}(1-\delta)\left[(\log A_S + \epsilon_S - \log b_S) - (\log A_M + \epsilon_M - \log b_M)\right] \tag{8}$$

In equation (8), relative employment distortions are proportional to the degree of nominal rigidity $1-\delta$. When prices are sticky the relative price of manufacturing fails to increase rapidly enough, leading to inefficiently high demand for manufacturing and the inelastically supplied capital that it utilizes. Relative employment distortions are also proportional to the demand elasticity θ, which governs how much consumers substitute away from manufacturing and towards services as the price of manufacturing rises.

Note that this happens even if the central bank implements a zero aggregate output gap. Closing the aggregate gap ensures a zero average employment gap, but it does not guarantee that all factor-level employment gaps are closed. Consequently, the economy produces the efficient aggregate output but employs an inefficient mix of primary factors.

Although employment gaps average out to zero, wage gaps do not. Specifically, we have $\ell_K > 0 \iff \tilde{w}_K > -\tilde{w}_L$. Capital is less elastic than labor, causing the price of capital to increase more than wages decline in response to demand distortions. This leads to higher average factor prices and hence higher average final prices for a given aggregate output, giving rise to an inflation-output tradeoff.

Implications for real wages Real wages are given by

$$\pi_{w_L} - \bar{\pi} = -\frac{1}{2}\left[\frac{\delta}{1-\delta}\frac{\varphi_K - \varphi_L}{1+\theta\delta\bar{\varphi}}\tilde{y} + \delta\frac{1+\theta\bar{\varphi}}{1+\theta\delta\bar{\varphi}}\left[(\epsilon_S - \log b_S) - (\epsilon_M - \log b_M)\right] + \frac{1-\delta}{1+\theta\delta\bar{\varphi}}(\log A_S - \log A_M)\right] \tag{9}$$

Equation (9) indicates that real wages decrease with a positive output gap ($\tilde{y} > 0$), or with bottleneck shocks ($d\log b_M > 0$, $\epsilon_S > \epsilon_M$ or $\log A_S > \log A_M$). Both labor and capital demand increase with an increase in aggregate demand ($\tilde{y} > 0$). However this triggers an asymmetric response in prices and quantities. Labor is more elastically supplied, hence its relative employment increases while its relative price declines. Equation (9) shows that wages decline relative to consumer prices as well.

Similarly, bottleneck shocks also reduce real wages. Whether it's a preference change favoring manufacturing over services or a decline in capital supply relative to labor, both shocks directly increase the relative price of capital. The role of productivity is more subtle. In equation (9), nominal rigidities ($1-\delta > 0$) result in declining real wages as the relative productivity of manufacturing diminishes ($\log A_S - \log A_M > 0$). This occurs because the relative price of manufacturing does not rise enough in the short run, maintaining an excessively high demand for capital, thereby driving up the price of capital and depressing real wages.

Equation (9) highlights that bottlenecks have the same effect on real wages as an increase in markups. Consequently, if changes in the price of capital are not precisely factored into marginal cost calculations, bottlenecks might be erroneously interpreted as "greedflation". In practice, measuring the price of capital is notoriously challenging (Karabarbounis and Neiman (2014)). Some components of the user cost of capital, such as changes in the shadow value of land and other fixed assets, are strongly affected by bottleneck shocks. Yet they are challenging to measure accurately, particularly when these assets are firm-specific and infrequently traded.

Filtering out the effect of bottlenecks on aggregate inflation In equation (7), bottlenecks result in higher relative prices of inelastic sectors (an increase in the price of manufacturing vs services in our example). A natural question arises: can one infer the effect of bottlenecks on inflation from the covariance between relative price changes and measures of industry-level supply elasticity? In Section 5 I show that this is indeed possible. Here I illustrate the concept within our simplified framework.

Combining equations (6) and (7), is easy to verify that the average inflation rate

$$\pi^{DC} \equiv \bar{\pi} - \frac{\theta\bar{\varphi}}{1+\theta\bar{\varphi}} Cov\left(\frac{\boldsymbol{\varphi}}{\bar{\varphi}}, \boldsymbol{\pi}\right) = \frac{\delta}{1-\delta}\bar{\varphi}\left[1 - \frac{1}{4}\frac{\theta\bar{\varphi}}{1+\theta\bar{\varphi}}\left(\frac{\varphi_L - \varphi_K}{\bar{\varphi}}\right)^2\right]\tilde{y}$$

only depends on the aggregate output gap. The inflation rate π^{DC} is lower than consumer price inflation when the relative price of manufacturing rises, indicating an increased demand for manufacturing or a decline in its productivity. Put differently, π^{DC} discounts the sector with a steeper supply curve, as prices in this sector are more responsive to shocks. With this adjustment, π^{DC} offers a proxy for the component of inflation driven by the aggregate output gap.

Remark 1. The simplified model in this section and the broader framework introduced in Section 3 both rely on a first-order approximation where factor supply elasticities remain locally constant. Hence factor supply curves have constant – albeit possibly heterogeneous – slopes. Shocks leading to a positive demand wedge for factors with steep supply curves create an inflation-output tradeoff. In practice, real-world supply elasticities may not remain constant and the supply slope might increase with employment. In the presence of variable supply elasticities, any large shock to relative productivity, demand, or factor supply across industries would induce a bottleneck through a non-linear effect, even if starting with ex-ante identical supply elasticities. That is, any large industry-specific shock would reduce the natural output y^{nat} and generate an inflation-output tradeoff. While in the linear approximation the inflation-output tradeoff depends on the covariance between factor supply elasticities and inflation rates, in a non-linear environment it would be associated with the variance of sectoral inflation rates.

3 Environment

This section sets up a New Keynesian model with monopolistic competition and sticky prices, incorporating multiple heterogeneous industries and primary factors. Sections 3.1 through 3.3 outline the assumptions about preferences, production and policy instruments, and the optimality conditions for consumers and producers. Section 3.4 defines the general equilibrium, and sections 3.5 and 3.6 describe the log-linearized model.

I allow for many dimensions of heterogeneity across households and industries, and a broad range of shocks. As a significant contribution, Corollary 1 narrows down the relevant dimensions of heterogeneity for the impact of sectoral shocks into aggregate inflation. It establishes that industry-specific shocks affect aggregate inflation only when good and factor prices have heterogeneous pass-through into aggregate prices, or when primary factors have heterogeneous supply elasticities. Furthermore, Proposition 1 demonstrates that the impact of any shock on inflation can be summarized by two sufficient statistics, capturing the response of aggregate output and relative prices in a counterfactual economy without nominal rigidities.

3.1 Final users and primary factors

The economy has a set of N_w household types, denoted by $\mathcal{N}_w$, and a set of N_f capital types, denoted by $\mathcal{N}_f$. Each household type has a corresponding consumer, and each capital type has a corresponding investment (or utilization) producer. Together the consumers, the investment producers, and the government form the set of final users. Likewise each household supplies a differentiated type of labor, and each capital type provides a distinct flow of services (the markets for labor and capital are fully segmented across types). The various labor and capital types form the set of primary factors. Final users are indexed by h or $f \in \{1, ..., N_w + N_f + 1\}$, and, with some abuse of notation, primary factors are also indexed by h or $f \in \{1, ..., N_w + N_f\}$. The price of factor h is W_h, and the quantity supplied is L_h.

Remark 2. Different types of labor and capital correspond to segmented factor markets. They need not be employed by different industries. For example, worker types could represent regions or occupations between which there are no worker flows at business cycle frequencies. Likewise, capital assets could represent equipment and structures which cannot be repurposed in the short run. Industries use a combination of all labor and capital types, in different proportions. Factor market segmentation will be key to understanding inflation dynamics.

3.1.1 Households

Preferences Household types are characterized by their preferences over consumption goods, their labor supply elasticity, and their ownership shares in industries and capital assets. Households face shocks to expenditure shares at constant prices (such as the shift from services to manufacturing during Covid), and shocks to their relative preference for leisure (for example, due to a desire to avoid Covid exposure).

Each household type $h \in \mathcal{N}_w$ has a representative agent, who supplies a distinct labor type and whose per-period preferences are described by the utility function

$$U_{ht} = \frac{C_{ht}^{1-\gamma_h}}{1-\gamma_h} - \frac{1}{\mathcal{E}_h^{\varphi_h}} \frac{L_{ht}^{1+\varphi_h}}{1+\varphi_h} \tag{10}$$

All households enjoy consumption (C) and dislike labor (L). The parameter γ_h governs both the intertemporal elasticity of substitution and the income effects in labor supply. The parameter φ_h is the inverse Frisch elasticity, which determines the slope of worker-specific labor supply curves. Households also face shocks $\mathcal{E}_h$ to their relative preference for leisure. Consumption aggregators $C_{ht} \equiv \mathcal{C}_{ht}\left(c_{1ht}, ..., c_{Nht}\right)$ are homothetic over the N goods produced in the economy, and can differ across households. Consumption preferences $\mathcal{C}_{ht}$ are time-varying, to account for changes in expenditure shares across goods at constant prices.

Consumers of type $h \in \mathcal{N}_w$ have elasticities of substitution

$$\theta_{ij}^h \equiv -\frac{d\log\frac{C_{hi}}{C_{hj}}}{d\log\frac{P_i}{P_J}}$$

between goods i and j.

Budget constraints Households maximize the present discounted value of per-period utility flows, with a common discount factor ρ, subject to type-specific budget constraints

$$P_{ht}^C C_{ht} \leq W_{ht}L_{ht} + \sum_{f\in\mathcal{N}_f} \mathcal{W}_{fh} W_f L_f + \sum_{i=1}^{N} \hat{\Xi}_{ih}\left(\Pi_{it} - \mathcal{T}_{it}\right) + T_{ht} \tag{11}$$

where P_{ht}^C is the price index implied by the consumption aggregator $\mathcal{C}_h$, W_{ht} is the nominal wage earned by workers of type h, and T_h is a lump-sum transfer received by type h.

Each household type h owns shares in all the N industries in the economy, as described by the ownership matrix $\hat{\Xi}$. The elements $\hat{\Xi}_{ih}$ denote the share of profits from industry i accruing to type-h agents. Profits Π_{it}, net of lump-sum taxes $\mathcal{T}_{it}$ paid by firms to the government, are allocated according to ownership shares. Agents also own shares of the N_f capital assets in the economy, as described by the matrix $\mathcal{W}$. The elements $\mathcal{W}_{fh}$ denote the share of income from asset f that is rebated to households of type h.

Remark 3. Below I assume that agents cannot borrow and lend between each other, so that the transfers $\{T_h\}_{h\in\mathcal{N}_w}$ are exogenously chosen by the government. The inflation decomposition in Section 5 would remain valid if these transfers were determined endogenously by the agents' borrowing and lending decisions.

Consumption-leisure tradeoff The optimal consumption-leisure tradeoff satisfies the first order condition

$$C_{ht}^{\gamma_h} L_{ht}^{\varphi_h} = \mathcal{E}_h^{\varphi_h} \frac{W_{ht}}{P_{ht}^C} \tag{12}$$

As explained in detail in Section 3.2 below, the flexible nominal wage W_{ht} is defined as the value W_{ht} which satisfies the consumption-leisure tradeoff (12) given consumption, labor demand, prices, and leisure shocks $\mathcal{E}_h$. Importantly,

W_{ht} has no direct data counterpart. The model counterpart of wages in the data is the sticky nominal wage paid by the firms, introduced in Section 3.2 below.

3.1.2 Capital utilization

I adopt a stylized model of capital utilization, with the objective to deliver capital supply curves with constant elasticity φ_f. Importantly, to simplify the analysis I abstract from the intertemporal dimension of the investment problem, and hence I abstract from the differential sensitivity of durable and non-durable goods to monetary policy.

Each capital asset f is produced by combining a fixed endowment ($\bar{K}_f$) with utilization U_f. At each period, the supply of asset f is given by (omitting time subscripts for legibility)

$$L_f = U_f \mathcal{E}_f^{\varphi_f} \bar{K}_f$$

where $\mathcal{E}_f$ is a shock to the endowment of capital. For convenience I assume that the utilization component U_f fully depreciates from one period to the next, while the endowment component $\bar{K}_f$ never depreciates.

In turn the production function for utilization is given by

$$U_f \equiv \left[(1+\varphi_f)\, I_f\right]^{\frac{1}{1+\varphi_f}}$$

where the utilization bundle I_f is produced with constant returns to scale, using as inputs a combination of primary factors (L_{fht}) and intermediate goods (X_{fit}):

$$I_{ft} = \mathcal{I}_{ft}(\{L_{fht}\}, \{X_{fit}\})$$

The production function $\mathcal{I}_{ft}$ is time-varying, to allow for changes in expenditure shares at constant prices. The elasticity of substitution between goods i and j in the utilization bundle of factor f is

$$\theta_{ij}^f \equiv -\frac{d \log \frac{X_{fi}}{X_{fj}}}{d \log \frac{P_i}{P_J}}$$

There are N_f industries that produce utilization for each asset type f, with marginal cost P_f^I, and sell the utilization bundle U_f to capital retailers. Retailers purchase capital endowments from the households, combine them with utilization, and sell capital services to the firms at a rental rate W_f in a perfectly competitive market. Capital retailers are owned by the agents in proportion to their ownership shares in the capital endowments, and rebate their profits accordingly.

Profit maximization by utilization producers yields the capital supply curves

$$U_f^{\varphi_f} = \frac{W_f}{P_f} \mathcal{E}_f^{\varphi_f} \bar{K}_f \tag{13}$$

Note that (13) implies a supply curve with constant elasticity φ_f. We will typically assume that capital assets are more inelastically supplied than labor ($\varphi_f > \varphi_h$ for $h \in \mathcal{N}_w$ and $f \in \mathcal{N}_f$). Profit maximization by utilization producers also implies that the payments $W_f L_f$ to factor f are divided between investment expenditures, given by

$$\frac{1}{1+\varphi_f} W_f L_f \tag{14}$$

and profits of the investment producers, given by

$$\frac{\varphi_f}{1+\varphi_f} W_f L_f \tag{15}$$

Profits in turn are rebated to households according to their ownership shares of investment firms, denoted by $\mathcal{Z}$. Hence the share of income from factor $f \in \mathcal{N}_f$ rebated to households of type $h \in \mathcal{N}_w$ is given by $\mathcal{W}_{fh} = \mathcal{Z}_{fh} \frac{\varphi_f}{1+\varphi_f}$, while the share of income from factor f that goes to investment into factor $f' \in \mathcal{N}_f$ is $\mathcal{W}_{ff'} = \frac{1}{1+\varphi_f}$ if $f = f'$ and $\mathcal{W}_{ff'} = 0$ otherwise.

Like the flexible wages in the consumption-leisure tradeoff (12), the flexible rental rates are defined as the value W_{ft} that satisfies the utilization equation (12) given capital demand and prices. These flexible rental rates have no direct data counterpart, while the model counterpart of rental rates measured in the data is the sticky rental rate paid by the firms, introduced in Section 3.2 below.

3.1.3 Government

The government implements transfers T_{ht} across consumers, and between itself and the consumers. We denote the total nominal transfer to the government by $G_t \geq 0$. The government's budget constraint is

$$-\sum_{h \in \mathcal{N}_w} T_{ht} = G_t \geq 0 \ \forall t$$

The government uses its revenue G to buy goods, with time-varying expenditure shares given by

$$\beta_{iGt} \equiv \frac{P_{it} G_{it}}{G_t}$$

3.2 Production

There are N good-producing industries in the economy (indexed by $i, j \in \{1, ..., N\}$). Within each industry there is a continuum of firms, producing differentiated varieties. I assume that each firm is owned by a single household type, and each household type h owns a share Ξ_{ih} of industry i.[4]

All firms z in industry i have the same constant returns to scale production function

$$Y_{izt} = A_{it} G_i(\{L_{ihzt}\}_{h \in \mathcal{N}_h \cup \mathcal{N}_f}, \{X_{ijzt}\}_{j=1}^{N}; \{\mathcal{X}_{ijt}\})$$

where L_{ihzt} is the quantity of primary factor h hired by firm z in industry i at time t, and X_{ijzt} is the quantity of intermediate input j used by the firm. The Hicks-neutral productivity shifter A_{it} is industry-specific, and varies over time. The matrix $\mathcal{X}$ denotes shocks to the production functions which determine exogenous changes in expenditure shares at constant prices:

$$\mathcal{X}_{ijt} \equiv d\log \omega_{ijt} \,|_{\text{const. prices}}$$

The elasticity of substitution between goods j and k in the production of good i is

$$\theta^i_{jk} \equiv -\frac{d\log \frac{X_{ij}}{X_{ik}}}{d\log \frac{P_j}{P_k}}$$

Customers (consumers and other producers) buy a CES bundle of sectoral varieties, with elasticity of substitution ϵ_i. The industry output is given by

$$Y_i = \left(\int Y_{if}^{\frac{\epsilon_i - 1}{\epsilon_i}} df \right)^{\frac{\epsilon_i}{\epsilon_i - 1}}$$

and the implied sectoral price index is

$$P_i = \left(\int P_{if}^{1-\epsilon_i} df \right)^{\frac{1}{1-\epsilon_i}}$$

The government provides time-varying proportional input subsidies, fully financed through lump-sum taxes on profits. I follow a standard practice in the literature, and assume that steady-state subsidies eliminate the markup distortions arising from monopolistic competition. Subsidies are such that at time t sector i pays a share

$$(1 - \tau_{it}) \frac{\epsilon_{it} - 1}{\epsilon_{it}}$$

of its actual marginal cost. With this notation, $\tau_i^* = 0$ in steady state guarantees efficiency.

All producers minimize costs given input prices. With constant returns to scale marginal costs are the same for all firms within a sector i, and they all use inputs in the same proportions. The marginal cost of sector i, denoted by

[4] This assumption only matters to define the discount factor in the price setting problem below, and it will be irrelevant in the log-linear approximation.

MC_i, is the solution of the cost minimization problem (omitting time subscripts for legibility)

$$MC_i = min_{\{X_{ij}\},\{L_{ih}\}} \sum_{h \in \mathcal{N}_h \cup \mathcal{N}_f} W_h L_{ih} + \sum_{j=1}^{N} P_j X_{ij} \;\; s.t. \; A_i G_i \left(\{L_{ih}\}_{h \in \mathcal{N}_h \cup \mathcal{N}_f}, \{X_{ij}\}_{j=1}^{N}; \{\mathcal{X}_{ij}\} \right) = 1 \tag{16}$$

Producers are subject to nominal rigidities, modeled à la Calvo. In every sector i and at each period t, only a randomly-selected fraction δ_i of the firms can update their price. The firms z who can adjust their price set it to maximize the present discounted value of future profits, conditional on being unable to update their price:

$$P^*_{izt} = \frac{\epsilon_{it}}{\epsilon_{it} - 1} \frac{\mathbb{E} \sum_s \left[SDF_{ht+s} \left(1 - \delta_i\right)^s Y_{izt+s}(P^*_{izt}) \left(1 - \tau_{it+s}\right) MC_{it+s} \right]}{\mathbb{E} \sum_s \left[SDF_{ht+s} \left(1 - \delta_i\right)^s Y_{izt+s}(P^*_{izt}) \right]} \tag{17}$$

where $SDF_{h,t+s} = \rho^s \frac{U_{ct+s}(C_{ht+s}.L_{ht+a})}{U_{ct}(C_{ht+s}.L_{ht+a})} \frac{P^c_{ht}}{P^c_{ht+s}}$ is the stochastic discount factor of the household h who owns the firm, and demand functions are given by $Y_{izt+s}(P_{zt}) = Y_{it+s} \left(\frac{P_{izt}}{P_{it+s}} \right)^{-\epsilon_{it}}$. The firms z who cannot adjust their price instead keep it unchanged ($P_{izt} = P_{izt-1}$), and must absorb any cost changes in their markup $\mathcal{M}_{izt}$.

