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# Why Did Putin Invade Ukraine? A Theory of Degenerate Autocracy

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## Abstract

Many, if not most, dictatorships end up with a series of disastrous decisions such as Hitler's attack on the Soviet Union or Hirohito's government launching a war against the United States in 1941 or Saddam Hussein's aggression against Kuwait in 1991. Even if a certain policy choice is not ultimately fatal for the regime, such as Mao's Big Leap Forward or the Pol Pot's collectivization drive, they typically involve both a miscalculation by the leader and an institutional environment in which better-informed subordinates have no chance to prevent the decision from being implemented. We offer a dynamic model of non-democratic politics, in which repression and bad decision-making are self-reinforcing. Repressions reduce the immediate threat to the regime, yet raise future stakes for the dictator; with higher stakes, the dictator puts more emphasis on loyalty than competence, which in turn increases the probability of a wrong policy choice. Our theory offers an explanation of how rational, calculating dictators end up in an informational bubble even in highly institutionalized authoritarian regimes.

**Keywords:** nondemocratic politics, authoritarian dynamics, repression, information.

**JEL Classification:** P16, C73, D72, D83.

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## Introduction

The war that Russia launched against Ukraine in February 2022 presents an intellectual puzzle for theorists of authoritarian regimes. A naive rationalist analysis prior to the war would have almost inevitably concluded that the outcome of brinkmanship would not involve actual warfare. Regardless of the estimates of each party's relative power, an actual war is associated with such costs for both sides that avoiding them seems to provide strong incentives to compromise (Fearon, 1995; Powell, 2000; Blattman, 2022). Not surprisingly, most analysts and public commentators who based their analysis on rationalist models were discussing Putin's strategic "bluff" on the brink of the invasion. Yet the Russia-Ukraine war has become, arguably, the bloodiest and costliest inter-state conflict since World War II, involving the use of modern war tools on a scale unheard of in decades.

Even more importantly, while it might take years to end the conflict and decades to fully evaluate the consequences, it is already clear that the decision to invade Ukraine has caused a lot of harm to Putin's regime and its functionaries; with a high probability, it will be the regime's undoing (Gomza, 2022; Stoner, 2022). Though at present there is no way to determine the exact outcome of the Russia-Ukraine war, we observe that over the last 100 years no war of conquest on a similar scale has led to anything rather than a defeat, with major internal repercussions, of the aggressor state. By 2024, almost a million Russians fled the country, twice as many as in any year following the collapse of the Soviet Union in 1991.

Putin's decision to invade Ukraine is just one of many examples of dictators' decisions that appear unforced errors in the hindsight.<sup>1</sup> Emperor Nicholas II's decision to enter World War I in 1914, in the absence of any threat to Russian Empire's core security interests, led to a revolution, the collapse of the empire, the nearly-total destruction of the elite, and an execution of the entire emperor's family (Lieven, 2015). Hitler's decision to invade the USSR after failing to defeat Britain and then declaring war against the USA, the world's largest industrial power, before defeating

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<sup>1</sup> See also "Putin's Colossal Intelligence Failure", *The Russia File*, Kennan Center, March 12, 2022; "Hubris and isolation led Vladimir Putin to misjudge Ukraine", *The Washington Post*, April 12, 2022; "How Putin's War in Ukraine Became a Catastrophe for Russia", *The New York Times*, December 16, 2022; "Putin, Isolated and Distrustful, Leans on Handful of Hard-Line Advisers", *The Wall Street Journal*, December 23, 2022; "What Russia Got Wrong", *Foreign Affairs*, February 8, 2023; "Putin Should Have Known His Invasion Would Fail", *Foreign Policy*, February 23, 2023.

the Soviets, made the war unwinnable for Germany as early as 1941 (Harrison, 1998; O'Brien, 2015). Saddam Hussein's decision to invade Kuwait in 1991 led to a highly predictable military defeat at the hands of an international coalition, reparation payments that have stretched over two decades, and ultimately, Hussein's fall and execution (Karsh and Rautsi, 2007). The 1982 decision by Argentinian military junta led by General Leopoldo Galtieri to invade the Falkland Islands led to a highly predictable military defeat, Galtieri's ouster, and the soon-to-follow demise of the regime (Lewis, 2002).<sup>2</sup> Weeks (2014) provides further examples of dictators' choices that led to wars with predictably disastrous consequences.

The disastrous decisions made by top leaders of authoritarian regimes not necessarily involve launching a war. For our purposes, these decisions simply serve as examples of choices that should not have been made by the respective leaders.<sup>3</sup> Economic campaigns can be just as ill-advised and self-defeating. Mao's Great Leap Forward from 1958 to 1962, which combined economic reforms with a political campaign to jump-start industrial development in then-predominantly agrarian China, led to mass famine, economic disaster, and nearly cost Mao his political preeminence (Meng, Qian and Yared, 2015; Shih, 2022). What is surprising is that the dictators that make these decisions are not ancient emperors whom their subjects cannot approach. Rather, they operate in institutionalized environments, with councils and advisors who presumably have specialized expertise, and their decisions are carried out by career professionals within structured hierarchies (Besley and Kudamatsu, 2009).

In this paper, we present a model of non-democratic regimes that addresses this apparent paradox. The regime is both institutionalized and personalized at the same time. The leader's decisions are based on input from subordinates, but the leader himself chooses the quality of ad-

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<sup>2</sup>Analyzing the origins of the Falklands War, there is an argument that Galtieri and other junta members had their reasons to expect a muted UK reaction. Arguments justifying, ex ante, the disastrous decisions exist for every historical episode we mentioned. We are certainly not claiming that outcomes have been predetermined. Still, these decisions were as close to unforced errors as any historical decision might be.

<sup>3</sup>For modern explanations of war, we refer to excellent surveys in Fearon (1995), Jackson and Morelli (2011), and Herrera, Morelli and Nunnari (2019), as well as recent work on "democratic peace" (Debs and Goemans, 2010; Bueno de Mesquita and Smith, 2012; Weeks, 2012). It is well-documented in studies of "democratic peace" that nearly all wars since 1945 were initiated by dictatorships. However, there is a prominent example of a modern war initiated by a functioning democracy, the U.S. invasion of Iraq in 2003. The decision to start the war relied, in part, on a dramatic underestimation of the future cost of the civil war and the U.S.-born cost of post-war reconstruction. However, from a purely military standpoint, the operation went according to the pre-war analysis. Despite the massive humanitarian cost for Iraqis, the war did not threaten US domestic stability in the way that Galtieri's, Hussein's, or Putin's wars affected their own countries.

vice that he receives, with the primary goal of maintaining power. Specifically, the leader appoints an agent or council with better expertise and relies on their advice in making the policy decision. The leader has the freedom to choose an agent of any level of information-processing quality, from one who knows the regime's vulnerabilities for certain to one who cannot differentiate at all. This is an institutional choice made by the leader.

The leader's power is at risk only if the regime is vulnerable and a wrong policy decision is made. In such an environment, it seems a no-brainer to appoint an advisor with the highest information-processing quality and to make the correct policy decision based on informed advice. The problem is that the advisor has an unobserved characteristic – their affinity with the opposition. This affinity is distributed independently of the lieutenant's quality; if it were, this would only strengthen the effect. When deciding what advice to provide to the leader, the advisor considers two factors: the vulnerability of the leader and their own prospects under a new regime if the leader falls. As the leader is not as well-informed as the advisor, the advisor has leverage and might falsely assure the leader of their safety, leading to a wrong policy decision by a vulnerable leader, which ultimately results in a change in leadership.

In our model, the vulnerability of the dictator leader is driven by external shocks. However, other factors also play a role in the leader's decision-making process. The dictator's decision to repress opposition reduces the likelihood of a challenge, but also increases the stakes in future power struggles. Specifically, if the dictator has repressed opposition in the past, then once overthrown, he represents a more serious threat for the new leader: if not repressed, he might regain power in the future, and with the reputation for repression he is more likely to repress those who overthrew him. Therefore, a leader with a history of repression is more likely to be repressed than a leader with no such history. This, in turn, affects the leader's choice of the level of repression and the quality of the advisor, and ultimately, his chances of survival.

Worsening survival prospects generate a vicious cycle. Once the leader set on the repression path, the stakes become higher. The increasing stakes – the fear to be tried and executed if dethroned – result in the leader choosing advisors with a lower information-processing capacity. Such advisors are more loyal in equilibrium. Even with a high affinity with the opposition, they have low ability to process information and are uncertain about the opposition's chances to oust

the dictator. Thus, they stay loyal. However, the quality of policy-making with such advisors becomes worse. As a result, a fully rational, strategic dictator who has chosen to repress opposition to reduce the probability of a strong challenge ends up in an “informational bubble”, surrounded by low-quality subordinates.

Our theory makes specific predictions about the dynamics of individual dictatorships.<sup>4</sup> A new leader might be surrounded by brilliant people when he comes to power. His tenure might follow either a bloody or peaceful path, depending on many circumstances, including both luck and rational decisions. The dictators that stay in power long enough to witness their power wane become fearful of losing their position. Because of this fear, they replace competent subordinates with loyal ones, which ultimately leads to poor advice and bad policy. As a result, the successes of a dictator’s earlier years fall victim of later years attempts (often futile) to hold on to power. Though there are individual examples of autocrats appointing competent reformers closer to the end of their tenure,<sup>5</sup> systematic evidence points out to the relationship that our model predicts: towards the end of their tenure, authoritarians increasingly pursue bad policy (see, e.g., [Besley and Kudamatsu, 2009](#)). [Jones and Olken \(2005\)](#) used unexpected deaths of leaders as a source of exogenous variation to demonstrate that negative effects of individual leaders are strongest for unconstrained autocrats. [Easterly and Pennings \(2017\)](#) replicated, using an expanded data set, the Jones and Olken’s results with respect to low growth episodes in autocracies.