Sticky factor prices To model sticky factor prices I assume that primary factors (workers and capital assets) are first purchased by marketplaces, who then sell their services to producers in the various industries. Each marketplace deals with only one primary factor.

Marketplaces are treated like any other industry. In particular there is a continuum of marketplaces for each factor, with fixed unit mass, facing Calvo-style price rigidities. The marketplaces rebate profits to final users according to their ownership shares of primary factors $\mathcal{W}$.

The markets for different primary factors are fully segmented, hence wages differ across primary factors. Nonetheless workers and capital suppliers can freely move across marketplaces, therefore all units of a given primary factor earn the same flexible wage. By contrast, different marketplaces who hire the same factor type might charge different prices, due to the Calvo friction. We assume that ownership of each marketplace is equally shared among the relevant final users, so that in practice all units of each primary factor h earn the same (sticky) wage, equal to the average price charged by h-specific marketplaces.

Retailers To streamline the notation, it is convenient to augment the set of industries with one retailer for each final user. Retailers assemble consumption, investment and government spending bundles and sell them to the relevant final user, to whom they also rebate their profits. Importantly, these fictitious retailers do not correspond to the actual retailer sector in the data. Their price is equal to the price index of the relevant final user. Without loss of generality, I model relative preference shocks for consumers, capital suppliers, or the government, as relative demand shocks in the corresponding retailers' production function.

Remark 4. Given the modeling of retailers and factor marketplaces, the overall number of industries in the economy is $\bar{N} = N + 2N_w + 2N_f + 1$ (the actual production sectors, the factor marketplaces for labor and capital, and the retailers for consumption, capital suppliers, and government spending).

3.2.1 Aggregation

In the next sections I relate changes in aggregate inflation, as measured by the GDP deflator, with changes in aggregate output, as measured by real GDP. Aggregate quantities are defined below. Changes in real GDP and the GDP deflator are measured relative to the initial steady-state, denoted by starred variables.

Definition 1. Nominal GDP is given by total final expenditures:

$$GDP = \sum_{h \in \mathcal{N}_w} P_h^C C_h + \sum_{f \in \mathcal{N}_f} P_f^I I_f + G_t$$

Definition 2. Infinitesimal changes in real GDP around the initial steady-state are denoted by $d\log Y_t$, and are equal to the share-weighted sum of changes in aggregate consumption, aggregate investment, and government spending:

$$d\log Y_t = \sum_{h \in \mathcal{N}_w} \frac{P_h^{*C} C_h^*}{GDP^*} d\log C_{ht} + \sum_{f \in \mathcal{N}_f} \frac{P_f^* I_f^*}{GDP^*} d\log I_{ft} + \frac{dG_t}{GDP^*}$$

Definition 3. Infinitesimal changes in the GDP deflator around the initial steady-state are denoted by $d\log P_t^Y$, and are equal to the share-weighted change in the price indices of the final users:

$$d\log P_t^Y = \sum_{h \in \mathcal{N}_w} \frac{P_h^{*C} C_h^*}{GDP^*} d\log P_{ht}^C + \sum_{f \in \mathcal{N}_f} \frac{P_f^{*C} I_f^*}{GDP^*} d\log P_{ft}^C + \frac{G}{GDP^*} d\log P_G$$

Remark 5. Around an efficient equilibrium, changes in real GDP equal the income-weighted sum of changes in the quantities of primary factors, plus the change in aggregate productivity:

$$d\log Y_t = \sum_{h \in \mathcal{N}_w \cup \mathcal{N}_f} \frac{W_h^* L_h^*}{\sum_{f \in \mathcal{N}_w \cup \mathcal{N}_f} W_f^* L_f^*} d\log L_{ht} + \sum_{i=1}^{N} \frac{P_i^* Y_i^*}{GDP^*} d\log A_{it}$$

3.3 Monetary policy

The Phillips curves in section 4 and the inflation decomposition in Section 5 do not depend on how monetary policy determines the output gap $\tilde{y}$. To derive the dynamic decomposition in Section 6 I assume that there is a representative households facing a standard Euler equation, and monetary policy sets the nominal interest rate following a Taylor rule (see Section 3.6 below).

3.4 Equilibrium

The equilibrium concept adapts the definition in Baqaee and Farhi (2020b) to account for the endogenous determination of markups given pricing frictions and shocks. Given sectoral markups, all markets must clear; and the evolution of markups must be consistent with Calvo pricing and the realization of monetary policy.

Definition 4. At each period t, for given sectoral probabilities of price adjustment δ_i, and all the shocks (monetary policy, transfers T_{ht}, productivity A_{it}, relative preference $\mathcal{X}_t$, factor supply $\mathcal{E}_{ft}$, demand elasticities ϵ_{it}), the general equilibrium is given by a vector of firm-level markups $\mathcal{M}_{ift}$, a vector of sectoral prices P_{it}, a vector of factor-specific nominal wages W_{ht}, a vector of factor supplies L_{ht}, a vector of sectoral outputs Y_{it}, a matrix of intermediate input quantities X_{ijt}, and a matrix of final demands C_{iht} such that: (i) a fraction δ_i of firms in each sector i charges the profit-maximizing price given by (17); (ii) the markup charged by adjusting firms is given by the ratio of the profit-maximizing price and marginal costs, while the markups of non-adjusting firms are such that their price remains constant; (iii) agents maximize utility subject to their budget constraint; (iv) producers in each sector i minimize costs and charge the relevant markup; and (v) markets for all goods and factors clear.

This equilibrium concept nests the standard one with flexible prices, which is obtained as a special case when $\delta_i = 1$ for every sector i.

3.5 Log-linearized model

The analysis of inflation in Sections 4 and 5 is based on a log-linear approximation of the model around an efficient equilibrium.

Assumption 1. *The model is approximated around an efficient equilibrium where all producers charge unit markups.*

In this Section I define the natural equilibrium, and introduce notation for the model variables and parameters. Vectors are boldfaced.

Steady-state, shocks, and natural equilibrium Steady-state variables are denoted with a star. I assume that steady-state sectoral productivities are equal to one ($\mathbf{A}^* = \mathbf{1}$), supply shifters are $\mathcal{E}_h^* = 1$ for all factors h, and nominal GDP is normalized to one ($M^* = \sum_h P_h^* C_h^* + \sum_f P_f^* I_f^* = 1$).

Table 1 lists all the exogenous shocks. Government spending G is not explicitly included in Table 1. This is without loss of generality, because changes in total spending and in its financing scheme can be represented as transfers $\mathbf{T}$ among final users, while changes in the sectoral allocation of spending can be represented as shocks $\mathcal{X}_{Gt}$ to the government's expenditure shares.

Definition 5 introduces the natural equilibrium, which is the benchmark against which output and employment gaps are computed. Gaps are log-deviations of endogenous variables with respect to the natural equilibrium. As

Industry-specific productivity	$\log \mathbf{A}_t \in \mathbb{R}^{\mathbb{N}}$
Factor supply	$\boldsymbol{\epsilon}_t \in \mathbb{R}^{N_w+N_f}, \ \epsilon_{ht} \equiv \log \mathcal{E}_{ht}$
Desired markups	$\boldsymbol{\mu}_t^D \in \mathbb{R}^N$
Input subsidies	$\boldsymbol{\tau}_t \in \mathbb{R}^N$
Expenditure shares at constant prices	$\mathcal{X}_t \in \mathbb{R}^{\bar{N}\times\bar{N}}$
Income transfers	$\mathbf{T} \in \mathbb{R}^{N_w+N_f}, \ \mathbf{T} = \left(T_1, ..., T_{N_w}, \mathbf{0}_{N_f}^T\right)^T$

Table 1: Exogenous shocks

it is usual, the natural benchmark is subject to the same productivity shocks as the actual economy. According to definition 5, the natural benchmark is also subject to the same transfers across final users (including government spending) and the same changes in desired markups. I discuss how this affects the inflation decomposition in Section 5 below.

Definition 5. The natural equilibrium is the equilibrium which would prevail in an economy with flexible prices, subject to the same shocks to productivity, desired markups, labor supply, relative demand, production subsidies, and lump-sum transfers as the sticky-price economy.

Definition 6. The aggregate output gap $\tilde{y}$ is the log difference between real GDP in the actual economy relative to the natural benchmark. Likewise, factor-specific employment gaps $\tilde{\boldsymbol{\ell}}$ are defined as the log-difference between employment in the actual economy and in the natural benchmark.

Endogenous variables Table 2 introduces the endogenous variables. I use lower case letters to denote log-deviations from the initial steady state, and tildes to denote gaps.

Employment gaps	$\tilde{\boldsymbol{\ell}}_t = \left(\tilde{\ell}_{1t} \quad ... \quad \tilde{\ell}_{N_w+N_f,t} \right)^T$
Factor price gaps	$\tilde{\mathbf{w}}_t = \left(\tilde{w}_{1t} \quad ... \quad \tilde{w}_{N_w+N_f,t} \right)^T$
Inflation rates	$\boldsymbol{\pi}_t = \left(\pi_{1t} \quad ... \quad \pi_{Nt} \right)^T, \ \pi_{it} \equiv p_{it} - p_{i,t-1}$

Table 2: Model variables

Remark 6. Table 2 includes employment and utilization gaps for primary factors, rather than for sectors. Knowing factor-level gaps is sufficient to characterize the evolution of inflation and output gaps in all sectors.

Income flows and income shares The income flows in the economy are determined by expenditure on primary factors, intermediate inputs, and final goods, income rebates from primary factors and firms, and transfers. I start by describing the notation for income rebates.

Factor income rebates are described by the matrix $\mathcal{W}$ introduced in Section 3.1.1. The shares of profits from industry i rebated to final user h are equal to the elements $\hat{\Xi}_{hi}$ of the ownership matrix $\hat{\Xi}$ introduced in Section 3.1.1. The assumptions about retailers and factor marketplaces imply $\hat{\Xi}_{hr_f} = \mathbb{I}(h = f)$ for $h, f \in \mathcal{N}_w \cup \mathcal{N}_f \cup \{G\}$ and $\hat{\Xi}_{hm_f} = \mathcal{W}_{hf}$ for $h, f \in \mathcal{N}_w \cup \mathcal{N}_f$, where I denoted by r_f the retailer specific to final user f and by m_f the marketplace specific to factor f. I assume that investment suppliers do not own shares in production industries ($\hat{\Xi}_{fi} = 0$ for $i = 1, ..., N,\ f \in \mathcal{N}_f$), and the government owns no shares in industries or primary factors.

In the derivations below, it is convenient to define a matrix $\Xi \in \mathbb{R}^{\bar{N}, N_w+N_f+1}$ which accounts for both income rebates and transfers, as follows:

$$\Xi \equiv \hat{\Xi} + \mathbf{T}^* \mathbf{1}^T$$

I now turn to expenditure shares. The matrix Ω encodes the producers' expenditure shares on intermediate inputs. Following Remark 4, I include factor marketplaces and retailers among the production sectors. I order the factor marketplaces first, then the good-producing sectors, and lastly the final consumption and investment retailers. The matrix Ω is then given by

$$\Omega \equiv \begin{bmatrix} \mathbb{O}_{N_w+N_f} & \mathbb{O}_{N_w+N_f,N} & \mathbb{O}_{N_w+N_f,N_w+N_f+1} \\ \left(\Omega_{ih} \equiv \frac{W_h L_{ih}}{MC_i Y_i}\right) & \left(\Omega_{ij} \equiv \frac{P_j X_{ij}}{MC_i Y_i}\right) & \mathbb{O}_{N,N_w+N_f+1} \\ \mathbb{O}_{N_w+N_f+1,N_w+N_f} & \left(\Omega_{hi} \equiv \frac{P_i C_{ih}}{P_h^C C_h}\right) & \mathbb{O}_{N_w+N_f+1} \end{bmatrix}$$

I also define the $\bar{N} \times N_w + N_f + 1$ matrix of final expenditure shares β, and the $\bar{N} \times N_w + N_f$ matrix of primary factor shares α, as follows.

$$\alpha = \begin{bmatrix} I_{N_w+N_f} \\ \mathbb{O}_{N,N_w+N_f} \\ \mathbb{O}_{N_w+N_f+1,N_w+N_f} \end{bmatrix}, \ \beta = \begin{bmatrix} \mathbb{O}_{N_w+N_f,N_w+N_f+1} \\ \mathbb{O}_{N,N_w+N_f+1} \\ I_{N_w+N_f+1} \end{bmatrix}$$

The matrix α has only one non-zero block, because only factor marketplaces hire primary factors directly. Moreover each marketplace only hires one primary factor, therefore the upper block of α is the identity matrix. Likewise, final consumers only purchase goods from their specific retailer, therefore β also has only one non-zero block, equal to the identity matrix.

Definition 7. The Leontief inverse is $\Psi \equiv (I - \Omega)^{-1}$. I denote the lowest $N_w + N_f + 1 \times \bar{N}$ block of the Leontief inverse by $\Psi_{C,:}$ (whose rows corresponds to the retailers), and I denote the leftmost $\bar{N} \times N_w + N_f$ block by $\Psi_{:,L}$ (whose columns correspond to the primary factors).

While the input-output matrix captures the *direct* elasticity of each sector i's cost to every other sector j's price, the Leontief inverse captures the *total* exposure, directly and indirectly through intermediate inputs. To see this, one can expand the Leontief inverse as a geometric sum, where the n-th term is the exposure of i to j through paths of length n:

$$\Psi_{ij} \equiv (I-\Omega)^{-1}_{ij} = I_{ij} + \Omega_{ij} + \left(\Omega^2\right)_{ij} + ...$$

Following the same logic, the lower block $\Psi_{C,:}$ of the Leontief inverse corresponds to the total content of good i in the bundle of final user h, while the leftmost block $\Psi_{:,L}$ denotes the total content of factor f in good i. In particular, the bottom-left block Ψ_{CL} is the total content of primary factors in final uses.

Remark 7. Under Assumption 1, the initial expenditure shares on inputs and primary factors must sum to one in every sector: $(\Omega + \alpha)\,\mathbf{1} = \mathbf{1}$. This implies $\Psi_L \mathbf{1} = \mathbf{1}$.[5]

Definition 8 and Lemma 1 relate the income shares of primary factors, industries, and final users.

Definition 8. Final use shares are defined as

$$s_h = \frac{P_h^C C_h}{nGDP}$$

and Domar weights are defined as

$$\bar{\Psi}_i = \frac{P_i Y_i}{nGDP}$$

for producers $i \in \{1, ..., N\}$ and

$$\bar{\Psi}_h = \frac{W_h L_h}{nGDP}$$

for primary factors $h \in \mathcal{N}_H \cup \mathcal{N}_F$.

Assumption 2. *Assume*

$$rank\left(I - \mathcal{W}\Psi_{CL}^T\right) = N_w + N_f$$

Lemma 1. *The steady-state income shares* $\mathbf{s}$ *of final users and the Domar weights* $\bar{\boldsymbol{\Psi}}$ *solve the following system:*

$$\begin{cases} \bar{\boldsymbol{\Psi}}^T = \mathbf{s}^T \Psi_{C,:} \\ \mathbf{s} = \mathcal{W}\bar{\boldsymbol{\Psi}}_L + \mathbf{T} \end{cases} \tag{18}$$

where $\bar{\boldsymbol{\Psi}}_L^T$ *denotes the components of* $\bar{\boldsymbol{\Psi}}$ *corresponding to the factor marketplaces. Under Assumption 2 there is a unique solution for* $\bar{\boldsymbol{\Psi}}$ *and* $\mathbf{s}$ *in (**??**) such that* $\sum_{h\in\mathcal{N}_H\cup\mathcal{N}_F} s_h + s_G = 1$.

The second equation in (18) states that, in a steady-state with zero profits, the income of final users consists of factor income rebates plus transfers. In turn, the first equation states that each producers' (primary factor's) income depends on the total content of this producer (primary factor) in final demand, weighting final users according to their income shares.

[5]Under Assumption 1 producers price at marginal cost, and $(\Psi_L)_{nh}$ is the total share (directly and through intermediate inputs) of primary factor h in n's cost, where n could be an industry or a final user. Remark 7 states that, with unit markups, primary factors must account for the entirety of the costs paid by producers and final users ($\Psi_L \mathbf{1}$).

Note that the system (18) is not invertible.[6] Nonetheless, Assumption 2 guarantees that it has a unique solution such that $\sum_{h\in\mathcal{N}_H\cup\mathcal{N}_F} s_h + s_G = 1$. The assumption is equivalent to imposing that, for every pair (h,f) of a primary factor h and a final user f, there is a connected path $(h, a_1, ..., a_K, f)$ between h and f, where a_k is connected to a_{k-1} if there is an income flow from a_k to a_{k-1}. This guarantees that relative prices and income shares are all well defined.

Table 3 summarizes the notation for income flows and income shares.

Rebates and transfers	$\Xi \in \mathbb{R}^{\bar{N},N_w+N_f+1}$
Input shares	$\Omega \in \mathbb{R}^{\bar{N}\times\bar{N}}$
Expenditure shares on primary factors	$\alpha \in \mathbb{R}^{\bar{N}\times N_w+N_f}$
Final use bundles	$\beta \in \mathbb{R}^{\bar{N}\times N_w+N_f+1}$
Leontief inverse	$\Psi \equiv (I-\Omega)^{-1}$
Income shares of final users	$\mathbf{s} \in \mathbb{R}^{N_w+N_f+1}$, $\mathcal{S} \equiv diag\,(\mathbf{s})$
Domar weights	$\bar{\mathbf{\Psi}}^T = \mathbf{s}^T\Psi_C$
Factor income shares	$\bar{\mathbf{\Psi}}_L^T \equiv \mathbf{s}^T\Psi_{CL}$, $\mathcal{L} \equiv diag\left(\bar{\mathbf{\Psi}}_L\right)$

Table 3: Input-output definitions and income rebates

Elasticities We introduced relative demand and factor supply elasticities in Sections 3.1.1, 3.1.2 and 3.2. The elasticity of sectoral prices with respect to own marginal costs is summarized by the elements Δ_{ii} of a diagonal price elasticity matrix Δ. Following the usual Phillips curve derivation (see Gali (2015); Woodford (2003)), Δ_{ii} is an increasing and convex function of the primitive Calvo parameter δ_i and of the discount factor ρ given by

$$\Delta_{ii}\,(\rho,\delta_i) \equiv \frac{\delta_i\,(1-\rho(1-\delta_i))}{1-\rho\delta_i(1-\delta_i)}$$

Table 4 summarizes the notation for all elasticities.

Aggregation Following definition 3, log-changes in the GDP deflator π^Y are given by a weighted average of sectoral inflation rates with weights equal to the aggregate final expenditure shares $\bar{\boldsymbol{\beta}}$:

$$\pi^Y = \sum_{i=1}^{\bar{N}} \bar{\beta}_i \pi_i$$

[6] The system implies

$$\left(I - \mathcal{W}\Psi_{CL}^T\right)\mathbf{s} = \mathbf{T}$$

The matrix $I-\mathcal{W}\Psi_{CL}^T$ is not invertible, because $\mathbf{1}^T\left(I-\mathcal{W}\Psi_{CL}^T\right) = \mathbf{1}^T - \mathbf{1}^T\Psi_{CL}^T\right) = \mathbf{1}^T - \mathbf{1}^T\Psi_{CL}^T = \mathbf{1}^T - \mathbf{1}^T = \mathbf{0}^T$ (following remark 7).