Some features of our model are fairly standard. For example, the within-period interaction is aligned with the now-standard theoretical approach to the loyalty vs. competence trade-off in authoritarian regimes ([Egorov and Sonin, 2011](#); [McMahon and Slantchev, 2015](#); [Zakharov, 2016](#); [Kosterina, 2017](#); [Hollyer, Rosendorff and Vreeland, 2018](#); [Tyson and Smith, 2018](#)); the presence of this trade-off has been confirmed empirically in [Jia, Kudamatsu and Seim \(2015\)](#); [Bai and Zhou \(2019\)](#); [Shih \(2022\)](#); [Mattingly \(2022\)](#). Still, our baseline model is different: in contrast with [Egorov and Sonin \(2011\)](#), it does not assume that the agent is fully informed when the dictator is not vulnerable; in our model, there is imperfect information in either state. In [McMahon and Slantchev](#)

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<sup>4</sup>There is a recent surge in literature that focuses on formal models of authoritarian dynamics: see, e.g., [Paine \(2021, 2022\)](#); [Meng and Paine \(2022\)](#); [Gratton and Lee \(2023\)](#); see also Subsection 5.

<sup>5</sup>Francisco Franco technocratic reforms in 1957-59 that laid foundation for the “Spanish economic miracle” is an example of competent appointments in a mature authoritarian regime. Still, consistent with our model, these reforms have followed a drastic fall in repression levels under Franco.

(2015), the ruler solves the loyalty vs. competence dilemma by appointing the most competent general and disciplining him by under-funding.

When it comes to dynamics, we seek to expand the methodology beyond the now-standard approach to authoritarian dynamics (Acemoglu and Robinson, 2001, 2005; Acemoglu, 2003; Lagunoff, 2009; Bai and Lagunoff, 2011; Gratton and Lee, 2023; see more references and discussion in Subsection 5). Our primary motivation is that the reliance on two-state dynamic models with Markovian restriction on possible strategies limits the ability of these theories to explicitly focus on mechanisms of path-dependence, first discussed by Douglass North (North, 1981). Any formal theory of this type is bound to capture path-dependence as multiple stable equilibria, saying very little about transitional dynamics. Our focus on reputational concerns allows us to go beyond the existing models by explicitly demonstrating the workings of such mechanism – while also staying away from “folk-theorem” type of results.

In this paper, we do not directly discuss authoritarian power-sharing (Gandhi and Przeworski, 2006; Svolik, 2009, 2012; Powell, 2013; Paine, 2021; Meng, Paine and Powell, 2022; Francois, Trebbi and Xiao, 2023). In Svolik (2012) dichotomy of “authoritarian power-sharing” vs. “authoritarian control”, our model is a theory of authoritarian control. Still, our model contributes to understanding of authoritarian power sharing as well. Specifically, relying on someone’s privately obtained information is, effectively, sharing power. If a dictator chooses to allow media freedom as in Egorov, Guriev and Sonin (2009); Lorentzen (2014); Tyson and Smith (2018), or Shadmehr and Bernhardt (2011), then the autocrat’s power is shared with whoever controls or influences the media. If the dictator relies, as he does in our model, on his subordinates for the information, then the power is shared with these subordinates, and these subordinates have “real authority” even if the dictator has the “formal” one (Aghion and Tirole, 1997).

Finally, our game-theoretic approach to study of authoritarian regimes is not an antithesis to the quantitative and qualitative studies in political science, economics, sociology, history and other disciplines. (See, e.g., Przeworski, 2022 for the recent critique of formal models of authoritarian regimes.) Game-theoretic models of politics do have limitations, and by construction they are bound to be simplistic representations of the reality. Choosing a narrow set of actors and their possible actions, which is necessary to make a game-theoretic model tractable, invariably

involves depriving all other potential actors of agency. This particular paper demonstrates the critical complementarity between the emphasis on loyalty and repression of opposition, which results, dynamically, in a “degenerate autocracy”. This could have been challenging to do, in a logically consistent way, without using game theory.

The rest of the paper is organized as follows. Section 2 introduces our theoretical model. Section 3 analyses decisions that the dictator makes within a period. Section 4 studies the joint dynamics of repression and policy choice. Section 5 briefly discusses the robustness of our model. Section 6 concludes.

## 2 Setup

Time is discrete,  $t = 1, 2, \dots \infty$ . The game is an infinite sequence of interactions between the leaders, their subordinates, and the opposition. Each incumbent leader decides whether or not to repress the opposition, determines the quality of information to base his policy choice on, and makes the policy choice. Depending on these decisions, the leader might remain in power or lose to an opponent. The power struggle is a lottery with the odds determined by the incumbent’s choices. If the opponent overthrows the incumbent, the now-former dictator becomes the opposition.

Every time there is a power struggle, its winner – either the former incumbent or the former opponent – decides whether or not to repress the opposition. If the opposition is not repressed, then the leader faces a challenge in the next period. If the opposition is repressed, then the dictator faces a challenge in the next period only with probability  $\mu < 1$ . When there is a challenge, the odds that the dictator faces in the struggle for power are determined as follows. The dictator may be vulnerable (with probability  $q$ ,  $0 < q < 1$ ) or safe; this is drawn independently in each period. The dictator may choose a strong policy  $d = \bar{P}$  or weak policy  $d = \underline{P}$ ; the former costs  $C > 0$  whereas the latter is cost-less. We assume that the dictator wins if either he is safe or the policy is strong; the dictator loses if he is vulnerable and the policy is weak.