Factor supply elasticities	$\Phi \in \mathbb{R}^{N_w+N_f},\ \Phi \equiv \Gamma\mathcal{S}^{-1}\left(\mathcal{W}+\frac{\mathbf{T}^*}{GDP}\mathbf{1}^T\right)\mathcal{L}+diag\left(\varphi_1,...,\varphi_{N_w+N_h}\right)$
Demand elasticities	$\theta^i_{jk} \equiv -\frac{d\log\frac{X_{ij}}{X_{ik}}}{d\log\frac{P_j}{P_k}},\ i,j,k \in \left\{1,...,\bar{N}\right\}$
Price elasticities	$\Delta \equiv diag\left(\Delta_{11},...,\Delta_{NN}\right)$

Table 4: Input-output definitions

In turn, aggregate final expenditure shares are a weighted average of expenditure shares across final users, with weights given by final use shares:

$$\bar{\beta}_i \equiv \sum_{h\in\mathcal{N}_w\cup\mathcal{N}_f\cup G} s_h\beta_{ih}$$

Following Definition 2 and Remark 5 , the aggregate output gap is the same as the aggregate factor employment gap. It is given by a weighted average of factor-level employment gaps, with weights equal to factor income shares:

$$\tilde{y} = \sum_{h\in\mathcal{N}_w\cup\mathcal{N}_f} \bar{\Psi}_h\tilde{\ell}_h$$

3.6 Log-linearized equilibrium conditions

This section presents the log-linearized equilibrium conditions, expressed in gaps relative to the natural allocation. The equilibrium of the economy is characterized by three equations, describing supply and relative demand for primary factors, and the evolution of prices. I introducing and explain each equation, and combine them to replace the supply equation with the Phillips curve.

Supply equation The factor supply equation (19) stacks together the log-linearized consumption-leisure tradeoff (12) and the optimal capital utilization (13):

$$\Phi\tilde{\boldsymbol{\ell}}_t = \tilde{\mathbf{w}}_t - \underline{\beta}^T\tilde{\mathbf{p}}_t \tag{19}$$

where

$$\underline{\beta}^T \equiv \beta^T - \Gamma\left(\beta^T - \mathcal{S}^{-1}\Xi^T diag\left(\bar{\boldsymbol{\Psi}}\right)\Psi^{-1}\right)$$

and Φ is defined in Table 4. Equation (19) indicates that factor supply is increasing in real wages, with elasticity Φ. The matrix Φ is not necessarily diagonal, due to wealth effects in labor supply from capital income. Additionally, the relevant price index for real wages is weighted by $\underline{\beta}^T$ rather than by the expenditure shares β^T. This weighting accounts for the disparity in income exposure to different industries through consumption prices (β^T) versus profits ($\mathcal{S}^{-1}\Xi^T diag\left(\bar{\boldsymbol{\Psi}}\right)\Psi^{-1}$) across households. In a representative agent economy these two exposures would cancel out, but with heterogeneous households individuals might purchase from firms they do not own and vice versa.

Equation (19) accounts for different supply curve slopes across primary factors, as reflected in the matrix Φ. This feature is crucial. In Section 4.2 I show that when supply elasticities are heterogeneous, relative demand and supply shocks across industries shocks can affect aggregate inflation.

Relative demand equation The relative demand equation, derived in Lemma 2, combines micro-level demand elasticities to compute the effect of relative price changes on relative factor demand.

Lemma 2. *Relative employment gaps and relative price gaps are related through the demand equation*

$$\tilde{\boldsymbol{\ell}} - \mathbf{1}\tilde{y} = \mathcal{D}_p \tilde{\mathbf{p}} \tag{20}$$

where

$$\mathcal{D}_p \equiv -\mathcal{L}^{-1}\left(I - Cov_{\mathbf{s}}\left(\Psi_{CL}^T, S^{-1}\mathcal{W}\right)\right)^{-1}\left[Cov_{\mathbf{s}}\left(\Psi_{CL}^T, \beta^T - \mathcal{S}^{-1}\Xi^T diag\left(\bar{\boldsymbol{\Psi}}\right)\Psi^{-1}\right) + \Theta\left(\Psi_{:,L}, I\right)\right]$$

and Θ is a substitution operator (defined below). It holds that $\bar{\boldsymbol{\Psi}}_L^T \mathcal{D}_p = \mathbf{0}^T$ and $\mathcal{D}_p \mathbf{1} = \mathbf{0}$.

Proof. See Appendix A. □

In Equation (20), increasing the output gap raises the demand for all primary factors proportionately, as captured by the vector $\mathbf{1}$. Employment gaps also respond to changes in relative prices, as described by the matrix $\mathcal{D}_p$. Changes in the price level have no effect on factor demand, for constant relative prices ($\mathcal{D}_p\mathbf{1} = \mathbf{0}$). Moreover, changes in relative prices only affect relative factor demand ($\bar{\boldsymbol{\Psi}}_L^T \mathcal{D}_p = \mathbf{0}^T$).

The matrix $\mathcal{D}_p$ isolates two channels through which changes in relative prices affect relative factor demand. First, with multiple sectors and input-output linkages, changes in relative prices determine expenditure switching at multiple levels: upstream producers substitute between primary factors, downstream producers substitute between inputs, and final users substitute between consumption goods. These many layers of substitution are summarized by the substitution operator Θ.

Definition 9. Given any two matrices $X \in \mathbb{R}^{\bar{N}\times H}$ and $Y \in \mathbb{R}^{\bar{N}\times M}$ (where M can take any value in $\mathbb{N}$), the substitution operator $\Theta\left(X, Y\right) : \mathbb{R}^{N_w + N_f \times \bar{N}} \times \mathbb{R}^{\bar{N}\times M} \mapsto \mathbb{R}^{H\times M}$ is defined as

$$\left[\Theta\left(X, Y\right)\right]_{hm} \equiv \frac{1}{2}\sum_{n=1}^{\bar{N}} \bar{\Psi}_n \sum_{i,j=1}^{\bar{N}} \Omega_{ni}\Omega_{nj}\theta_{ij}^n \left(X_{ih} - X_{jh}\right)\left(Y_{im} - Y_{jm}\right)$$

for all $h \in \{1, ..., N_w + N_f\}$ and $m \in \{1, ..., M\}$.

In the substitution operator $\Theta\left(\Psi_{:,L}, I\right)$ each primary factor h is affected by a change in the price of good i according to i's network-adjusted expenditure share on h, Ψ_{ih}, and the amount of substitution is governed by the relevant elasticities θ_{ij}^{n}. Note that the vector $\tilde{\mathbf{p}}$ also accounts for changes in the (sticky) factor prices charged by factor marketplaces.

Second, changes in relative prices reallocate real income across final users with different consumption baskets. Relative demand increases for primary factors that are used more intensively in the consumption baskets which became relatively cheaper. This is captured by the covariance between factor contents in consumption (Ψ_{CL}^{T}) and changes in real income $(\mathcal{S}^{-1}\Xi^{T} diag\left(\bar{\boldsymbol{\Psi}}\right)\Psi^{-1} - \beta^{T})$, and by the multiplier $\left(I - Cov_{s}\left(\Psi_{CL}, \mathcal{S}^{-1}\mathcal{W}\right)\right)^{-1}$. The multiplier captures a feedback loop between changes in relative factor demand and changes in the relative income of different final users, which in turn feed back into relative factor demand, and so on.

Pricing equation Prior to introducing the pricing equation, I define the price wedges $\boldsymbol{\chi}_t$ which summarize the effect of all shocks accounted for in the natural equilibrium.

Definition 10. I denote the wedges between sectoral lagged prices $\mathbf{p}_{t-1}$ and natural prices $\mathbf{p}_{t}^{nat}$ by

$$\boldsymbol{\chi}_t = \mathbf{p}_{t-1} - \mathbf{p}_{t}^{nat}$$

The natural prices $\mathbf{p}_{t}^{nat}$ are a function of all the shocks that are accounted for in the natural equilibrium. For example, a negative productivity shock to some sector i opens a price wedge $\chi_i < 0$. Past prices $\mathbf{p}_{t-1}$ instead are a state variable, capturing a persistent effect of past shocks (monetary policy, productivity, preferences, markups, etc.) on relative prices at time t.

The pricing equation is derived by log-linearizing equation (17), and it relates good prices with factor prices $\tilde{\mathbf{w}}$, price wedges $\boldsymbol{\chi}$, and expected inflation $\rho\mathbb{E}\boldsymbol{\pi}_{t+1}$:

$$\tilde{\mathbf{p}}_t = \mathcal{P}_L \tilde{\mathbf{w}}_t + \left[I - \mathcal{P}\Psi^{-1}\right]\left(\boldsymbol{\chi}_t + \rho\mathbb{E}\boldsymbol{\pi}_{t+1}\right) \tag{21}$$

The pass-through matrix $\mathcal{P} \in \mathbb{R}^{\bar{N}\times\bar{N}}$ is defined as

$$\mathcal{P} \equiv \Delta\left(I - \Omega\Delta\right)^{-1}$$

The (i,j)-th element $\mathcal{P}_{ij}$ describes the propagation of a cost shock to sector j into the price of sector i. I also denote by

$$\mathcal{P}_L \equiv \mathcal{P}\alpha$$

the $\bar{N} \times N_H + N_F$ block of this matrix corresponding to primary factors, and by

$$\mathcal{P}_{C,:} \equiv \beta^{T}\mathcal{P},\ \underline{\mathcal{P}}_{C,:} \equiv \underline{\beta}^{T}\mathcal{P}$$

the $N_H + N_F \times \bar{N}$ blocks of this matrix corresponding to final users.

In the definition of $\mathcal{P}$, the price elasticity Δ dictates how prices respond to changes in marginal costs. The matrix $(I - \Omega\Delta)^{-1}$ resembles the Leontief inverse, but it discounts sticky-priced suppliers. The (i,j)-th element of this matrix represents the transmission of a cost shock in sector j into the marginal cost of sector i, either directly or through a chain of intermediate suppliers, suppliers' suppliers, and so forth.

The matrix $\mathcal{P}_L$ describes the pass-through of factor prices into good prices, both directly and through input-output linkages. Instead, the matrix $\mathcal{P}\Psi^{-1}$ describes the response of good prices to price wedges. The matrix Ψ^{-1} maps price wedges into the underlying changes in marginal costs, while the matrix $\mathcal{P}$ describes how changes in marginal costs propagate into prices. An economy with fully flexible prices has $\mathcal{P}\Psi^{-1} = \mathbb{I}$, meaning that changes in the natural prices do not open a price gap $\tilde{\mathbf{p}}_t$, and inflation expectations do not affect current prices. In sticky price economies it is easy to verify that prices adjust by less than with flexible prices $((\mathcal{P}\Psi)_{ij} < \mathbb{I}_{ij}\ \forall i,j)$. Thus changes in the natural prices open a price gap $\tilde{\mathbf{p}} \neq \mathbf{0}$. In the extreme case of fully rigid prices ($\mathcal{P} = \mathbb{O}$), price wedges translate one-for-one into price gaps ($\tilde{\mathbf{p}}_t = \boldsymbol{\chi}_t$).

Aggregate output gap and monetary policy The aggregate output gap is a free parameter determined by monetary policy. The Phillips curve in Section 4 and the static inflation decomposition in Section 5 hold regardless of how monetary policy rule pins down the aggregate output gap.

The illustrative examples throughout the text are based on a static version of the model, with $\rho = 0$. The economy enters period 0 in steady-state with uniform pre-set prices $\mathbf{p}_{t-1}$ across producers within each sector, and a fraction δ_i of producers in each industry i can update prices after observing the realized shocks. Monetary policy determines the output gap through a cash-in-advance constraint

$$P_t Y_t \leq M_t \tag{22}$$

whereby nominal GDP cannot exceed the money supply M_t chosen by the central bank.[7] Log-linearizing equation (22) yields

$$\pi_t^Y + \tilde{y}_t = m_t - p_{t-1}^Y$$

To derive the dynamic decomposition in Section 6 instead I assume that there is a representative household facing a standard Euler equation, and monetary policy sets the nominal interest rate following a Taylor rule

$$i_{t+1} = r_{t+1}^{nat} + \phi_\pi \pi_t^Y + \phi_y \tilde{y}_t$$

where ϕ_π and ϕ_y are parameters that govern the responsiveness of interest rates to aggregate inflation and the output gap, and r_{t+1}^{nat} is the natural real interest rate.

[7] I assume that seignorage revenues are distributed in proportion to the agents' consumption shares, so that – to a first order – seignorage rebates are exactly equal to the amount of new money that the agents need to purchase in order to finance consumption, and the two cancel out from the budget constraint.

Specifically, in Section 6 the representative household has expenditure shares $\boldsymbol{\beta}_h$, income share s_h, and intertemporal elasticity of substitution γ. The Euler equation is

$$\tilde{c}_{h,t} = \mathbb{E}\tilde{c}_{h,t+1} - \frac{1}{\gamma}\left(i_{t+1} - \mathbb{E}\pi^C_{t+1} - r^{nat}_{h,t+1}\right) \tag{23}$$

where $\mathbb{E}\pi^C_{t+1} \equiv \boldsymbol{\beta}_h^T \mathbb{E}\boldsymbol{\pi}_{t+1}$, i_{t+1} is the nominal risk-free rate and $r^{nat}_{h,t+1} \equiv r^{nat}_{t+1} - \left[\boldsymbol{\beta}_h^T - \frac{\bar{\boldsymbol{\beta}}^T + \boldsymbol{\Xi}^T_{h,L}\mathcal{L}\mathcal{D}_p}{s_h}\right]\left(\mathbb{E}\mathbf{p}^{nat}_{t+1} - \mathbf{p}^{nat}_t\right)$.

We can further express the Euler equation (23) in terms of the aggregate output gap using the budget constraint (11) and the relative demand equation (20), as follows:

$$\tilde{y}_t = \mathbb{E}\tilde{y}_{t+1} - \frac{1}{\gamma}\left(i_{t+1} - \hat{\boldsymbol{\beta}}^T \mathbb{E}\boldsymbol{\pi}_{t+1} - r^{nat}_{t+1}\right) \tag{24}$$

where

$$\hat{\boldsymbol{\beta}}^T \equiv (1-\gamma)\,\boldsymbol{\beta}_C^T + \frac{\gamma}{s_h}\left[\bar{\boldsymbol{\beta}}^T + \boldsymbol{\Xi}^T_{h,L}\mathcal{L}\mathcal{D}_p\right]$$

4 Aggregate and cross-sectional drivers of inflation

This section focuses on sector-level and aggregate Phillips curves. Proposition 1 expresses sectoral Phillips curves as a function of the aggregate output gap, relative price wedges $\boldsymbol{\chi}$, and expected inflation ($\rho\mathbb{E}\boldsymbol{\pi}_{t+1}$). This enables us to formalize the distinction between an aggregate inflation driver (corresponding to the output gap), and a cross-sectional driver corresponding to the relative price wedges $\boldsymbol{\chi}$. Section 4.1 then discusses the effect of relative price wedges on industry-level prices, and Section 4.2 establishes conditions for relative price wedges to also affect aggregate inflation.

Proposition 1. *The evolution of sectoral inflation rates is described by the Phillips curves*

$$\boldsymbol{\pi}_t - \rho\mathbb{E}\boldsymbol{\pi}_{t+1} = \boldsymbol{\kappa}\tilde{y}_t - \mathcal{V}\left(\boldsymbol{\chi}_t + \rho\mathbb{E}\boldsymbol{\pi}_{t+1}\right) \tag{25}$$

The slope $\boldsymbol{\kappa}$ is given by

$$\boldsymbol{\kappa} \equiv \left(I - \mathcal{P}_L\left(\underline{\beta}^T + \Phi\mathcal{D}_p\right)\right)^{-1}\mathcal{P}_L\Phi\mathbf{1}$$

The matrix

$$\mathcal{V} \equiv I - \left(I - \mathcal{P}_L\left(\underline{\beta}^T + \Phi\mathcal{D}_p\right)\right)^{-1}\left(I - \mathcal{P}\Psi^{-1}\right) \tag{26}$$

is such that $\sum_j \mathcal{V}_{ij} = 0$ and $-1 \leq \mathcal{V}_{ij} \leq 0\ \forall i \neq j$, $0 \leq \mathcal{V}_{ii} \leq 1$.

Proof. To derive Equation (25) I first combine the supply equation (19), which expresses real wages as a function of factor-specific gaps, with the relative demand equation (20), which relates factor-specific gaps with the aggregate gap and relative prices. This allows me to express real wages as a function of the aggregate output gap and relative prices. I then combine this relation with the pricing equation (21), which expresses prices as a function of wages, price wedges and inflation expectations, to solve out for wages and express prices as a function of the aggregate output gap, $\boldsymbol{\chi}$, and $\rho \mathbb{E} \boldsymbol{\pi}_{t+1}$. Detailed derivations are in Appendix A. □

The slope $\boldsymbol{\kappa}$ is the elasticity of sectoral prices to the output gap, while the matrix $\mathcal{V}$ is the elasticity of sectoral prices to price wedges and inflation expectations. Proposition 1 highlights that $\mathcal{V}$ has zero-sum rows, implying that only shocks impacting *relative* price gaps across sectors can influence inflation. Once the output gap $\tilde{y}$ is controlled for, shocks to the *aggregate* price wedges ($\boldsymbol{\chi} \propto \mathbf{1}$) have no effect. Since $\mathcal{V}$ filters out aggregate shocks, it cannot span the entire space of all possible relative changes. Specifically, Section 5 shows that the slope $\boldsymbol{\kappa}$ does not belong to the image of $\mathcal{V}$. This implies that relative price movements induced by the aggregate output gap $\tilde{y}$ and by relative price wedges $\boldsymbol{\chi}$ are never collinear. This observation is crucial to disentangling the contribution of the output gap vs relative price wedges to aggregate inflation (see Section 5).

Remark 8. In the Phillips curve equation (25), the price wedges $\boldsymbol{\chi}$ are a sufficient statistic for the inflation response to all the shocks accounted for in the natural equilibrium (productivity, relative demand, factor supply, government spending, transfers, desired markups). For instance, higher productivity in sector i corresponds to a lower efficient price p_i^{nat}. Assuming that initial prices are at their steady state value ($p_{i,t-1} = 0$), an increase in productivity implies a positive price wedge $\chi_i = -p_i^{nat}$. Vice versa, an increase in the relative demand for sector i would imply a higher natural price and therefore a negative price wedge $\chi_i = -p_i^{nat}$. It is important to note that the price wedges $\boldsymbol{\chi}$ also depend on past shocks – including past monetary shocks – through the lagged relative prices $\mathbf{p}_{t-1}$, so that past shocks can have a persistent effect on inflation at time t.

Moving from Proposition 1, Sections 4.1 and 4.2 illustrate the effect of industry-specific shocks on relative prices and aggregate inflation.