The dictator himself does not know whether or not he is vulnerable in a given period, yet can gather information by appointing a lieutenant of competence  $\theta \in [\frac{1}{2}, 1]$ . Let  $s = v$  denote the

state of the world, in which the dictator is vulnerable; otherwise, the state of the world is  $s = n$ ,  $\Pr(s = v) = q$ . If a lieutenant of competence  $\theta$  is appointed, the lieutenant gets informative signal  $\widehat{s} \in \{n, v\}$ ,

$$\Pr(\widehat{s} = v | s = v) = \Pr(\widehat{s} = n | s = n) = \theta.$$

That is, the lieutenant's competence  $\theta$  is the precision of the signal that he receives about the dictator's vulnerability. Each lieutenant is a strategic player, yet they are short-lived; in other words, their behavior correspond to a perfect Bayesian equilibrium of a one-period game.

If the dictator had full information about his vulnerability in a given period, he would prefer to choose the strong policy  $d = \bar{P}$  if and only if  $s = v$ . The lieutenant, however, may choose to betray the dictator – that is, to misinform the leader about the signal he received, which changes the dictator's odds of survival. (Assuming that the dictator prefers to have a lieutenant to not having one, he would always follow the lieutenant's advice, so misinforming does change the odds.) The signal that the lieutenant received and the action,  $d \in \{\bar{P}, \underline{P}\}$ , that he recommended to the dictator, becomes observable to the winner of the power struggle *ex post*.

If the dictator wins, the lieutenant gets wage  $w$  if he did not betray and suffers punishment  $-\pi$  if he did. If the opponent wins, the lieutenant gets reward  $R$ . Thus,  $R$  parameterizes the lieutenant's affinity with the opposition. Values  $w$  and  $\pi$  are fixed and known to everyone;  $R$  is a random variable, which becomes known to the lieutenant before he makes decision. Assume that  $R$  is distributed on  $(0, \infty)$  with c.d.f.  $F(x)$ , p.d.f.  $f(x)$  such that  $f(x) > 0$  and  $f'(x) < 0$  for  $x > 0$ . For simplicity and without loss of generality, we assume that if the lieutenant is indifferent whether to obey or betray, he obeys the dictator. Lieutenants live for one period; assuming otherwise overburdens the model without bringing significant insights to the issues we focus on.

The dictator  $i$  maximizes his life-time utility

$$U(i) = \sum_{\tau=1}^{\infty} \beta^{\tau} U_{\tau}(i),$$

where  $U_{\tau}(i)$  is the instantaneous utility he  $i$  receives in period  $t$ ,  $\beta < 1$  is the discount factor, common for all dictators. The winner of the power struggle gets one-period utility  $H$ ; the loser gets  $-T < 0$  when repressed. When a player dies peacefully, which happens with probability  $\delta$ ,  $K > 0$  is subtracted from their utility. We assume  $K < T$ ,  $\delta K < H$ . In all other circumstances,

one-period utility equals 0.

In each period  $t$ , the timing of the stage game is as follows.

1. the dictator leader appoints a lieutenant of competence  $\theta_t$ .
2. the dictator faces an opponent with probability  $\mu \in (0, 1)$  if the opposition was repressed, and with probability 1 otherwise. If there is no opponent, the current incumbent remains in power and the game moves to stage 6.
3. The state of the world  $s_t \in \{n, v\}$  is realized. If there is an opponent, the dictator's lieutenant learns the realization of the signal about the dictator's vulnerability,  $\hat{s}_t \in \{n, v\}$  and of the lieutenant's affinity with the opponent  $R_t \in [0, +\infty)$ . After that, the lieutenant chooses recommendation  $d_t \in \{\bar{P}, \underline{P}\}$ .
4. The outcome of the power struggle is determined, depending on the state of the world and the dictator's decision based on the lieutenant's advice.
5. The winner of the power struggle decides whether or not to repress the opposition.
6. The opposition leader, if not repressed, and the dictator die independently with probability  $\delta \in (0, 1)$ . If only the winner dies, the loser (if alive) becomes the next dictator automatically, and faces an opponent next period. If only the loser dies, a new opponent is chosen for the dictator. If both die, a new dictator and a new opponent are chosen. If none die, the winner is the dictator of the next period, and the loser (if exists) is the next opponent.

In a generic dynamic game, strategies may depend on the whole history, and this leads to a large number of equilibria, many unrealistic. We focus on a class of symmetric equilibria where the dictator's actions in any period may depend on minimal information about the opponent, specifically, whether this opponent repressed opposition in the past.<sup>6</sup> If so, dictator's own history of repressions (whether he repressed in the past or not) also becomes payoff-relevant, because

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<sup>6</sup>Admittedly, our assumption that the players' strategies might depend on "reputation" state variables, the reputation of the winner and the reputation of the loser, is a short-cut. It is possible to do the same model with incoming leaders having, with some probability, a commitment type that always represses the opposition, and other players' having uncertainty about whether or not the leader has this commitment type as in the canonical models of reputation (Kreps and Wilson, 1982; Milgrom and Roberts, 1982). Our approach allows to carry out the basic intuition and economize on notation. See also Section 5.

the opponent, if he wins the struggle for power, would use that variable to determine the current incumbent's fate. In other words, we focus on equilibria where the strategies may depend on whether or not both the dictator and the opponent ever resorted to repression. Conceptually, this corresponds to the notion of *[imperfect] public monitoring*. (See [Mailath and Samuelson, 2006](#), for the definition and foundational results in the context of repeated games; our game is not a repeated game as there might be a new challenger arriving after an execution or a natural death in the previous period.)

We relegate a full formal definition of strategies, as well as a full formal definition of equilibria we are interested in, to the Appendix; the Appendix also contains a list of notation. Here, we focus on essential details. Let us say that a leader has good reputation ( $G$ ) if he has never resorted to repression before, and has bad reputation ( $B$ ) otherwise. Thus, all possible combinations of the winner's and the loser's types belong to the set  $\Lambda = \{(B, G), (G, B), (G, G)\}$ . Indeed, if the dictator repressed before, then the opponent is a "novice", and therefore  $(B, B)$  is impossible for any history, on or off equilibrium path.

**Definition 1.** A tuple  $(\alpha^*, \theta^*, d^*)$ , where  $\alpha^*$  and  $\theta^*$  are mappings from  $\Lambda$  to  $[0, 1]$  and the lieutenant's decision function  $d^*$  is a mapping from  $[0, 1] \times \{n, v\} \times [0, \infty)$  to  $\{\bar{P}, \underline{P}\}$  is called an equilibrium if and only if

- (a) for any  $(X, Y) \in \Lambda$ , choosing repression with probability  $\alpha^*((X, Y))$  is weakly optimal for a winner with reputation  $X$  if loser's reputation is  $Y$ ;
- (b) for any  $(X, Y) \in \Lambda$ , choosing a lieutenant with competence  $\theta^*((X, Y))$  is weakly optimal for a dictator with reputation  $X$  if next opponent's reputation is  $Y$ ;
- (c) for a lieutenant of competence  $\theta$  who received signal  $\hat{s}$  and has learned the reward  $R$ , it is optimal to recommend the strong costly policy ( $d^*(\theta, s, R) = \bar{P}$ ) if the expected payoff exceeds the expected payoff of recommending the weak policy;
- (d) for a dictator that appointed a lieutenant of competence  $\theta^*$ , it is optimal to follow the lieutenant's recommendation about the policy.

### 3 Static Regime Formation

To make our analysis tractable, we split it into several steps. We start with analyzing the choice of the regime by the dictator leader. First, we study the lieutenant's behavior, since it is least connected with past and future decisions of players. Then, treating lieutenants' behavior as given, we find dictator's optimal choice of lieutenant's competence; these will depend on dictator's expected continuation utilities, which we will for a moment treat as given. After analyzing the static institutional choice, we will characterize dictator's utilities in the case of no repression and repression, again treating future behavior of all players, including himself, as given. This will allow us to find out in when players choose the repression regime. The next important step is to find dictators' best responses if they correctly predict future winners' decisions on repression, but consider the odds of winning or losing struggle for power to be fixed arbitrarily. We will say that the corresponding strategy profiles result in *dynamically consistent paths*. Finally, to find equilibria, we check strategy profiles that result in dynamically consistent paths, on which future dictators do hire lieutenants, who are short-lived, of the quality expected by the current incumbent.

#### 3.1 The Information Gathering Trade-off

We begin by studying the behavior of a lieutenant of a fixed type  $\theta$  who has received a noisy signal  $\hat{s} \in \{n, v\}$  about the dictator's vulnerability and the value of potential reward  $R$  from the opponent if the dictator is overthrown.

The first observation is that for the dictator, it does not make sense to appoint a lieutenant and then *not* to follow the lieutenant's recommendations. (This parallels the *revelation principle* in mechanism design.) Appointing a lieutenant of competence  $\theta$  makes sense for the dictator if and only if the dictator's payoff with such a lieutenant exceeds both the expected payoff with a weak policy and the expected payoff with strong policy regardless of circumstances; we focus on this situation. Thus, if the lieutenant gets signal  $\hat{s} = n$ , then the agent recommends the weak policy as this is the optimal, from the dictator's standpoint, policy and recommending the strong policy cannot increase the agent's payoff.

The only situation when the lieutenant behaves opportunistically is when the signal is  $\hat{s} = v$ .

The agent betrays the dictator if the expected utility from betrayal exceeds that in the case of no betrayal. Both expectations are conditional on the agent's signal  $\hat{s} = v$  and thus are functions of the agent's competence  $\theta$ . To calculate them, the agent uses the Bayes formula:

$$\begin{aligned} \Pr(s = v | \hat{s} = v) &= \frac{\Pr(\hat{s} = v | s = v) \Pr(s = v)}{\Pr(\hat{s} = v | s = v) \Pr(s = v) + \Pr(\hat{s} = v | s = n) \Pr(s = n)} \\ &= \frac{q\theta}{q\theta + (1 - \theta)(1 - q)}. \end{aligned}$$

So, after receiving signal  $\hat{s} = v$  and learning the value of the reward  $R$ , the lieutenant's betrayal yields, in expectation

$$R \frac{q\theta}{q\theta + (1 - \theta)(1 - q)} - \pi \frac{(1 - \theta)(1 - q)}{q\theta + (1 - \theta)(1 - q)},$$

while staying loyal results in  $w$ . Therefore, the lieutenant betrays the incumbent, by recommending the weak policy when  $\hat{s} = v$ , if and only if

$$R > w + \frac{(1 - q)(1 - \theta)}{q\theta} (w + \pi).$$

This gives us the following formal result. (Since lieutenants are short-lived, we describe perfect Bayesian equilibria of the one-period game between the dictator and the lieutenant that the dictator has chosen.)