4.1 Sector-level inflation

In equation (25), the slope $\boldsymbol{\kappa}$ accounts for the effect of the aggregate output gap on sectoral inflation through changes in factor prices.[8] Specifically, implementing a 1% increase in the output gap would require increasing the employment of all primary factors by 1% (as captured by the vector $\mathbf{1}$). This in turn requires raising real factor prices by $\Phi\mathbf{1}$. Factor prices are then passed through into good prices, increasing them by $\mathcal{P}_L \Phi \mathbf{1}$. This sets off a feedback loop between factor prices and good prices, captured by the multiplier

$$\left(I - \mathcal{P}_L \left(\underline{\beta}^T + \Phi \mathcal{D}_p\right)\right)^{-1} = I + \mathcal{P}_L \left(\underline{\beta}^T + \Phi \mathcal{D}_p\right) + \left[\mathcal{P}_L \left(\underline{\beta}^T + \Phi \mathcal{D}_p\right)\right]^2 + ...$$

[8] See Rubbo (2023a) for an in-depth discussion of how changes in aggregate demand affect the relative employment and prices of different primary factors.

The multiplier captures two forces. First, as good prices experience positive inflation $\boldsymbol{\pi}$, nominal factor prices must also increase by $\underline{\beta}^T\boldsymbol{\pi}$ in order to maintain real factor prices constant. Second, good prices do not change proportionately across industries. Changes in relative good prices then feed back into relative factor demand, which changes by $\mathcal{D}_p\boldsymbol{\pi}$, and factor prices, which change by $\Phi\mathcal{D}_p\boldsymbol{\pi}$.

The matrix $\mathcal{V}$ instead describes the response of inflation to relative price wedges $\boldsymbol{\chi}$, under a zero aggregate output gap. In the expression for $\mathcal{V}$, the identity matrix captures desired price changes at constant employment, which are proportional to the price wedges. The remaining terms instead account for changes in marginal costs that occur when the employment of primary factors adjusts to the replicate natural level of output. For example if the natural level of output declines, employment and factor prices must also decline. The change in sectoral real marginal costs caused by price wedges is given by $I - \mathcal{P}\Psi^{-1}$, while the change in real marginal costs caused by the implied changes in factor prices is given by $I - \mathcal{P}_L\left(\underline{\beta}^T + \Phi\mathcal{D}_p\right)$. Thus the product $\left[\left(I - \mathcal{P}_L\left(\underline{\beta}^T + \Phi\mathcal{D}_p\right)\right)^{-1}\left(I - \mathcal{P}\Psi^{-1}\right)\right]_{ij}$ is greater than the identity $\mathbb{I}_{ij}$ if a price wedge in sector j increases the real marginal cost of sector i by more than it reduces its real factor input costs, when factor employment adjusts to keep aggregate output at potential.

In the baseline New Keynesian model with only one sector and flexible wages, closing the output gap determines a wage adjustment that fully offsets the changes in marginal costs induced by exogenous shocks. Formally, it holds that $\mathcal{V} = 0$ for $\bar{N} = N_w = 1$ and $N_f = 0$. This is the well-known divine coincidence. With multiple sectors, instead, the divine coincidence cannot hold at the sector level. As sectors face different demand and productivity changes, relative prices must also adjust. Therefore inflation cannot be zero in all sectors.

Furthermore nominal rigidities slow down relative price adjustments, creating relative price distortions which monetary policy alone cannot eliminate. In turn, relative price distortions imply relative factor demand distortions. Consequently monetary policy can close the aggregate output gap (i.e set $\tilde{y} = 0$), but it cannot close the employment gaps of all primary factors. In other words, monetary policy can ensure that aggregate output remains at potential, but it cannot ensure that output is produced using the efficient combination of primary factors. As I discuss in Section 4.2, relative demand distortions across primary factors are a major reason why industry-specific demand and supply disruptions can generate an aggregate inflation-output tradeoff.

4.2 Cross-sectional shocks and aggregate inflation

Having characterized the effect of relative price wedges on sectoral inflation rates in Section 4.1, I now turn to their effect on aggregate inflation. I first provide a general expression that holds for any aggregate inflation index. I then restrict attention to the GDP deflator, showing that negative price wedges ($\boldsymbol{\chi} < 0$) in sectors with flexible prices or inelastic factor supply cause an inflation-output tradeoff in the aggregate Phillips curve.

Definition 11. Given a vector of weights $\boldsymbol{\phi} = (\phi_1, ..., \phi_{\bar{N}})^T$, such that $\sum_{i=1}^{\bar{N}} \phi_i = 1$, the aggregate inflation index

$$\pi^\phi \equiv \sum_i \phi_i \pi_i$$

is the weighted average of sectoral inflation rates according to $\boldsymbol{\phi}$.

Following Definition 11 one can construct infinitely many inflation indices, each characterized by the weights that it assigns to the various sectors. Corollary 1 below provides a general expression for the response of any aggregate inflation index to cross-sectional shocks. It then specializes the formula to the GDP deflator.

Corollary 1. *The Phillips curve for the aggregate inflation index* $\pi^{\phi} \equiv \sum_i \phi_i \pi_i$ *is*

$$\pi^{\phi} - \rho \mathbb{E} \pi^{\phi}_{t+1} = \kappa^{\phi} \tilde{y}_t + u^{\phi}_t$$

where

$$\kappa^{\phi} \equiv \boldsymbol{\phi}^T \boldsymbol{\kappa}$$

is the slope and

$$u^{\phi}_t = \boldsymbol{\phi}^T \mathcal{V} \left(\boldsymbol{\chi}_t + \rho \mathbb{E} \boldsymbol{\pi}_{t+1} \right)$$

is an endogenous inflation-output tradeoff driven by the cross-sectional component of $\boldsymbol{\chi}_t$ *and* $\rho \mathbb{E} \boldsymbol{\pi}_{t+1}$. *As a special case, the Phillips curve for the GDP deflator is*

$$\pi^{Y} - \rho \mathbb{E} \pi^{\bar{\beta}}_{t+1} = \kappa^{\bar{\beta}} \tilde{y}_t + u^{\bar{\beta}}_t \tag{27}$$

When there are no wealth effects in labor supply ($\Gamma = \mathbb{O}$)*, the cost-push shock in the GDP deflator is given by*

$$\begin{aligned} u^{\bar{\beta}}_t = & \left[Cov_{\bar{\boldsymbol{\beta}}} \left(\mathbf{PT}, -I \right) + Cov_{\mathbf{s}} \left(\mathbf{PT}_L \mathcal{S}^{-1}, \beta^T \right) + \mathbb{E}_{\mathbf{PT}_L} \left(\boldsymbol{\varphi} \right) Cov_{\bar{\boldsymbol{\Psi}}_L} \left(\mathbf{PT}_L \mathcal{L}^{-1}, \mathcal{D}_p \left(I - \mathcal{V} \right) \right) \right. \\ & \left. + Cov_{\mathbf{PT}_L} \left(\boldsymbol{\varphi}, \mathcal{D}_p \left(I - \mathcal{V} \right) \right) \right] \left(\boldsymbol{\chi}_t + \rho \mathbb{E} \boldsymbol{\pi}_{t+1} \right) \end{aligned} \tag{28}$$

where

$$PT_i \equiv \frac{\left[\bar{\boldsymbol{\beta}}^T \left(I - \mathcal{P}_L \underline{\beta}^T \right)^{-1} \mathcal{P} \Psi^{-1} \right]_i}{\bar{\beta}_i}$$

is the pass-through of a cost shock to sector i *into the GDP deflator relative to sector* i*'s aggregate consumption share, and*

$$PT_{Lh} \equiv \left[\bar{\boldsymbol{\beta}}^T \left(I - \mathcal{P}_L \underline{\beta}^T \right)^{-1} \mathcal{P}_L \right]_h$$

is the pass-through of a change in the wage of factor h *into the GDP deflator.*

The covariances on the right-hand-side of (28) capture two dimensions of heterogeneity through which relative price wedges affect aggregate inflation: heterogeneous pass-through of wage and price changes into the GDP deflator, and heterogeneous factor supply elasticities. The pass-through of good and factor prices into final prices is governed by the matrix $\mathcal{P}$, which depends on both input-output linkages and price adjustment frequencies. Example 2 illustrates the role of each.

The term

$$-Cov_{\bar{\boldsymbol{\beta}}}\left(\mathbf{PT}, \boldsymbol{\chi}_t + \rho\mathbb{E}\boldsymbol{\pi}_{t+1}\right)$$

indicates that there is a worse inflation-output tradeoff when $\chi_i < 0$ (productivity declines, or relative demand increases) in sectors i with larger cost pass-through into final prices. A sector i has a large cost pass-through if its own price is flexible, or if its downstream customers, its customers' customers, etc., have flexible prices, as captured by the matrix $\mathcal{P}$. This term is present even in economies with only one primary factor (see Rubbo (2023b)).

The following three terms are present only when there are multiple primary factors. The covariances

$$Cov_{\mathbf{s}}\left(\mathbf{PT}_L\mathcal{S}^{-1}, \underline{\beta}^T\right)$$

and

$$Cov_{\bar{\boldsymbol{\Psi}}_L}\left(\mathbf{PT}_L\mathcal{L}^{-1}, \mathcal{D}_p\left(I-\mathcal{V}\right)\right)$$

indicate that there is a worse inflation-output tradeoff when relative demand is inefficiently high ($\left[\mathcal{D}_p\left(I-\mathcal{V}\right)\left(\boldsymbol{\chi}_t + \rho\mathbb{E}\boldsymbol{\pi}_{t+1}\right)\right]_h > 0$) for factors h with high wage pass-through into the GDP deflator, or when the relevant price indices for the real returns to these primary factors are inefficiently high ($\underline{\boldsymbol{\beta}}_h^T\left(\boldsymbol{\chi}_t + \rho\mathbb{E}\boldsymbol{\pi}_{t+1}\right) > 0$),

The last covariance

$$Cov_{\frac{\mathbf{PT}_L^T}{\mathbf{PT}}}\left(\boldsymbol{\varphi}, \mathcal{D}_p\left(I-\mathcal{V}\right)\left(\boldsymbol{\chi}_t + \rho\mathbb{E}\boldsymbol{\pi}_{t+1}\right)\right)$$

instead indicates that there is a worse inflation-output tradeoff when the relative demand for inelastically supplied factors is inefficiently high ($\left[\mathcal{D}_p\left(I-\mathcal{V}\right)\left(\boldsymbol{\chi}_t + \rho\mathbb{E}\boldsymbol{\pi}_{t+1}\right)\right]_h > 0$ for factors h with large inverse Frisch φ_h).

In turn, the relative demand for a primary factor is inefficiently high when the sectors that use this factor more intensively have a negative price gap (i.e. nominal rigidities prevented prices from increasing in response to shocks). Importantly, relative demand gaps cancel out on average when aggregate output is at potential ($\tilde{y} = 0$).[9] However factor price gaps do not cancel out, because the price of inelastically supplied factors responds more strongly to changes in demand ($\bar{\boldsymbol{\Psi}}_L^T\Phi\mathcal{D}_p \neq \mathbf{0}^T$). Therefore average factor prices increase if and only if the relative demand for inelastic factors is inefficiently high. This in turn raises average good prices, causing an inflation-output tradeoff.

Corollary 2 provides sufficient conditions under which endogenous cost-push shocks are never present. All primary factors must have the same supply elasticity, and all goods and factor prices must have the same pass-through into final prices. These conditions are fairly restrictive, as they are satisfied only by Armington-style production networks where all sectors have the same price adjustment probabilities, and there are no non-labor primary factors (or all primary factors are fully elastically supplied).

Corollary 2. *Assume that there are no wealth effects in labor supply ($\Gamma = \mathbb{O}$). Then there is no inflation-output tradeoff in the GDP deflator ($u^{\bar{\beta}} \equiv 0$) if the following conditions hold: (1) all sectors have the same pass-through into the GDP deflator ($\mathbf{PT} \propto \mathbf{1}$); (2) final expenditure shares and factor income shares are equal ($\mathbf{s} = \bar{\boldsymbol{\Psi}}_L$), and all*

[9] This follows immediately from the condition $\bar{\boldsymbol{\Psi}}_L^T\mathcal{D}_p = \mathbf{0}^T$.

primary factors have the same wage pass-through into the GDP deflator ($\mathbf{PT}_L \propto \mathbf{s}$); (3) all primary factors have the same Frisch elasticity.

Example 1 shows how a shock to the supply of some primary factor can inefficiently increase the price of other primary factors that are its close substitutes, thereby creating an inflation-output tradeoff. Example 2 instead illustrates the role of price adjustment frequencies and input-output linkages in determining the pass-through of good and factor prices into the GDP deflator.

Example 1. Domestic spillovers of import price shocks

This Example illustrates how supply shocks to imported inputs can lead to distortions in the demand for domestic factors, further contributing to domestic inflation.

Consider a slightly richer version of the economy in Section 2 , with two final goods. For illustrative purposes let's call them "transportation" (T) and "other services" (OS). Consumers have equal expenditure shares on the two final goods, and substitution elasticity σ. Transportation is produced using either new or used cars, with expenditure shares Ω_N and $\Omega_U = 1 - \Omega_N$ and substitution elasticity θ. Used cars (U) are an endowment, while new cars are produced using imported semiconductors (SC). I model imports as a primary factor as well, and for simplicity assume that semiconductors are the only input in the production of new cars. The service sector instead uses only labor as an input. Used cars and semiconductors are both inelastically supplied – with the same inverse Frisch elasticity φ_K – while labor is elastically supplied ($\varphi_L < \varphi_K$).

Let's study the effect of a negative supply shock to semiconductors, particularly focusing on the spillovers on used cars and how they translate into aggregate inflation. To simplify the math assume that all factor prices and car prices are flexible while the prices of transportation and other services are sticky, with the same Calvo parameter δ.

Using Proposition 1 we can compute the effect of the shock on relative factor prices:

$$w_{SC} - \bar{w} = \left(w_{SC}^{nat} - \bar{w}^{nat}\right) + \frac{1}{4}(1-\delta)\frac{\sigma\varphi_K}{1+\sigma\delta\bar{\varphi}}\left(p_T^{nat} - p_{OS}^{nat}\right)$$

$$w_U - \bar{w} = \left(w_U^{nat} - \bar{w}^{nat}\right) + \frac{1}{4}(1-\delta)\frac{\sigma\varphi_K}{1+\sigma\delta\bar{\varphi}}\left(p_T^{nat} - p_{OS}^{nat}\right)$$

Like in section 2, the relative price of transportation vs other services does not increase enough due to nominal rigidities. The missing price adjustment is proportional to the degree of nominal rigidity $1 - \delta$. Importantly, the missing price adjustment leads to inefficiently high demand for used cars and semiconductors relative to labor. As a result, the relative price of both semiconductors and used cars becomes inefficiently high (as $p_T^{nat} - p_{OS}^{nat} > 0$). In other words, a shortage of semiconductors spills over into a shortage of used cars.

The relative demand distortion results in an aggregate cost-push shock, similar to section 2, which is given by

$$u^Y = \frac{1}{4}\frac{\sigma\delta\bar{\varphi}}{1+\sigma\delta\bar{\varphi}}\frac{\varphi_K - \varphi_L}{\bar{\varphi}}\left(p_T^{nat} - p_{OS}^{nat}\right)$$

or equivalently

$$u^Y = \frac{\delta}{1-\delta}\left(1-\frac{\varphi_L}{\varphi_K}\right)\left\{\Omega_N\left[(w_{SC}-\bar{w})-\left(w_{SC}^{nat}-\bar{w}^{nat}\right)\right]+(1-\Omega_N)\left[(w_U-\bar{w})-\left(w_U^{nat}-\bar{w}^{nat}\right)\right]\right\}$$

This second expression highlights how spillovers on the price of domestic resources (used cars in our example) further contribute to increasing domestic inflation.

Example 2. Input-output linkages, price adjustment frequencies, and cost pass-through

Consider a horizontal economy with two sectors (sector *Flex* and sector *Sticky*) and one primary factor (labor), as in Figure **??**. Sectors face different productivity shocks $d\log A_{Flex}$ and $d\log A_{Sticky}$, and have different price

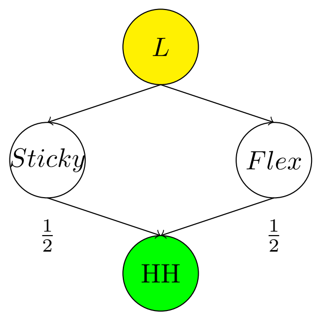

Figure 1: Two-sector horizontal economy

adjustment probabilities $\delta_{Flex} > \delta_{Sticky}$. Consumers have equal expenditure shares on the two goods.

In this economy, heterogeneous frequencies of price adjustment determine a different pass-through of cost shocks into final prices across industries.

The Phillips curve for the GDP deflator is

$$\pi^Y = \kappa^Y \tilde{y} - \frac{Cov\left(\delta, d\log A\right)}{1-\bar{\delta}} \tag{29}$$

while relative prices evolve according to

$$\pi_{Flex}-\pi_{Sticky} = \frac{1}{2}\frac{\delta_{Flex}-\delta_{Sticky}}{\bar{\delta}}\kappa^Y\tilde{y} - \left[\bar{\delta}+\frac{1}{4}\frac{\left(\delta_{flex}-\delta_{sticky}\right)^2}{1-\bar{\delta}}\right]\left(\log A_{Flex}-\log A_{Sticky}\right) \tag{30}$$

If all sectors have the same price adjustment probability – so that $\delta_{flex}-\delta_{sticky}=0$ and $Cov_{\beta}\left(\delta, d\log A\right)=0$ – the GDP deflator only depends on the aggregate output gap, while relative prices only depend on relative productivity shocks. With heterogeneous price stickiness, instead, the GDP deflator depends on both the aggregate output gap and relative productivity shocks. Specifically, there is a "bad" cost-push shock whenever the relative productivity of the flex-price sector declines. In this case marginal costs increase in the flex-price sector and decline in the sticky price one. Given that the flex-price sector has a higher cost pass-through into the GDP deflator, given by $\delta_{Flex} > \delta_{Sticky}$, it also has a disproportionate influence on final prices.

Likewise, if all sectors have the same price adjustment probability then relative prices only depend on relative productivity. If instead sectors have heterogeneous frequencies of price adjustment relative prices also depend on the aggregate output gap. This is because wages are increasing in the output gap, and prices in the flexible sector respond more strongly to changes in wages.

Besides heterogeneous frequencies of price adjustment, input-output linkages are another important source of heterogeneity in cost pass-through. Consider a vertical economy with an intermediate input supplier (I) and a final good producer (F), as in Figure **??**. Assume that price adjustment frequencies are the same in both industries,

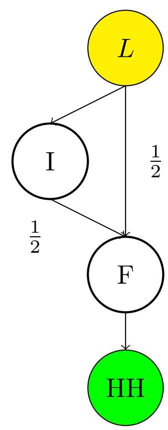

Figure 2: Two-sector vertical economy

equal to δ. The Phillips curve for the GDP deflator is

$$\pi^Y = \kappa^Y \tilde{y} - \frac{1}{2} \frac{\delta (1-\delta)}{1-\bar{\delta}} \left(d \log A_F - d \log A_I \right) \tag{31}$$

while relative prices evolve according to

$$\pi_F - \pi_I = -\frac{1}{2} \frac{\delta (1-\delta)}{\bar{\delta}} \kappa^Y \tilde{y} - \delta \left[1 + \frac{1}{2} \frac{1-\delta}{1+\frac{1}{2}\delta} \right] \left(\log A_F - \log A_I \right) \tag{32}$$

where we denoted by

$$\bar{\delta} = \frac{1}{2} \delta (1+\delta)$$

Note that equation (31) is identical to (29), setting $\delta_{Flex} = \delta$ and $\delta_{Sticky} = \delta^2$. That is we replaced the frequencies of price adjustment in the horizontal economy with the pass-through of productivity shocks into final prices, which is equal to δ for the final good sector and δ^2 for the intermediate good sector. From equation (31) we see that consumer inflation increases under zero output gap when the relative productivity of the final good sector declines, because this sector is closer to final consumers and therefore cost shocks to this sector have a higher pass-through into final prices.