**Proposition 1.** *If the lieutenant's signal is that the leader is not vulnerable,  $\hat{s} = n$ , the lieutenant stays loyal regardless of the affinity with the opposition. If the signal is that the leader is vulnerable,  $\hat{s} = v$ , the lieutenant stays loyal if and only if*

$$R \leq R^*(\theta) \equiv w + \frac{(1 - q)(1 - \theta)}{q\theta} (w + \pi).$$

*The threshold level  $R^*(\theta)$  of reward that is required by the lieutenant of a fixed type  $\theta$ , having received signal  $\hat{s} = v$ , to "betray" the dictator, increases with the lieutenant's wage  $w$ , the level of punishment for treason  $\pi$ , and decreases with the ex ante probability of the dictator being vulnerable  $q$ . In particular, a more competent lieutenant, one with a higher  $\theta$ , betrays the dictator for lower values of the reward.*

The intuition behind the comparative statics in Proposition 1 is as follows. When the lieutenant of competence  $\theta$  receives the signal that the dictator is vulnerable, there is a trade-off be-

tween the probability of the reward from a victorious opponent and the probability of a punishment from a surviving incumbent. A lieutenant of high competence has a very precise signal: such a lieutenant knows with near-certainty that the dictator is vulnerable. Thus, a competent lieutenant might accept a lower reward for misinforming the dictator about the signal, which results in the poor policy choice, and, by doing so, the lieutenant guarantees the dictator's loss.

Both a higher reward for remaining loyal and a higher punishment for the opposite increase the lieutenant's incentives to be loyal. An increase in  $q$  leads to an increase of the probability that conditions are favorable for the enemy, as perceived by the lieutenant. This, in turn, decreases lieutenant's fear of being punished, and makes him more likely to betray. Finally, though a smarter lieutenant receives a signal that the enemy is likely to win less frequently than a less competent one does, once he does, he is more sure that the enemy will win if he betrays, which also decreases his fear of punishment.

### 3.2 Equilibrium Choice of Loyalty vs. Competence

The dictator does not observe the affinity between the lieutenant and the opponent (the value of the reward for betrayal  $R$ ), but knows its distribution. From the leader's standpoint, the probability of betrayal (i.e., recommending  $d = \underline{P}$  when  $\hat{s} = v$ ) conditional on the fact that the agent gets signal  $\hat{s} = v$  is

$$\Pr(d = \underline{P} | \hat{s} = v) = 1 - F(R^*(\theta)).$$

The probability of losing the struggle is therefore

$$p(\theta) = q\theta(1 - F(R^*(\theta))) + q(1 - \theta),$$

where  $q\theta(1 - F(R^*(\theta)))$  is the probability that the incumbent is actually vulnerable ( $s = v$ ), the lieutenant got the right signal ( $\hat{s} = v$ ) and chooses to betray, while  $q(1 - \theta)$  is the probability that the incumbent is vulnerable ( $s = v$ ) and the lieutenant gets the wrong signal ( $\hat{s} = n$ ).

The reason why the dictator appoints the better-informed lieutenant is efficiency. Decisions based on the informed advice save the cost of the strong policy. The lieutenant recommends strong policy ( $d = \bar{P}$ ) with probability

$$r(\theta) = (q\theta + (1 - \theta)(1 - q))F(R^*(\theta)).$$

It is straightforward that  $(\theta)$  is decreasing with respect to  $\theta$ .

Being aware of the constraints imposed by the lieutenant's possible disloyalty, the dictator faces the following maximization problem. Let the expected utilities of winning and losing in the current power struggle be denoted by  $U$  and  $V$ , respectively. Then the dictator's optimization problem is

$$\max_{\theta} \{(1 - p(\theta)) U + p(\theta) V - r(\theta) C\}.$$

The dictator's solution of this maximization problem is given by the following proposition.

**Proposition 2.** *There is a unique perfect Bayesian equilibrium in the one-period-choice-of-a-lieutenant game. The dictator chooses a lieutenant characterized by  $\theta^*$ , who is more competent, i.e.,  $\theta^*$  is higher, when the stakes are lower for the dictator ( $U - V$  is lower) and the measures that have to be taken are more costly ( $C$  is higher).*

Basically, this proposition says that an insecure dictator, e.g., the one that fears that he will be executed upon removal from power, is bound to select less competent lieutenants. Indeed, as we know from Proposition 1, a more competent lieutenant is more likely to betray the dictator. With higher stakes, loyalty, the flip side, in equilibrium, of competence, becomes relatively more important for the dictator.

Proposition 2 is an important building block of our dynamic story: it shows that when the stakes for the dictator leader are high, the leader chooses to select a less competent lieutenant, thus increasing the probability of a bad policy choice. When are the stakes high for the dictator? When his reputation has deteriorated as a result of past repression decisions. So, repressing the opposition does reduce the probability of a challenge in the current period, yet has an endogenous opportunity cost – a leader with a bad reputation is bound to select less competent subordinates, increasing, ultimately, the probability of a fatal policy mistake.

## 4 Joint Dynamics of Repression and Bad Policy

In the previous section, we analyzed the leaders' choice of his information-gathering institution. Now, we are going to analyze how this choice and the policy choice evolve over time, responding

to the leader's choice of the repressiveness of his regime. At the end of the section, we illustrate the equilibrium dynamics with a simple simulation exercise.

To study formally the dynamics of the regime, we write down the continuation values that correspond to different choices of the winner of a power struggle. As before,  $(X, Y) \in \Lambda$  describes the history of the winner-loser pair at the point when the winner makes the repression decision. Then  $U_{XY}^E$  is the continuation value of the winner when the choice is to repress the opposition;  $U_{XY}^S$  – when the decision is not to repress the opposition, and  $U_{XY} = \max(U_{XY}^E, U_{XY}^S)$  is the optimal choice.  $V_{XY}$  is the continuation value of the loser of the power struggle, and  $W_{XY}$  is the result of the optimal choice of the lieutenant's competence. It is straightforward to demonstrate, and this is done formally in the Appendix, that any of the values  $U_{XY}, U_{XY}^E, U_{XY}^S, V_{XY}, W_{XY}$  is not greater than  $-T$ , and is smaller than  $\frac{H-\delta K}{1-(1-\delta)\beta}$ , because a player may lose at most  $-T$ , and only once in his life, and likewise, he may not expect to get more than  $H - \delta K$  each period.

Now let us write down the equations that link these expected utilities to each other. Suppose for a moment that  $\alpha_{XY}$ , the probability of repressions when the dictator-opponent reputation is  $(X, Y)$ , and  $p_{XY}$ , the probability that the power struggle is won, which is a function of  $\theta$ , are given. Then utilities must satisfy the following conditions.

$$U_{XY} = \max(U_{XY}^E, U_{XY}^S) \quad (1)$$

$$U_{XY}^E = H + (1 - \delta) \beta ((1 - \mu) (H + (1 - \delta) \beta W_{BG} - \delta K) + \mu W_{BG}) - \delta K \quad (2)$$

$$U_{XY}^S = H + (1 - \delta) \beta ((1 - \delta) W_{XY} + \delta W_{XG}) - \delta K \quad (3)$$

$$V_{XY} = (1 - \alpha_{XY}) (1 - \delta) \beta ((1 - \delta) ((1 - p_{YX}) V_{XY} + p_{YX} U_{YX}) + \delta W_{YG}) - (1 - \alpha_{XY}) \delta K - \alpha_{XY} D \quad (4)$$

$$\begin{aligned} W_{XY} &= \max_{\theta} W_{XY}(\theta) \\ &= \max_{\theta} \{(1 - p(\theta)) U_{XY} + p(\theta) V_{YX} - r(\theta) C\} \end{aligned} \quad (5)$$

The first equation simply says that the winner of the power struggle maximizes his expected utility when he decides whether or not to repress the opposition. If the opposition is repressed, he earns bad reputation, but his next opponent will necessarily have good reputation – as this would be the opponent's first entry. Following repression, this opponent will appear in the next period with

probability  $\mu$ , and after one period with probability  $1 - \mu$ . If the opposition is not repressed, the reputation of the winner (the new does not change, and he will face an opponent (if he does not die), who will be the same, unless the opponent dies (in this case, a new opponent with a good reputation emerges). The loser, in his turn, expects to be repressed with probability  $\alpha_{XY}$ , and even if he is not, he may die with a certain probability. However, if he survives, he has a chance to regain power – either through a struggle, or simply because the winner peacefully dies himself. Finally,  $W_{XY}$  is just a weighted average of utilities from winning and from losing (accounting for costs of a policy choice, of course).

In the Appendix, we show that if we fix any  $\alpha_{XY}$  and  $p_{XY}$  in  $[0, 1]$ , equations (1) – (5) have a unique solution  $(U_{XY}, U_{XY}^E, U_{XY}^S, V_{XY}, W_{XY})$  – 15 variables in total – which continuously depend on all parameters:  $\alpha_{XY}, p_{XY}, \beta, \delta, H, C, K, T$ . (See Subsection A1 for the list of notation.) This makes analysis of utilities if  $\alpha_{XY}$  and  $p_{XY}$  are fixed more or less straightforward, and that is what we will start with. In the full game, however, values  $\alpha_{XY}$  and  $p_{XY}$  are determined endogenously rather than fixed. To form an equilibrium, they must satisfy the following conditions. First,

$$\alpha_{XY} \in \begin{cases} \{0\}, & \text{if } U_{XY}^E < U_{XY}^S; \\ [0, 1], & \text{if } U_{XY}^E = U_{XY}^S; \\ \{1\}, & \text{if } U_{XY}^E > U_{XY}^S. \end{cases} \quad (6)$$

Second,

$$p_{XY} = p(\theta_{XY}), \quad (7)$$

where

$$\begin{aligned} \theta_{XY} &= \arg \max_{\theta} W_{XY}(\theta) \\ &= \arg \max_{\theta} ((1 - p(\theta)) U_{XY} + p(\theta) V_{YX} - r(\theta) C) \end{aligned}$$

(this complies with the definition of  $\theta_{XY}$  above).