On the other hand the loading of relative prices on the output gap in equation (32) is like equation (30) in the horizontal economy, replacing the frequencies of price adjustment with the pass-through of wage changes into sectoral prices. This pass-through is equal to δ for the intermediate good sector and δ^2 for the final good sector. The output gap affects marginal costs by changing wages, and final prices respond less to wages than intermediate prices because the final good sector has a smaller labor share.

5 Dissecting inflation

Equation (27) isolates two drivers of the GDP deflator: the aggregate output gap, and the relative price wedges determining the endogenous residual $u_t^{\bar{\beta}}$. In practice, distinguishing these drivers in the data poses a challenge because measuring the output gap, the price wedges $\boldsymbol{\chi}$, and inflation expectations in real-time is difficult. To address this challenge, Section 5.1 derives an inflation proxy for the aggregate output gap. Subsequently , Section 5.2 builds on this result to isolate the aggregate and cross-sectional drivers of inflation. Finally, Section 5.4 extends the argument to construct inflation proxies for factor-specific gaps and regional output gaps in a currency union.

5.1 The divine coincidence index

Proposition 2 characterizes the divine coincidence index as the unique inflation index which is not affected by relative price wedges.

Proposition 2. *There exists a unique vector $\boldsymbol{\xi}_{DC}$ such that $\boldsymbol{\xi}_{DC}^T \mathcal{V} = \mathbf{0}^T$. The inflation index $\pi_t^{DC} = \sum_i \xi_{i,DC}\pi_{it}$ does not depend on the relative price wedges $\boldsymbol{\chi}$, and its Phillips curve is given by*

$$\pi_t^{DC} - \rho \mathbb{E}\pi_{t+1}^{DC} = \kappa^{DC}\tilde{y} \tag{33}$$

where

$$\kappa^{DC} \equiv \boldsymbol{\xi}_{DC}^T \boldsymbol{\kappa} \neq 0$$

When there are no wealth effects in labor supply and all final users have the same expenditure shares, the divine coincidence weights are

$$\boldsymbol{\xi}_{DC}^T \propto \left[\bar{\boldsymbol{\Psi}}^T - Cov_{\bar{\boldsymbol{\Psi}}_L}\left(\boldsymbol{\varphi}, \hat{\mathcal{D}}_p \Psi\right)\right] (I - \Delta)\, \Delta^{-1} \tag{34}$$

where $\hat{\mathcal{D}}_p$ is defined as

$$\left[I - \mathcal{D}_p \Psi_L \Phi\right]^{-1} \mathcal{D}_p$$

Appendix A derives $\boldsymbol{\xi}_{DC}$ as a function of primitives in the general case.

Recall from Proposition 1 that the elasticity $\mathcal{V}$ of prices with respect to the wedges $\boldsymbol{\chi}$ satisfies $\mathcal{V}\mathbf{1}=\mathbf{0}$, indicating that only the cross-sectional component of $\boldsymbol{\chi}$ affects inflation. Proposition 2 shows that these cross-sectional shocks have no effect on an appropriate weighted average inflation rate. Formally, $\mathcal{V}\mathbf{1}=\mathbf{0}$ implies that there exists a vector $\boldsymbol{\xi}_{DC}$ such that $\boldsymbol{\xi}_{DC}^T\mathcal{V}=\mathbf{0}^T$. Hence the vector $\boldsymbol{\xi}_{DC}$ defines a weighted average inflation index which filters out the effect of relative price wedges.

Proposition 2 also states that $\boldsymbol{\kappa}^{DC}\neq 0$. That is, while the divine coincidence weights $\boldsymbol{\xi}_{DC}$ are always orthogonal to the response of inflation to $\boldsymbol{\chi}$, they are not orthogonal to the slope $\boldsymbol{\kappa}$. In turn, this implies that the response of inflation to the aggregate gap is never collinear with its response to the price wedges $\boldsymbol{\chi}$.

Proposition 2 provides an explicit expression for the divine coincidence weights $\boldsymbol{\xi}_{DC}$. The divine coincidence weights can differ significantly from final expenditure shares (used in the GDP deflator) or consumption shares (used in the consumer price index or in the PCE price index), explaining why relative demand and supply shocks can substantially influence these other measures of aggregate inflation. The divine coincidence weights (34) discount sectoral inflation rates by a decreasing function $(I-\Delta)\,\Delta^{-1}$ of price adjustment probabilities, to account for heterogeneous price stickiness. Moreover they weigh sectors according to their total sales shares, to account for input-output linkages. These two aspects of the weighting scheme are the same as in an economy with only one primary factor (see Rubbo (2023b)).

The term $Cov_{\bar{\boldsymbol{\Psi}}_L}\left(\boldsymbol{\varphi},\hat{\mathcal{D}}_p\Psi\right)$ instead is novel, and it is present only in economies with multiple primary factors. The matrix $\hat{\mathcal{D}}_p$ captures the effect of changes in sectoral prices on relative factor demand, so that we have $\left(\hat{\mathcal{D}}_p\Psi\right)_{h,i}>0$ when sector i (or its suppliers, its suppliers' suppliers, etc.) use factor h more intensively than the average. Therefore the covariance $Cov_{\bar{\boldsymbol{\Psi}}_L}\left(\boldsymbol{\varphi},\left(\hat{\mathcal{D}}_p\Psi\right)_{:,i}\right)$ is positive for sectors i which use inelastic factors more intensively. These sectors get discounted in the divine coincidence index because they have steeper supply curves, and therefore their prices respond more to idiosyncratic shocks.

To better understand this result, it is important to note that – assuming $\tilde{y}=0$ – a sector i experiences positive inflation when its initial relative price $p_{i,t-1}$ is below the natural one. Whenever this is the case, demand is inefficiently high for the primary factors that sector i uses more intensively. For example, high inflation in a sector i which uses inelastically supplied primary factors is a symptom of inefficiently high demand for inelastic factors. When $\tilde{y}=0$ employment gaps cancel out across primary factors, but the resulting factor price gaps do not, because the price of inelastic factors is more responsive to their employment gap. Therefore average factor prices and good prices increase when the demand for inelastic factors is too high, even under a zero aggregate gap. To net out this effect, the divine coincidence index discounts inflation in sectors which use inelastic primary factors.

The component of the divine coincidence weights which corrects for heterogeneous supply elasticities sums to zero across sectors, weighted according to their total expenditure shares on primary factors:

$$\sum_i Cov_{\bar{\boldsymbol{\Psi}}_L}\left(\boldsymbol{\varphi},\left(\hat{\mathcal{D}}_p\Psi\right)_{:,i}\right)\bar{\alpha}_i=\sum_i Cov_{\bar{\boldsymbol{\Psi}}_L}\left(\boldsymbol{\varphi},\left(\hat{\mathcal{D}}_p\Psi_L\mathbf{1}\right)_{:,i}\right)=0 \tag{35}$$

The first equality in (35) follows from the definition of Ψ_L, while the second equality follows from the result $\Psi_L \mathbf{1} = \mathbf{1}$, stated in Remark (7). Equation (35) also implies that heterogeneous supply elasticities do not cause an inflation-output tradeoff when sectoral cost changes are proportional to total factor input shares $\bar{\boldsymbol{\alpha}}$.

5.2 Relative price changes and aggregate inflation

Using equations (27) and (33) we can express the Phillips curve for the GDP deflator as a function of the divine coincidence index and relative price wedges:

$$\pi_t^Y - \rho \mathbb{E} \pi_{t+1}^Y = \kappa^{\bar{\beta}} \frac{\pi_t^{DC} - \rho \mathbb{E} \pi_{t+1}^{DC}}{\kappa^{DC}} + u_t^{\bar{\beta}} \tag{36}$$

The first term on the right hand side of (36) is the component of the GDP deflator driven by the current output gap, while the residual $u_t^{\bar{\beta}}$ is driven by relative price wedges. Equation (36) allows us to distinguish the two components while relying only on inflation data. This result builds on the crucial observation that the response of inflation to relative price wedges is never parallel to the slope $\boldsymbol{\kappa}$ of sectoral Phillips curves, and relative price wedges do not affect the divine coincidence index.

Remark 9. The ratio

$$\frac{\pi_t^{DC} - \rho \mathbb{E} \pi_{t+1}^{DC}}{\kappa^{DC}} = \frac{\boldsymbol{\xi}_{DC}^T \left(\boldsymbol{\pi}_t - \rho \mathbb{E} \boldsymbol{\pi}_{t+1} \right)}{\boldsymbol{\xi}_{DC}^T \boldsymbol{\kappa}}$$

in equation (36) is equal to the projection of the inflation vector $\boldsymbol{\pi}_t - \rho \mathbb{E} \boldsymbol{\pi}_{t+1}$ on the slope $\boldsymbol{\kappa}$ with respect to a basis of $\mathbb{R}^{\bar{N}}$ formed by $\boldsymbol{\kappa}$ and any basis of the orthogonal space to $\boldsymbol{\xi}_{DC}$. The slope $\boldsymbol{\kappa}$ determines the response of relative prices to the aggregate output gap, while $\boldsymbol{\xi}_{DC}^{\perp}$ is spanned by the response of inflation to relative price wedges, therefore projecting on $\boldsymbol{\kappa}$ allows us to recover the aggregate component. It is important to note that this basis is not orthogonal in general, therefore a principal component decomposition would not recover the aggregate and cross-sectional drivers in equation (**??**).

5.3 Discussion

Implementing the decomposition (36) in practice poses two challenges. First, we have no measure of expected divine coincidence inflation $\mathbb{E} \pi_{t+1}^{DC}$. Second, the price wedges in the residual $u_t^{\bar{\beta}}$ depend on past output gaps as well as current shocks. Hence the residual is not exogenous to (past) monetary policy. To address these concerns, in the quantitative section 6 I propose a dynamic decomposition of consumer price inflation between 2019 and 2022 which takes into account the effect of output gaps on future price wedges and inflation expectations.

However the dynamic decomposition still has some limitations. First, the output gap and the relative price wedges $\boldsymbol{\chi}$ depend on several shocks which we cannot identify separately using only price data. Specifically the output gap includes pure monetary shocks as well as changes in the aggregate natural output that are not accommodated by monetary policy. Moreover, changes in the natural output themselves depend on a variety of shocks (TFP, desired markups, government spending, etc). Likewise the price wedges capture changes in the efficient relative prices driven by a combination of relative TFP, demand, and markup shocks. The decomposition in (**??**) does not allow us to identify each shock separately. It only allows us to distinguish between shocks that change the output gap, and shocks that change the efficient relative prices.

Second, in my environment price wedges are exogenous to monetary policy once we control for past output gaps and inflation expectations. However some components of the price wedges $\boldsymbol{\chi}$ might be endogenous to monetary policy in a more general framework. For example monetary policy may induce endogenous transfers $\mathbf{T}$ across households through asset trades, and this in turn may affect the relative demand for goods and factors. Similarly, if preferences were non-homothetic monetary policy would create endogenous relative demand shocks across industries by changing the households' real incomes. Moreover, the model abstracts from the fact that the relative demand for durable vs non-durable goods is sensitive to interest rates. If this were the case, monetary policy also causes relative demand shocks between durables and non-durables at constant within-period prices. All these relative demand changes would manifest in sectoral price wedges – and in an implied cross-sectional inflation driver – that are endogenous to monetary policy.

In spite of its shortcomings, the decomposition proposed in this paper is nonetheless useful to identify time periods when inflation is driven by demand and supply disturbances that are plausibly exogenous to monetary policy. In Section 6 I find that consumer price inflation comoves closely with its aggregate component in normal times, while the two diverged in 2020. This suggests that inflation during the early phases of the Covid pandemic was mostly driven by industry-specific demand and supply shocks, independent of monetary policy.

5.4 Local divine coincidence indices

Proposition 3 constructs inflation proxies for any linear combination of factor-specific employment gaps. This result is particularly useful in a currency union, where it allows us to proxy for regional output gaps.

Proposition 3. *For any weighted average employment gap*

$$\tilde{y}_\zeta \equiv \sum_h \zeta_h \tilde{\ell}_h$$

with $\zeta_h \in [0,1]$, $\sum_h \zeta_h = 1$, *there exists a unique inflation index*

$$\pi_\zeta^{DC} \equiv \boldsymbol{\xi}_\zeta^T \boldsymbol{\pi}$$

such that the Phillips curve

$$\pi_{\zeta,t}^{DC} - \rho \mathbb{E} \pi_{\zeta,t+1}^{DC} = \kappa_\zeta^{DC} \tilde{y}_{\zeta,t}$$

does not depend on relative price wedges. Appendix A reports an expression for $\boldsymbol{\xi}_\zeta$ in the general case. When there are no wealth effects in labor supply ($\Gamma = \mathbb{O}$), final expenditure shares are the same across all final users, and $\mathbf{T}^* = \mathbf{0}$ in steady-state, the divine coincidence weights $\boldsymbol{\xi}_\zeta$ are given by

$$\boldsymbol{\xi}_\zeta^T = \left[\bar{\boldsymbol{\Psi}}^T - Cov_{\bar{\boldsymbol{\Psi}}_L}\left(\boldsymbol{\varphi}, \hat{\mathcal{D}}_{\zeta,p}\right) - \mathbb{E}_{\bar{\boldsymbol{\Psi}}_L}\left(\boldsymbol{\varphi}\right) Cov_{\boldsymbol{\zeta}}\left(\frac{\bar{\boldsymbol{\Psi}}_L}{\boldsymbol{\zeta}}, \hat{\mathcal{D}}_{\zeta,p}\right)\right] (I - \Delta)\,\Delta^{-1} \tag{37}$$

where

$$\hat{\mathcal{D}}_{\zeta,p} \equiv \left[I - \left(I - \mathbf{1}\boldsymbol{\zeta}^T\right)\mathcal{D}_p \Psi_L \Phi\right]^{-1}\left(I - \mathbf{1}\boldsymbol{\zeta}^T\right)\mathcal{D}_p \Psi$$

The weighted average employment gap $\tilde{y}_\zeta$ in Proposition 3 can be any combination of factor employment gaps. In a currency union setting, $\tilde{y}_\zeta$ can represent the output gap of a region r. Regional output gaps are computed with weights $\boldsymbol{\zeta}$ equal to total factor income shares, multiplied times the share of each primary factor owned by agents in r relative to r's overall factor income share.

The local divine coincidence weights in (37) are similar to the aggregate weights in (34). Local indices however assign a higher weight to sectors which use local factors more intensively. This is captured by the term $Cov_{\boldsymbol{\zeta}}\left(\frac{\bar{\boldsymbol{\Psi}}_L}{\boldsymbol{\zeta}}, \mathcal{D}_{\zeta,p}\right)$, which discounts sectors that hire factors with small local weight ζ_h relative to the aggregate weight $\bar{\boldsymbol{\Psi}}_h$. Additionally, the relative demand coefficient $\mathcal{D}_{\zeta,p}$ rescales the matrix $\mathcal{D}_p$ to express factor-level employment gaps relative to the local gap $\tilde{y}_\zeta$, instead of the aggregate gap $\tilde{y}$.

Examples 3 through 5 below illustrate the local divine coincidence indices in simple economies.

Example 3. Symmetric two-region economy

Consider an economy with two goods and two primary factors, H and F. Each good uses a different primary factor. Both factors have the same supply elasticity φ, and both goods have the same price adjustment probability δ. All consumers have equal expenditure shares on the two goods, with elasticity of substitution θ. The local divine coincidence index for factor H is given by

$$\pi_H^{DC} = \bar{\pi}^{DC} + \frac{1}{2}\frac{1-\delta}{\delta}\frac{\theta\varphi}{1+\theta\varphi}\left(\pi_H - \pi_F\right) \tag{38}$$

As expected, the local divine coincidence index in equation (38) assigns a higher weight to inflation in the home region compared to the aggregate DC index. The relative inflation term is scaled by the demand elasticity θ. In fact, if home and foreign goods were perfect complements, the home and foreign employment gaps would be constrained to change proportionately.

Example 4. Sticky-price and flex-price region

Consider the same economy as in example 3, but now assume that the two goods have different price adjustment probabilities δ_H and δ_F. The divine coincidence index for factor H is

$$\pi_H^{DC} = \bar{\pi}^{DC} + \frac{1}{2}\frac{\theta\varphi}{1+\theta\varphi}\left(\frac{1-\delta_H}{\delta_H}\pi_H - \frac{1-\delta_F}{\delta_F}\pi_F\right) \tag{39}$$

Like in example 3 the local divine coincidence index places a higher weight on home inflation, and the relative price term is scaled by the demand elasticity θ. With heterogeneous price adjustment frequencies, the divine coincidence index rescales local inflation rates to discount the more flexible region.

Example 5. Elastic and inelastic region

Consider the same economy as in example 3, but now assume that the two factors have different supply elasticity φ_H and φ_F. The divine coincidence index for factor H is

$$\pi_H^{DC} = \frac{1-\delta}{\delta}\left[\pi^Y + \frac{1}{2}\frac{\theta\varphi_F}{1+\theta\varphi_F}(\pi_H - \pi_F)\right] \tag{40}$$

Like in example 3 the local divine coincidence index places a higher weight on home inflation, and the relative price term is scaled by the demand elasticity θ. To better compare the local with the aggregate divine coincidence index, we can rewrite (40) as

$$\pi_H^{DC} = \bar{\pi}^{DC} + \frac{1}{2}\frac{1-\delta}{\delta}\frac{\theta\bar{\varphi}}{1+\theta\bar{\varphi}}\frac{1+\theta\varphi_F\frac{\varphi_H}{\bar{\varphi}}}{1+\theta\varphi_F}(\pi_H - \pi_F)$$

Notably, when the home region is more inelastic ($\varphi_H > \bar{\varphi}$), the home inflation gets discounted by less in the local divine coincidence index than in the aggregate index.

6 Drivers of inflation during the Covid pandemic

In this section I calibrate the model using data from the US input-output tables, and construct a time series of the aggregate component of inflation by combining sectoral inflation series from the Producer Price Index (PPI) database. Section 6.1 describes the data, and Section 6.2 presents the results. Appendix C contains additional robustness checks and calibration results.

6.1 Data

In the calibration I consider: 22 worker types, corresponding to the 22 major occupations in the Standard Occupation Classification (SOC) by the BLS; 31 capital asset types, corresponding to the equipment, structures, and intellectual property categories in the NIPA tables; and 71 production industries, as in the BEA input-output tables.

The parameters describing workers, capital assets, industries, and their interactions are reported below.

6.1.1 Expenditure shares

Input-output network I calibrate the input-output matrix Ω based on input-output tables reported by the Bureau of Economic Analysis (BEA), for the year 2012. I compute input shares as the ratio between industry-level expenditures on each intermediate input and industry-level costs. Industry-level costs are the sum of expenditures on intermediate inputs, labor, and capital assets. I describe the methodology to measure expenditures on primary factors next.

Factor input shares I compute the 71×31 component of the matrix α of industry-level expenditures on capital assets ($W_f L_f$, $f \in \mathcal{N}_f$ in our notation) by combining industry-by-asset stock values ($P_f L_{if}$) from the NIPA tables with the values of asset-level rental rates relative to prices ($\frac{W_f}{P_f}$) computed by vom Lehn and Winberry (2021). For both, I use the 2012 values.

The 71×22 component of the matrix α corresponding to labor contains the expenditure shares of each industry on each occupation. I use data from the American Community Survey (ACS) for the year 2012, to compute the share of each industry's wage payroll that goes to each occupation, and I calibrate the overall labor share in total costs at the industry level using the BEA input-output tables.

Final expenditure shares I calibrate the 71×31 component of the matrix β corresponding to capital utilization based on the investment network constructed by vom Lehn and Winberry (2021), who provide bridge tables containing the expenditure shares on each good in the production of each capital assets.

I calibrate the components of the matrix β corresponding to final consumption and government spending from the personal consumption expenditures column of the use table provided by the BEA. I assume that in steady-state government spending is fully financed by taxing households.

6.1.2 Price adjustment probabilities and other parameters

I use the price adjustment probabilities computed by Pasten et al. (2019) based on micro-data underlying the Producer Price Index.

All other parameters are calibrated to consensus values in the literature, reported in Table 6.1.2.

6.1.3 Time series for prices and wages

The time series for the Consumer Price Index (CPI) and the Personal Consumption Expenditure price index less food and energy (core PCE) are from the St Louis Fed database (FRED).

Discount factor	$\rho = 0.9975$
Wealth effects in labor supply	$\gamma = 1$
Inverse Frisch elasticity (labor)	$\varphi_L = 2$
Inverse Frisch elasticity (structures)	$\varphi_S = 20$
Inverse Frisch elasticity (heavy equipment)	$\varphi_{EH} = 10$
Inverse Frisch elasticity (R&D)	$\varphi_{RD} = 5$
Inverse Frisch elasticity (light equipment software, artistic originals)	$\varphi_{EL} = 2$
Substitution between labor types	$\theta^i_{k,h} = 0.1\ \forall k, h \in \mathcal{N}_w, i \in \{1, ..., N\}$
Substitution between capital types	$\theta^i_{k,h} = 0.1\ \forall k, h \in \mathcal{N}_f, i \in \{1, ..., N\}$
Substitution between labor and capital	$\theta^i_{k,h} = 0.1\ k \in \mathcal{N}_w, h \in \mathcal{N}_f, i \in \{1, ..., N\}$
Substitution between factors and intermediates	$\theta^i_{h,j} = 0.1\ h \in \mathcal{N}_w \cup \mathcal{N}_f, i, j \in \{1, ..., N\}$
Substitution between intermediates	$\theta^i_{j,n} = 0.5\ i, j, n \in \{1, ..., N\}$
Substitution between final goods	$\theta^h_{i,j} = 0.9\ h \in \mathcal{N}_w, i, j \in N_f \cup \{1, ..., N\}$
Ownership shares of firms and assets (Ξ, $\mathcal{W}$)	Survey of consumer finances
Persistence of monetary shocks	$\eta_y = 0.5$
Persistence of shocks to $\mathbf{p}_{nat}$	$\Theta_p = 0.5I$

Table 5: Model calibration

I use industry-level price series from the BLS PPI and CPI datasets, and time series for occupation-level wages from the CPS.

Figure **??** plots the correlation between inflation rates, price adjustment frequencies, and factor supply elasticities across industries and over time. The figure illustrates that inflation in 2020 was mainly driven by inelastic sectors with flexible prices on the top right in the graph – such as commodity related sectors and primary manufacturing (oil and gas extraction, petroleum and coal manufacturing, chemical manufacturing, primary metal manufacturing, wood manufacturing and paper manufacturing) – which get discounted in the divine coincidence index. Starting in 2021 inflation became more broad based, spreading to more elastically supplied sectors with stickier prices on the bottom left in the graph.

6.2 Quantitative inflation decomposition

Together the sectoral Phillips curves (**??**) and the aggregate Euler equation (**??**) form a system of second-order difference equations, perturbed by monetary shocks (ϵ_y in equation (**??**)) and by changes in the natural prices $\mathbf{p}^{nat}$ (which affect the price wedges $\boldsymbol{\chi}$ in the Phillips curves). Computing the impulse response functions for this system, and using the divine coincidence result (36), I back out the unique combination of shocks to ϵ_y and $\mathbf{p}^{nat}$ that is consistent with observed sectoral inflation rates. This allows me to construct the counterfactual consumer price inflation rate which would have prevailed had monetary policy closed the output gap at all periods. The difference between actual and counterfactual inflation is a measure of the component of inflation due to non-zero output gaps, accounting for their effect through inflation expectations and the price wedges $\boldsymbol{\chi}$.

I describe the dynamic decomposition in detail in Appendix B.

Figure 4 applies the decomposition to consumer price inflation between January 2019 and July 2022. The blue line is the official CPI, and the red line is the core PCE. The black line plots the component of the CPI due to the aggregate output gap, which is equal to the difference between the actual CPI inflation and the counterfactual inflation under a zero output gap. The yellow bars instead represent the component of CPI inflation driven by exogenous shocks to the relative price wedges. The green bars represent the component of inflation coming from the current aggregate gap, while the purple bars represent the component due to aggregate inflation expectations.

All inflation series are normalized to zero in January 2020, to be consistent with the model which does not account for positive steady-state inflation.

In Figure 4, exogenous relative demand and supply shocks where the main driver of consumer price inflation in the early phase of the Covid pandemic (from February to October 2020). These shocks also affected the core PCE, which thus overstated the role of aggregate demand. Starting in late 2020, the core PCE has been moving closely with the aggregate component from my decomposition. Even in these later phases there still remained a sizable gap between CPI and core PCE, likely due to higher energy prices.

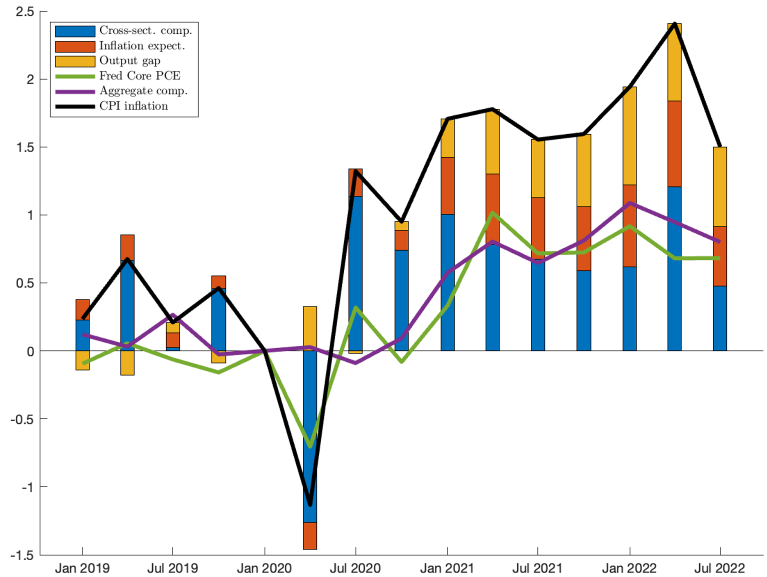

Figure 4: Inflation decomposition in the baseline calibration.

As a robustness check, Figure 5 replicates Figure 4 assuming flexible wages. Wage inflation has a high weight in the divine coincidence index, and we know that measures of wage inflation during the early phases of the pandemic were biased upwards because of composition effects (low-wage workers were more likely to lose their job). This in turn could lead us to underestimating the decline in aggregate demand in early 2020. Assuming flexible wages amounts to excluding wage inflation from divine coincidence index. This modification implies a larger aggregate demand decline in the first half of 2020, although the aggregate component remains well above the core PCE.

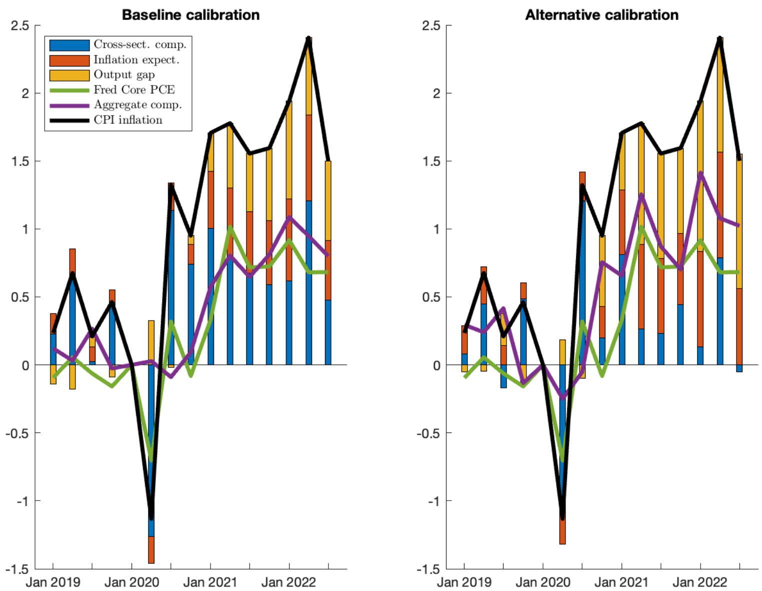

Figure 5: Inflation decomposition in the baseline model and in a counterfactual model with flexible wages.

Heterogeneous price adjustment frequencies and supply elasticities Figure 6.2 replicates Figure 4 in an alternative calibration where all primary factors have the same supply elasticity (calibrated to the share-weighted average across primary factors). Figure 6.2 computes the counterfactual impulse response of consumer inflation to the shocks estimated in the baseline model.

The calibration with uniform supply elasticities implies a larger variance in the aggregate component compared to the baseline. This is because inflation in this period was concentrated in capital-intensive manufacturing sectors, such as oil and gas extraction, chemical manufacturing, primary metal manufacturing, and wood manufacturing which rely on inelastically-supplied equipent and raw materials. For the same reason, the estimated sectoral shocks would have led to lower inflation in a world with homogeneous supply elasticities.

Figure 6.2 instead replicates Figure 4 in an alternative calibration where all industries have the same price adjustment probability (calibrated to the median). Figure 6.2 computes the counterfactual impulse response of consumer inflation to the shocks estimated in the baseline model.

Ignoring heterogeneous price adjustment frequencies implies a larger decline in aggregate demand in early 2020, and a larger increase starting mid 2021. In these time periods oil and oil-related industries were an important driver of aggregate inflation, and these industries have much more flexible prices than the median.

Appendix C presents further robustness checks and results, where I compare the static and dynamic decompositions, discuss the role of input-output linkages, and present the estimated shocks to natural prices by industry.

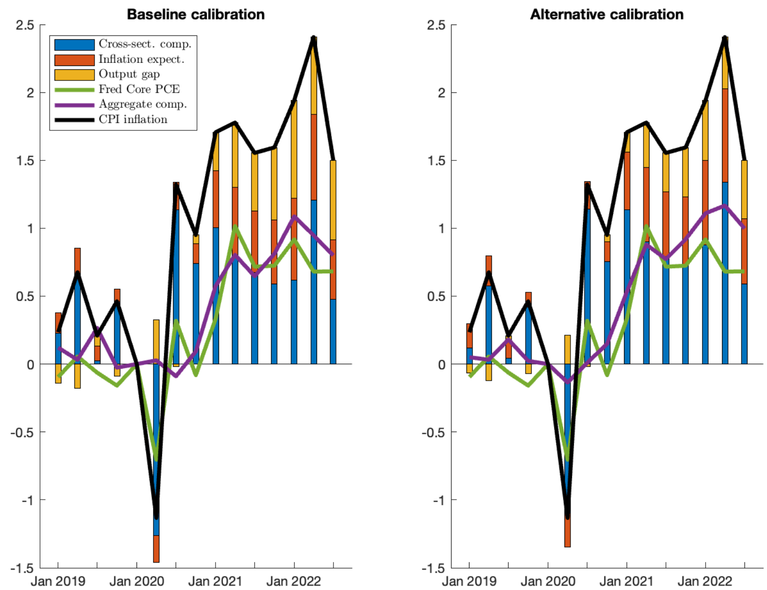

Figure 6: Inflation decomposition in the baseline model and in a counterfactual model with homoegenous supply elasticities

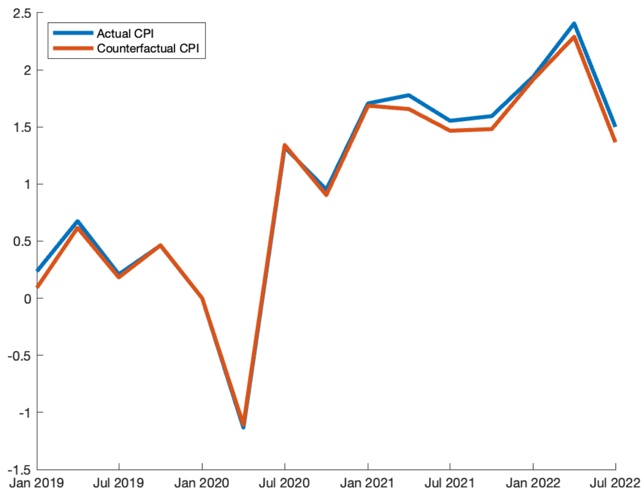

Figure 7: Inflation in a counterfactual model with homogeneous supply elasticities, subject to the same shocks as the actual economy

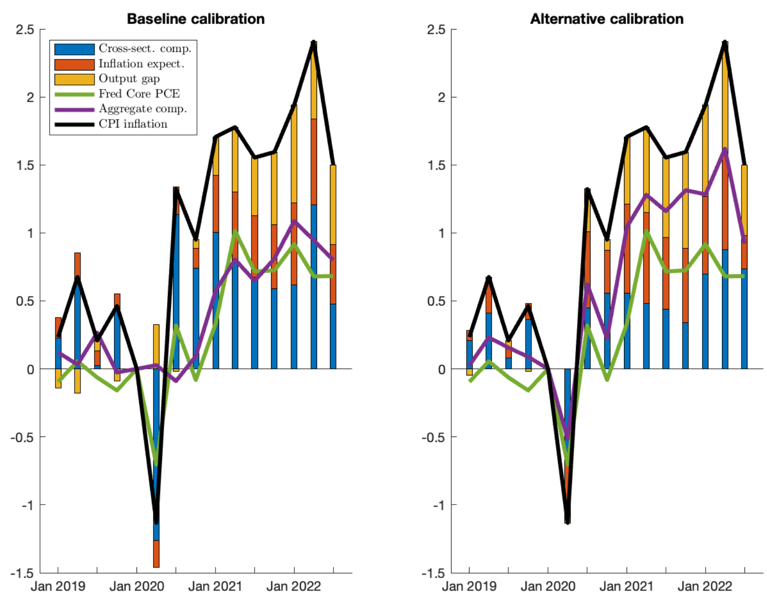

Figure 8: Inflation decomposition in the baseline model and in a counterfactual model with homoegenous price adjustment frequencies

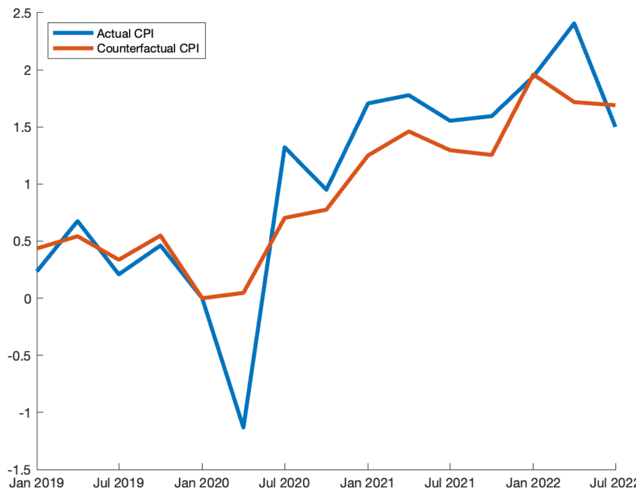

Figure 9: Inflation in a counterfactual model with homogeneous price adjustment frequencies, subject to the same shocks as the actual economy

7 Conclusion

In this paper I examine the impact of relative sectoral demand and supply shocks on inflation in a New Keynesian framework that accounts for multiple industries and primary factors. I argue that industry-specific shocks which increase the relative demand for, or reduce the relative productivity of, industries with more flexible prices or less elastic primary factors can create an inflation-output tradeoff. That is, final prices increase even if monetary policy keeps aggregate output at potential.

I also elaborate a method to measure the contribution of industry-specific shocks to aggregate inflation, based on observed relative price movements. Applying my decomposition to inflation in the United States during and after the Covid-19 pandemic, I find that during the early phases of the pandemic (up until October 2020) consumer price inflation was mostly driven by industry-specific shocks, whereas aggregate factors played an important role in increasing inflation in 2021-2022.

Overall I establish that relative sectoral demand and supply shocks are an important driver of inflation dynamics, and they impose a tradeoff on policymakers seeking to stabilize consumer price inflation and aggregate output at the same time.

References

Ball, L., D. Leigh, P. Mishra, and A. Spilimbergo (2021): "Measuring U.S. Core Inflation: The Stress Test of COVID-19," Tech. Rep. w29609, National Bureau of Economic Research, Cambridge, MA.

Ball, L. and N. G. Mankiw (1995): "Relative-Price Changes as Aggregate Supply Shocks," *The Quarterly Journal of Economics*, 110, 161–193.

Baqaee, D. and E. Farhi (2020a): "Supply and Demand in Disaggregated Keynesian Economies with an Application to the Covid-19 Crisis," Working Paper 27152, National Bureau of Economic Research.

Baqaee, D. R. and E. Farhi (2020b): "Productivity and Misallocation in General Equilibrium," *The Quarterly Journal of Economics*, 135, 105–163.

Carvalho, C. (2006): "Heterogeneity in Price Stickiness and the Real Effects of Monetary Shocks," *The B.E. Journal of Macroeconomics*, 6, 1–58.

Comin, D. A., R. C. Johnson, and C. J. Jones (2023): "Supply Chain Constraints and Inflation," .

di Giovanni, J., S. Kalemli-Ozcan, A. Silva, and M. A. Yildirim (2022): "Global Supply Chain Pressures, International Trade, and Inflation," Series: Working Paper Series Type: Working Paper.

Gali, J. (2015): *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*, Princeton ; Oxford: Princeton University Press, second edition ed.

Karabarbounis, L. and B. Neiman (2014): "The Global Decline of the Labor Share*," *The Quarterly Journal of Economics*, 129, 61–103.

LaO, J. and A. Tahbaz-Salehi (2019): "Optimal Monetary Policy in Production Networks," *SSRN Electronic Journal*.

Nakamura, E. and J. Steinsson (2010): "Monetary Non-Neutrality in a Multisector Menu Cost Model," *Quarterly Journal of Economics*, 125, 961–1013.

Pasten, E., R. Schoenle, and M. Weber (2019): "The propagation of monetary policy shocks in a heterogeneous production economy," *Journal of Monetary Economics*.

Rubbo, E. (2023a): "Monetary Non-Neutrality in the Cross-Section," *Working Paper*.

——— (2023b): "Networks, Phillips Curves, and Monetary Policy," *Econometrica*, 91, 1417–1455.

——— (2024): "Aggregation and Redistribution in General Equilibrium," *Working Paper*.

Ruge-Murcia, F., A. L. Wolman, and Federal Reserve Bank of Richmond (2022): "Relative Price Shocks and Inflation," *Federal Reserve Bank of Richmond Working Papers*, 22, 1–39.

Shapiro, A. H. (2020): "A Simple Framework to Monitor Inflation," *Federal Reserve Bank of San Francisco, Working Paper Series*, 1.000–34.000.

——— (2022): "How Much Do Supply and Demand Drive Inflation?" *FRBSF Economic Letters*, 2022.

vom Lehn, C. and T. Winberry (2021): "The Investment Network, Sectoral Comovement, and the Changing U.S. Business Cycle," *The Quarterly Journal of Economics*, 137, 387–433.

Werning, I. and G. Lorenzoni (2023): "Wage-price spirals," Publisher: Brookings Institution.

Woodford, M. (2003): *Interest and prices: foundations of a theory of monetary policy*, Princeton, N.J. ; Woodstock, Oxfordshire [England]: Princeton University Press.

Appendix

A. Proofs

Proof of Lemma 2

To derive the relative demand equation (20) start by computing the changes in factor income shares, in gaps with respect to the natural benchmark:

$$\left(I-\Psi_{CL}^{T}\mathcal{W}\right)\mathcal{L}\left(d\log\bar{\mathbf{\Psi}}_{L}-d\log\bar{\mathbf{\Psi}}_{L}^{nat}\right)=\left[d\left(\Psi_{CL}^{T}\right)-d\left(\Psi_{CL}^{T,nat}\right)\right]\mathbf{s}+\Psi_{CL}^{T}\Xi^{T}diag\left(\bar{\mathbf{\Psi}}\right)\tilde{\boldsymbol{\mu}}$$

where

$$\left[d\left(\Psi_{CL}^{T}\right)-d\left(\Psi_{CL}^{T,nat}\right)\right]\mathbf{s}=\Psi_{:L}^{T}\left[d\Omega^{T}-d\Omega^{T,nat}\right]|_{switch}\ \bar{\mathbf{\Psi}}-\Psi_{:L}^{T}diag\left(\bar{\mathbf{\Psi}}\right)\tilde{\boldsymbol{\mu}}$$

and

$$\Psi_{:L}^{T}\left[d\Omega^{T}-d\Omega^{T,nat}\right]|_{switch}\ \bar{\mathbf{\Psi}}=\sum_{i,j,k}\bar{\Psi}_{i}\omega_{ij}\omega_{ik}\left(1-\theta_{jk}^{i}\right)\left[\Psi_{:L}\right]_{j,:}\left(\tilde{p}_{j}-\tilde{p}_{k}\right)=$$

$$=\sum_{i,j,k}\bar{\Psi}_{i}\omega_{ij}\omega_{ik}\left[\Psi_{:L}\right]_{j,:}\left(\tilde{p}_{j}-\tilde{p}_{k}\right)-\Theta\left(\Psi_{:L}^{T},\tilde{\mathbf{p}}\right)=$$

$$=\left[\Psi_{:L}^{T}\left(I-\bar{\beta}\mathbf{1}^{T}\right)diag\left(\bar{\mathbf{\Psi}}\right)\Psi^{-1}-Cov_{s}\left(\Psi_{CL},\beta^{T}\right)-\Theta\left(\Psi_{:L},I\right)\right]\tilde{\mathbf{p}}$$

Therefore

$$\left(I-\Psi_{CL}^{T}\mathcal{W}\right)\mathcal{L}\left(d\log\bar{\mathbf{\Psi}}_{L}-d\log\bar{\mathbf{\Psi}}_{L}^{nat}\right)=\left[\Psi_{:L}^{T}\left(I-\bar{\beta}\mathbf{1}^{T}\right)diag\left(\bar{\mathbf{\Psi}}\right)\Psi^{-1}-Cov_{s}\left(\Psi_{CL},\beta^{T}\right)-\Theta\left(\Psi_{:L},I\right)\right]\tilde{\mathbf{p}}-\Psi_{:L}^{T}\left(I-\beta\Xi^{T}\right)diag\left(\right.$$

Solving for factor income shares and markups as a function of prices, wages, and aggregate GDP implies

$$\left(I-\Psi_{CL}^{T}\Xi_{L}\right)\mathcal{L}\left(\tilde{\boldsymbol{\ell}}-\mathbf{1}\tilde{y}\right)=\left[Cov_{s}\left(\Psi_{CL},\mathcal{S}^{-1}\Psi_{CL}^{T}\Xi^{T}diag\left(\bar{\mathbf{\Psi}}\right)\Psi^{-1}-\beta^{T}\right)-\Theta\left(\Psi_{:L},I\right)\right]\tilde{\mathbf{p}}$$

where Ξ_L denotes the columns of the matrix Ξ corresponding to factor marketplaces. Finally, noting that

$$\mathbf{1}^{T}\mathcal{L}\left(\tilde{\boldsymbol{\ell}}-\mathbf{1}\tilde{y}\right)=0$$

we can rewrite the right-hand-side as

$$\left(I-\Psi_{CL}^{T}\Xi_{L}\right)\mathcal{L}\left(\tilde{\boldsymbol{\ell}}-\mathbf{1}\tilde{y}\right)=\left(I-\left(\Psi_{CL}^{T}\Xi_{L}-\bar{\mathbf{\Psi}}_{L}\mathbf{1}^{T}\right)\right)\mathcal{L}\left(\tilde{\boldsymbol{\ell}}-\mathbf{1}\tilde{y}\right)$$

and invert to obtain

$$\tilde{\boldsymbol{\ell}} - \mathbf{1}\tilde{y} = \mathcal{L}^{-1}\left(I - Cov_{\mathbf{s}}\left(\Psi_{CL}^T, \mathcal{S}^{-1}\Xi_L\right)\right)^{-1}\left[Cov_s\left(\Psi_{CL}, \mathcal{S}^{-1}\Psi_{CL}^T\Xi^T diag\left(\bar{\boldsymbol{\Psi}}\right)\Psi^{-1} - \beta^T\right) - \Theta\left(\Psi_{:L}, I\right)\right]\tilde{\mathbf{p}}$$

Proof of Proposition 1

To derive the Phillips curves (25), start by combining the relative demand equation (20) with the supply equation (19):

$$\Phi\left[\mathbf{1}\tilde{y} + \mathcal{D}_p\tilde{\mathbf{p}}\right] = \tilde{\mathbf{w}} - \underline{\beta}^T\tilde{\mathbf{p}}_t \tag{41}$$

Then subtract $\mathcal{P}_L\underline{\beta}^T\tilde{\mathbf{p}}$ from both sides of (21) to make real wages appear:

$$\left[I - \mathcal{P}_L\underline{\beta}^T\right]\tilde{\mathbf{p}}_t = \mathcal{P}_L\left(\tilde{\mathbf{w}}_t - \underline{\beta}^T\tilde{\mathbf{p}}_t\right) + \left[I - \mathcal{P}\Psi^{-1}\right]\left(\boldsymbol{\chi}_t + \rho\mathbb{E}\boldsymbol{\pi}_{t+1}\right)$$

and substitute for real wages using (19):

$$\left[I - \mathcal{P}_L\left(\underline{\beta}^T + \Phi\mathcal{D}_p\right)\right]\tilde{\mathbf{p}}_t = \mathcal{P}_L\Phi\left(\mathbf{1} - \boldsymbol{\tau}\right)\tilde{y} + \left[I - \mathcal{P}\Psi^{-1}\right]\left(\boldsymbol{\chi}_t + \rho\mathbb{E}\boldsymbol{\pi}_{t+1}\right)$$

Finally, use the relation $\boldsymbol{\pi}_t = \tilde{\mathbf{p}}_t + \boldsymbol{\chi}_t$ to check that

$$\boldsymbol{\kappa} = \left(I - \mathcal{P}_L\left(\underline{\beta}^T + \Phi\mathcal{D}_p\right)\right)^{-1}\mathcal{P}_L\Phi\left(\mathbf{1} - \boldsymbol{\tau}\right)$$

and

$$\mathcal{V} = I - \left[I - \mathcal{P}_L\left(\underline{\beta}^T + \Phi\mathcal{D}_p\right)\right]^{-1}\left[I - \mathcal{P}\Psi^{-1}\right]$$

Let's then verify that $\mathcal{V}\mathbf{1} = \mathbf{0}$:

$$\begin{aligned}
\mathcal{V}\mathbf{1} &= \left[I - \left[I - \mathcal{P}_L\left(\underline{\beta}^T + \Phi\mathcal{D}_p\right)\right]^{-1}\left[I - \mathcal{P}_L\right]\right]\mathbf{1} \\
&= \left[I - \mathcal{P}_L\left(\underline{\beta}^T + \Phi\mathcal{D}_p\right)\right]^{-1}\mathcal{P}_L\left[I - \left(\underline{\beta}^T + \Phi\mathcal{D}_p\right)\right]\mathbf{1} \\
&= \left[I - \mathcal{P}_L\left(\underline{\beta}^T + \Phi\mathcal{D}_p\right)\right]^{-1}\mathcal{P}_L\left[\mathbf{1} - \mathbf{1}\right] = \mathbf{0}
\end{aligned}$$

where the first equality follows from the $\Psi^{-1}\mathbf{1} = \alpha\mathbf{1}$, as implied by Remark 7, and the second follows from $\underline{\beta}^T\mathbf{1} = \mathbf{1}$ and $\mathcal{D}_p\mathbf{1} = \mathbf{0}$.

Proof of Propositions 2 and 3

To prove Propositions 2 and 3 I derive a general expression for the divine coincidence weights corresponding to any output gap $\tilde{y}_\zeta \equiv \boldsymbol{\zeta}^T\tilde{\boldsymbol{\ell}}$.

Start from the demand and supply equations

$$\begin{cases} \tilde{\mathbf{w}}_t - \underline{\beta}^T \tilde{\mathbf{p}}_t = \Phi \tilde{\boldsymbol{\ell}}_t & \text{factor supply} \\ \left(I - \mathcal{L}^{-1} \Psi_{CL}^T \mathcal{W} \mathcal{L}\right) \tilde{\boldsymbol{\ell}}_t = \mathcal{D}_p \tilde{\mathbf{p}}_t & \text{factor demand} \end{cases}$$

Add $\mathbf{1}\tilde{y}_\zeta$ on both sides both sides of the demand equation and multiply times $\left(I - \left(\mathcal{L}^{-1}\Psi_{CL}^T \mathcal{W}\mathcal{L} - \mathbf{1}\boldsymbol{\zeta}^T\right)\right)^{-1}$ to obtain

$$\begin{cases} \tilde{\mathbf{w}}_t - \underline{\beta}^T \tilde{\mathbf{p}}_t = \Phi \tilde{\boldsymbol{\ell}}_t & \text{factor supply} \\ \tilde{\boldsymbol{\ell}}_t = \mathbf{1}\tilde{y}_{\zeta,t} + \mathcal{D}_{p,\zeta} \tilde{\mathbf{p}}_t & \text{factor demand} \end{cases}$$

where

$$\mathcal{D}_{p,\zeta} \equiv \left(I - \mathbf{1}\boldsymbol{\zeta}^T\right) \mathcal{D}_p$$

Next express price gaps as a function of wage gaps and markups

$$\tilde{\mathbf{p}}_t = \Psi_L \tilde{\mathbf{w}}_t + \Psi \boldsymbol{\mu}_t$$

plug into the demand and supply equations, and combine them to find

$$\Phi \mathbf{1} \tilde{y}_{\zeta,t} = \left[I - \left(\underline{\Psi}_{CL} + \Phi \mathcal{D}_{p,\zeta} \Psi_L\right)\right] \tilde{\mathbf{w}}_t - \left(\underline{\Psi}_{C,:} + \Phi \mathcal{D}_{p,\zeta} \Psi\right) \boldsymbol{\mu}_t \tag{42}$$

Note that

$$\bar{\boldsymbol{\Psi}}_L^T \left[I - \left(\left(\underline{\Psi}_{CL} - \mathbf{1}\bar{\boldsymbol{\Psi}}_L^T\right) + \Phi \mathcal{D}_{p,\zeta} \Psi_L\right)\right]^{-1} \left[I - \left(\underline{\Psi}_{CL} + \Phi \mathcal{D}_{p,\zeta} \Psi_L\right)\right] =$$

$$\bar{\boldsymbol{\Psi}}_L^T \left(I - \mathbf{1}\bar{\boldsymbol{\Psi}}_L^T\right) = \mathbf{0}^T$$

Therefore we can pre-multiply both sides of (42) times

$$\bar{\boldsymbol{\Psi}}_L^T \left[I - \left(\left(\underline{\Psi}_{CL} - \mathbf{1}\bar{\boldsymbol{\Psi}}_L^T\right) + \Phi \mathcal{D}_{p,\zeta} \Psi_L\right)\right]^{-1}$$

to eliminate the wage term and solve for the aggregate gap $\tilde{y}_\zeta$ as a function of markups:

$$\tilde{y}_{\zeta,t} = -\frac{\bar{\boldsymbol{\Psi}}_L^T \left[I - \left(\left(\underline{\Psi}_{CL} - \mathbf{1}\bar{\boldsymbol{\Psi}}_L^T\right) + \Phi \mathcal{D}_{p,\zeta} \Psi_L\right)\right]^{-1} \left(\underline{\Psi}_{C,:} + \Phi \mathcal{D}_{p,\zeta} \Psi\right)}{\bar{\boldsymbol{\Psi}}_L^T \left[I - \left(\left(\underline{\Psi}_{CL} - \mathbf{1}\bar{\boldsymbol{\Psi}}_L^T\right) + \Phi \mathcal{D}_{p,\zeta} \Psi_L\right)\right]^{-1} \Phi \mathbf{1}} \boldsymbol{\mu}_t \tag{43}$$

Finally, standard Phillips curve derivations (Gali (2015); Woodford (2003)) imply the relation

$$\boldsymbol{\pi}_t = \Delta \left(\mathbf{mc}_t - \mathbf{p}_{t-1}\right) + \left(I - \Delta\right) \rho \mathbb{E} \boldsymbol{\pi}_{t+1}$$

from which we can express markups $\boldsymbol{\mu}_t$ as a function of current and expected inflation:

$$\boldsymbol{\mu}_t = -\left(I - \Delta\right)\Delta^{-1}\left(\boldsymbol{\pi}_t - \rho\mathbb{E}\boldsymbol{\pi}_{t+1}\right)$$

Plugging this relation into (43) yields the divine coincidence weights

$$\boldsymbol{\xi}_\zeta^T = \frac{\bar{\boldsymbol{\Psi}}_L^T\left[I - \left(Cov_{\bar{\boldsymbol{\Psi}}_L}\left(\underline{\Psi}_{CL}\mathcal{L}^{-1}, I\right) + \Phi\mathcal{D}_{p,\zeta}\Psi_L\right)\right]^{-1}\left(\underline{\Psi}_{C,:} + \Phi\mathcal{D}_{p,\zeta}\Psi\right)}{\bar{\boldsymbol{\Psi}}_L^T\left[I - \left(Cov_{\bar{\boldsymbol{\Psi}}_L}\left(\underline{\Psi}_{CL}\mathcal{L}^{-1}, I\right) + \Phi\mathcal{D}_{p,\zeta}\Psi_L\right)\right]^{-1}\Phi\mathbf{1}}\left(I - \Delta\right)\Delta^{-1}$$

B. Dynamic inflation decomposition

The dynamic inflation decomposition in Section **??** assumes a representative household, supplying undifferentiated labor to all sectors. Consumption shares are set equal to the aggregate personal consumption expenditure. The household has access to risk-free bonds in zero net supply, with interest rate i_{t+1}. The central bank sets the risk-free rate according to a Taylor rule. The model equations are

$$\begin{cases} \boldsymbol{\pi}_t - \rho\mathbb{E}\boldsymbol{\pi}_{t+1} = \boldsymbol{\kappa}\tilde{y}_t - \mathcal{V}\left(\boldsymbol{\chi}_t + \rho\mathbb{E}\boldsymbol{\pi}_{t+1}\right) & \text{Phillips curves} \\ \mathbb{E}\tilde{y}_{t+1} = \tilde{y}_t + \frac{1}{\gamma}\left(i_{t+1} - r_t^{nat} - \mathbb{E}\hat{\boldsymbol{\beta}}^T\boldsymbol{\pi}_{t+1}\right) & \text{Euler equation} \\ i_{t+1} = r_t^{nat} + \phi_\pi\bar{\boldsymbol{\beta}}^T\boldsymbol{\pi}_t + \phi_y\tilde{y}_t + \gamma\epsilon_{yt} & \text{Taylor rule} \end{cases}$$

Past relative prices are a state variable, and enter the system through the price wedges $\boldsymbol{\chi}$. The exogenous shocks are ϵ_{yt} (the monetary shock) and the natural prices p^{nat}, which also enter the system through the price wedges $\boldsymbol{\chi}$. I assume that both follow autoregressive processes

$$\epsilon_{y,t} = \eta_y\epsilon_{y,t-1} + u_{y,t}$$

and

$$\mathbf{p}_t^{nat} = \Theta_p\mathbf{p}_{t-1}^{nat} + \boldsymbol{u}_{p,t}$$

with persistence η_y and Θ_p respectively.

Denote by

$$\begin{pmatrix} \mathbf{p}_t^{nat} \\ \epsilon_{y,t} \\ \mathbf{p}_{t-1} \end{pmatrix} = H\begin{pmatrix} \mathbf{p}_{t-1}^{nat} \\ \epsilon_{y,t-1} \\ \mathbf{p}_{t-2} \end{pmatrix} + \begin{pmatrix} \mathbf{u}_{p,t} \\ u_{y,t} \\ \mathbf{0} \end{pmatrix}$$

the impulse response functions for state variables and shocks, and denote by

$$\begin{pmatrix} \boldsymbol{\pi}_t \\ \tilde{y}_t \end{pmatrix} = G \begin{pmatrix} \mathbf{p}_t^{nat} \\ \epsilon_{y,t} \\ \mathbf{p}_{t-1} \end{pmatrix}$$

the impulse response functions for jump variables. The divine coincidence result implies

$$G_{y,:} \begin{pmatrix} \mathbf{p}_t^{nat} \\ \epsilon_{y,t} \\ \mathbf{p}_{t-1} \end{pmatrix} = \frac{\boldsymbol{\xi}_{DC}^T}{\kappa^{DC}} \left[\boldsymbol{\pi}_t - \rho G_{\pi,:} \cdot H \begin{pmatrix} \mathbf{p}_t^{nat} \\ \epsilon_{y,t} \\ \mathbf{p}_{t-1} \end{pmatrix} \right]$$

and we can invert the Phillips curves to infer shocks to the natural relative prices

$$\left(I - \mathbf{1}\bar{\boldsymbol{\beta}}^T\right) \mathbf{p}_t^{nat} = \left(I - \mathbf{1}\bar{\boldsymbol{\beta}}^T\right) \mathbf{p}_{t-1} + \left(\mathcal{V} + \mathbf{1}\bar{\boldsymbol{\beta}}^T\right)^{-1} \left[\boldsymbol{\pi}_t - \rho \left(I - \mathcal{V}\right) G_\pi \cdot H \begin{pmatrix} \mathbf{p}_t^{nat} \\ \epsilon_{y,t} \\ \mathbf{p}_{t-1} \end{pmatrix} - \boldsymbol{\kappa} G_{y,:} \begin{pmatrix} \mathbf{p}_t^{nat} \\ \epsilon_{y,t} \\ \mathbf{p}_{t-1} \end{pmatrix} \right]$$

We can then solve for the current $\epsilon_{y,t}$ and natural prices $\mathbf{p}_t^{nat}$ as follows:

$$\begin{pmatrix} \mathbf{p}_t^{nat} \\ \epsilon_{y,t} \end{pmatrix} = \mathcal{A}^{-1} \mathcal{B} \begin{pmatrix} \boldsymbol{\pi}_t \\ \mathbf{p}_{t-1} \end{pmatrix} \tag{44}$$

where

$$\mathcal{A} \equiv \begin{pmatrix} I - \mathbf{1}\bar{\boldsymbol{\beta}}^T + \left(\mathcal{V} + \mathbf{1}\bar{\boldsymbol{\beta}}^T\right)^{-1} \left(\boldsymbol{\kappa} G_{y,p^{nat}} + \rho \left(I - \mathcal{V}\right) G_\pi H_{:,p^{nat}}\right) & \left(\mathcal{V} + \mathbf{1}\bar{\boldsymbol{\beta}}^T\right)^{-1} \left(\boldsymbol{\kappa} G_{y,y} + \rho \left(I - \mathcal{V}\right) G_\pi H_{:,y}\right) \\ G_{y,p^{nat}} + \rho \frac{\boldsymbol{\xi}_{DC}^T}{\kappa^{DC}} G_{\pi,:} H_{:,p^{nat}} & G_{y,y} + \rho \frac{\boldsymbol{\xi}_{DC}^T}{\kappa^{DC}} G_{\pi,:} H_{:,y} \end{pmatrix}$$

and

$$\mathcal{B} \equiv \begin{pmatrix} \left(\mathcal{V} + \mathbf{1}\bar{\boldsymbol{\beta}}^T\right)^{-1} & I - \mathbf{1}\bar{\boldsymbol{\beta}}^T - \left(\mathcal{V} + \mathbf{1}\bar{\boldsymbol{\beta}}^T\right)^{-1} \left(\boldsymbol{\kappa} G_{y,p} + \rho \left(I - \mathcal{V}\right) G_\pi H_{:,p}\right) \\ \frac{\boldsymbol{\xi}_{DC}^T}{\kappa^{DC}} & -\left[G_{y,p} + \rho \frac{\boldsymbol{\xi}_{DC}^T}{\kappa^{DC}} G_{\pi,:} H_{:,p}\right] \end{pmatrix}$$

I assume $\mathbf{p}_{-1} = \mathbf{0}$ in January 2019. Once we estimate the shocks $\epsilon_{y,t}$ and $\mathbf{p}_t^{nat}$, we can also derive

$$\tilde{y}_t = G_{y,:} \begin{pmatrix} \mathbf{p}_t \\ \mathbf{p}_t^{nat} \\ \epsilon_{y,t} \end{pmatrix}$$

$$\boldsymbol{\epsilon}_{p,t} = \mathbf{p}_t^{nat} - \Theta_p \mathbf{p}_{t-1}^{nat}$$

In the dynamic decomposition, the aggregate component is given by the difference between the actual inflation rate and a the counterfactual inflation rate had the central bank kept a zero output gap at all periods (but subject to the same shocks to natural prices). The remaining component is due to different price wedges different inflation expectations in the actual vs the counterfactual economy.

C. Additional calibration results

Static inflation decomposition Figure 10 applies the static decomposition in (**??**) to consumer price inflation between January 2019 and July 2022, and compares it with the dynamic decomposition. The blue line is the aggregate component in the beaseline dynamic decomposition, while the red line is the aggregate component in the static decomposition. The static and dynamic decompositions are fairly similar, (Figure 4), suggesting that lagged price wedges and cross-sectional differences in inflation expectations are not a significant driver of $\boldsymbol{\chi}$ over our sample period.

Capital market segmentation Figure 7 replicates the inflation decomposition from Figure 4 in an alternative calibration where the markets for capital assets are fully segmented across industries. In the alternative calibration each industry has its own specific capital asset, with supply elasticity equal to the share-weighted average of the relevant micro-elasticities. The two calibrations imply a similar inflation decomposition, likely due to the fact that the baseline calibration already allows for 31 capital assets, so that in practice industries use fairly distinct sets of assets.

Input-Output linkages Figure 7 replicates the inflation decomposition from Figure 4 in a counterfactual calibration with no input-output linkages, but still accounting for heterogeneous nominal rigidities and capital intensity. Figure 7 plots the counterfactual response of consumer inflation to the shocks estimated in the baseline model.

Input-output linkages dampen the pass-through of shocks into downstream prices (see example 2). In Figure 7, inflation would have been more volatile without input-output linkages, conditional on the same underlying shocks. Moreover, in Figure 7, the estimated aggregate component would be excessively volatile and negatively correlated with aggregate inflation in 2020. This is because the observed relative price movements would imply an even larger effect on aggregate inflation without input-output linkages, therefore the model interprets more moderate movements in aggregate inflation as coming from offsetting output gaps.

Shocks to natural prices Figure 7 plots estimated shocks to relative natural prices by industry, using pre-pandemic levels (December 2019) as a benchmark. In Figure 7, the natural relative prices declined for all commodities and manufacturing industries early in the pandemic, as well as for leisure and hospitality. This is consistent with a negative demand shock concentrated on commodities and manufacturing. In 2021 the natural relative price of oil started to increase again, followed by manufacturing, consistent with a shift in spending away from services (such as leisure and hospitality) and towards manufacturing goods like home office furniture. By the end of 2021 the natural relative price had increased for all non-agricultural commodities and manufacturing, as well as for hospitality services.

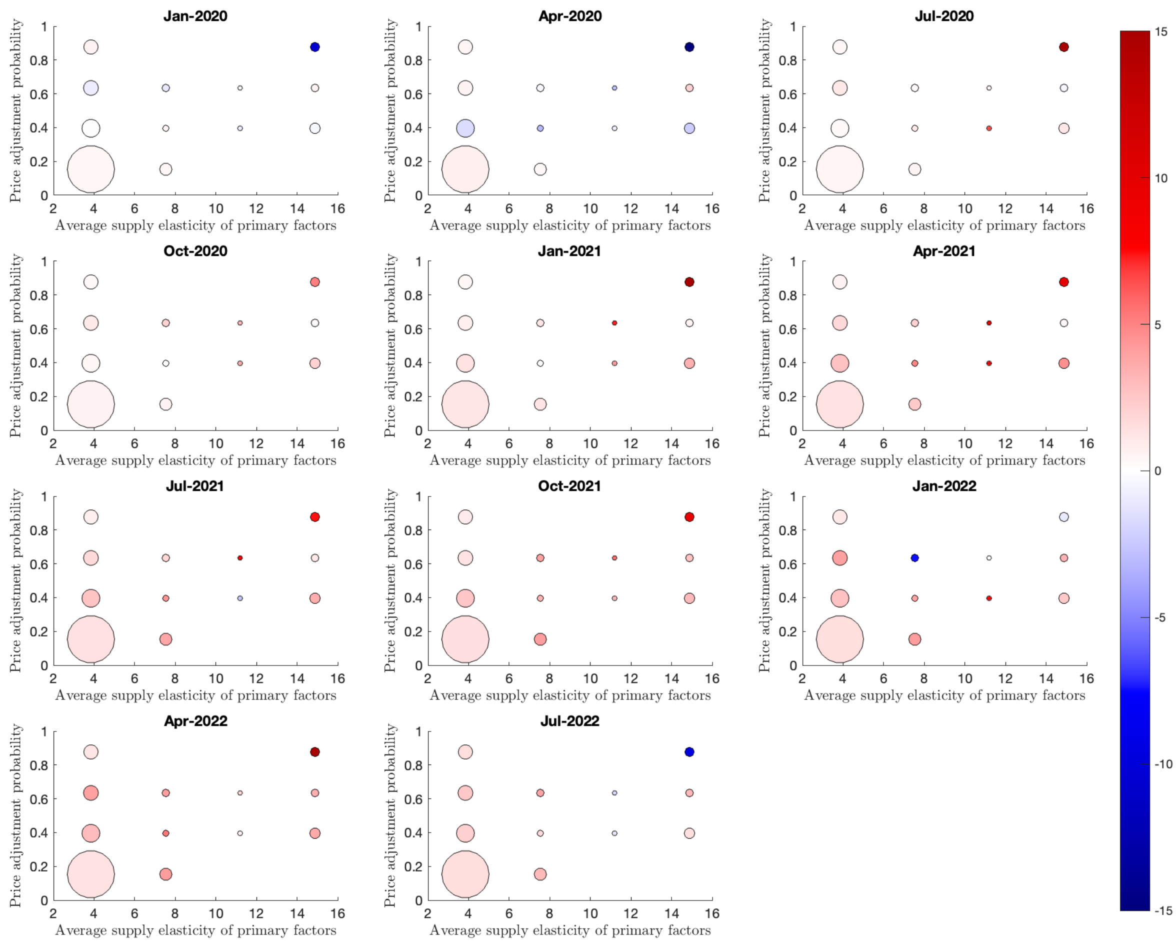

Figure 3: Correlation between inflation, price adjustment frequencies, and factor supply elasticity. Sectors are binned according to their price adjustment frequencies and factor supply elasticity, and larger dots correspond to bins with more industries (weighted by sales shares).

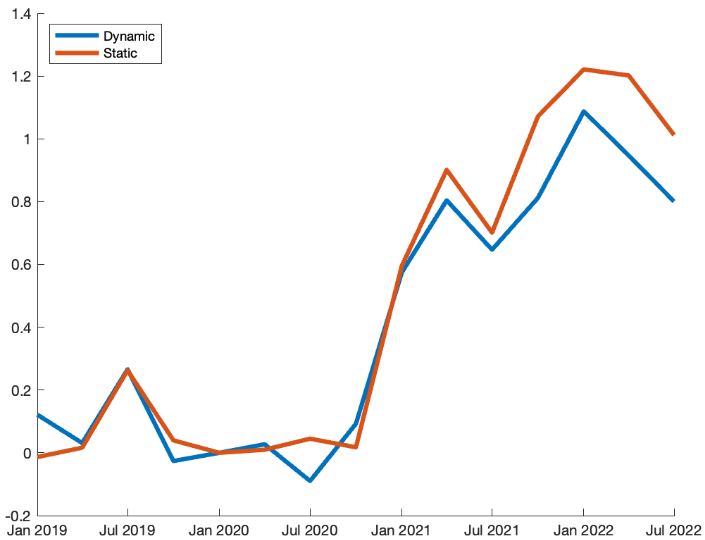

Figure 10: Static vs dynamic inflation decomposition (aggregate component)

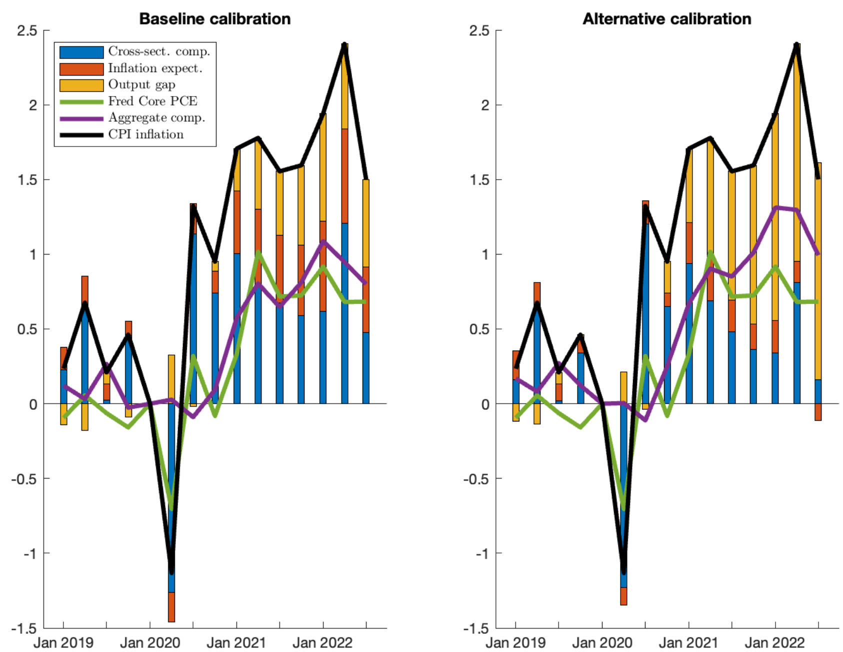

Figure 11: Inflation decomposition in the baseline model and in a counterfactual model with fully segmented capital markets

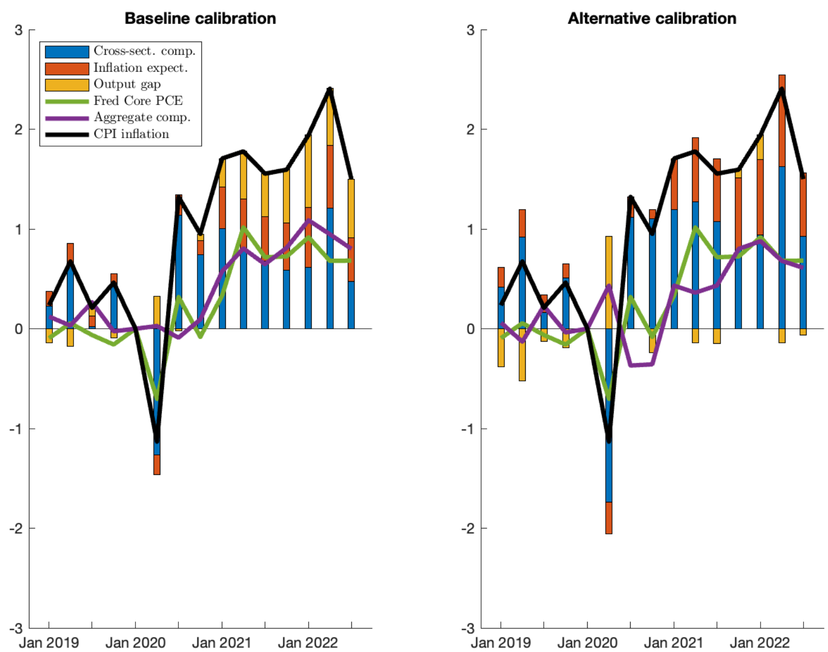

Figure 12: Inflation decomposition in the baseline model and in a counterfactual model with no input-output linkages

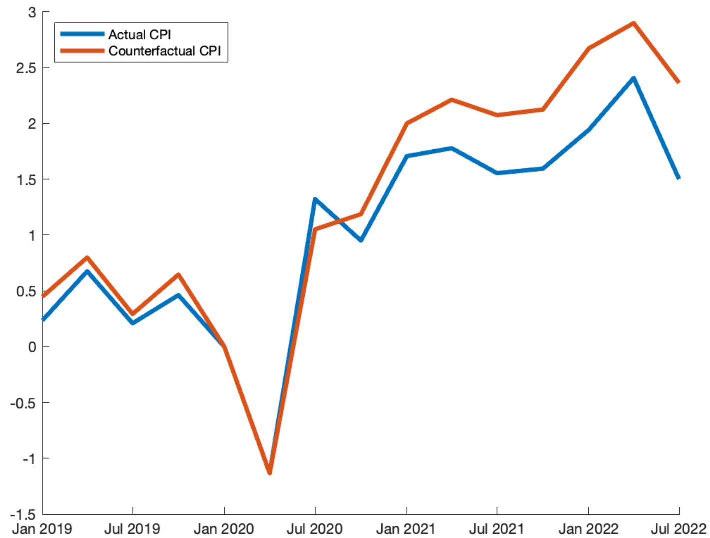

Figure 13: Inflation in a counterfactual model with no input-output linkages, subject to the same shocks as the actual economy

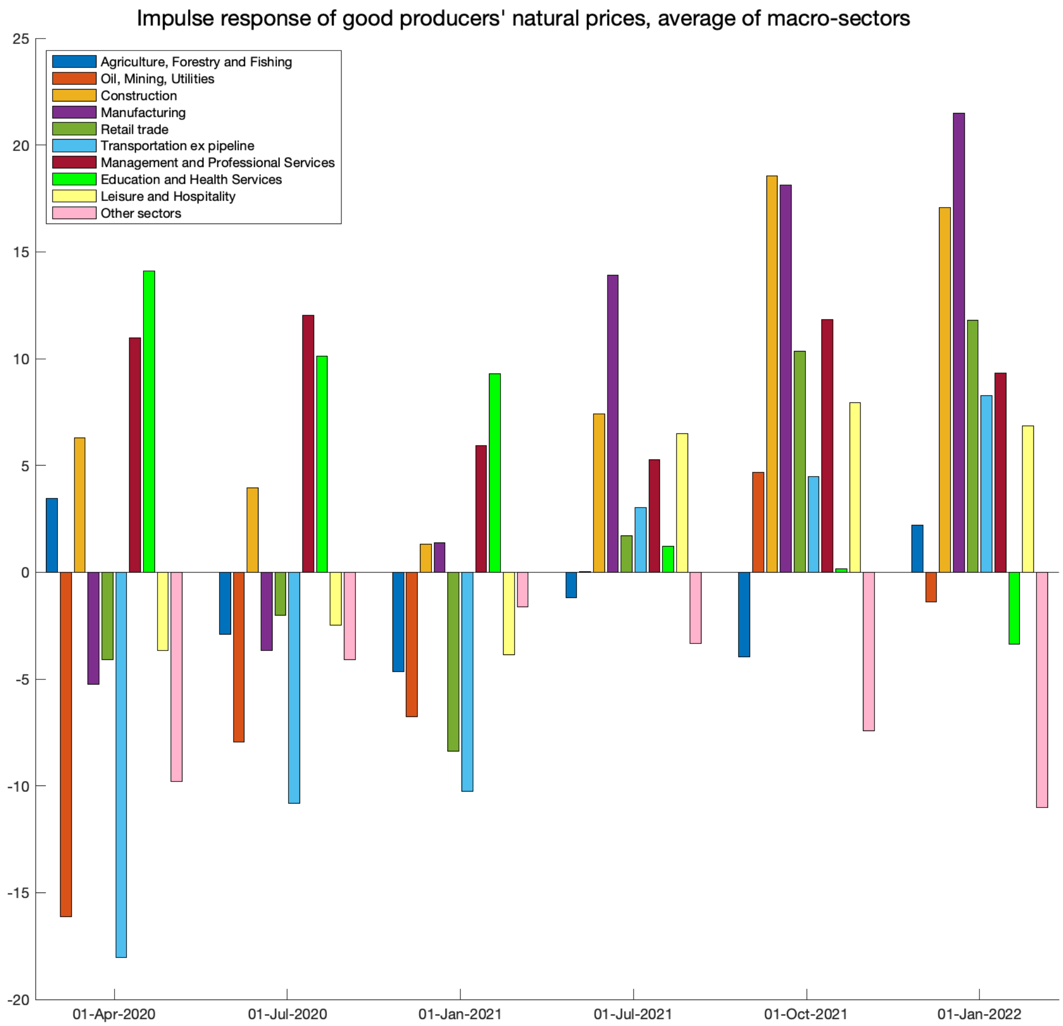

Figure 14: Estimated shocks to natural relative prices