To proceed, it is helpful to incorporate these conditions in stages. If a set of utilities and  $\alpha_{XY}$ 's satisfies (1) – (5) and (6) for a certain given set of  $p_{XY}$ 's, we say that it forms a dynamically consistent path. More precisely, we define it as follows.

**Definition 2.** A vector of state-dependent probabilities of repression and odds of winning the power struggle  $(\alpha_{XY}, p_{XY}) \in [0, 1]^6$  is said to form a dynamically consistent path if utilities

$(U_{XY}, U_{XY}^E, U_{XY}^S, V_{XY}, W_{XY})$ , uniquely identified by equations (1) – (5), satisfy conditions on  $\alpha_{XY}$ 's (6).

With this definition, an equilibrium is a strategy profile that generates dynamically consistent path for which (7) holds, because rationality of lieutenants is already incorporated in equations (1) – (5). Analytically, it is convenient to focus, for the time being, on dynamically consistent paths, which allows us to take the probabilities of power transition as given. We proceed by characterizing the behavior of those who kept or won power in different situations, starting with the case, in which either the leader or the opposition leader already has a bad reputation.

**Proposition 3.** *Suppose that in a power struggle there is a politician with a good reputation and a politician with reputation of repression:*

(i) *A politician with past experience of repression (B) always represses opposition with no experience of repression (G).*

(ii) *A politician with no past experience of repression (G) represses an opposition leader with a bad reputation (B), provided that  $\delta$ , the risk of natural death, is sufficiently small.*

(iii) *In each of these cases, lieutenants hired are of equal competence. More precisely, on any dynamically consistent path,  $\theta_{BG} = \theta_{GB}$ , and these values do not depend on  $p_{XY}$ 's. Furthermore, in any equilibrium,  $p_{BG} = p_{GB}$ , and these values may be found as  $p(\theta_{BG})$ .*

The dynamic mechanism behind this proposition is intuitive. A leader who has repressed in the past cannot undo the bad reputation. Therefore, he does not suffer any negative effects from repression, while repression results in a safe period with a positive probabilities. In terms of  $\alpha$ 's, this means  $\alpha_{BG} = 1$ , and therefore  $V_{BG} = -T$ . Now, if the loser is committed to repress the winner if spared and then returned to power at some point, he has to be repressed. Naturally, the condition that the probability of exogenous death,  $\delta$ , is sufficiently small ensures that it does not make sense not to repress the loser with a bad reputation in the hope that he will die on his own. As a consequence, the lieutenants chosen in both cases will have equal competence. The reason for this is the dictator's understanding (in either of these cases) that he will not survive if he loses. For that reason, he is not interested in the chance of coming back to power, and therefore chooses his lieutenant without taking any of  $p_{XY}$ 's into account. Furthermore, (7) implies that

that  $p_{BG} = p_{GB}$  in any equilibrium. Denote them, for simplicity, by  $\hat{p}$ . It is also evident that  $W_{BG}$  are the same on all dynamically consistent paths, and thus  $U_{BG}^E = U_{GB}^E = U_{GG}^E$  are also the same; we denote this value by  $\hat{U}^E$ .

#### 4.1 Characterizing Repression

Our next goal is to find whether or not a winner with a good reputation represses the loser with a good reputation. This is critical for the stability of the good (no repression) equilibrium. Technically, the question is what values  $\alpha_{GG}$  may take in an equilibrium. First, note that  $U_{GG}^E$  is known, since it equals  $U_{BG}^E$  (see (2)). It is easy to see that  $\alpha_{GG} = 1$  always forms a dynamically consistent path (and if  $p_{GG}$  equals  $p_{BG}$ , it is a real equilibrium). In other words, if one expects to be repressed regardless of his past actions, he opts to repression when he has a chance himself. Therefore, there always exists a ‘repressive’ equilibrium, which is uniquely defined by condition  $U_{GG}^E > U_{GG}^S$ . However, there may exist other equilibria. Consider the case  $U_{GG}^E \leq U_{GG}^S$ , or equivalently,  $U_{GG} = U_{GG}^S$ . Denote  $U_{GG}^S$  as a function of  $\alpha_{GG}$  (holding all  $p_{XY}$ ’s and  $\alpha_{BG} = \alpha_{GB} = 1$  constant, and replacing  $U_{GG} = U_{GG}^S$ ) with  $U_{GG}^S(\alpha_{GG})$  in (1). In the Appendix, we prove that  $U_{GG}^S(\alpha_{GG})$  is a continuous strictly decreasing function of  $\alpha_{GG}$ , and  $U_{GG}^S(1) < \hat{U}^E$ . Now we formulate the following proposition.

**Proposition 4.** *The following is true about any dynamically consistent path:*

- (a) *For any  $p_{XY}$ ’s there always exists a unique dynamically consistent path in which  $\alpha_{BG} = \alpha_{GB} = \alpha_{GG} = 1$ . If  $p_{XY}$ ’s are set to be equal to  $p^*$ , then it is an equilibrium, and moreover, it is the only equilibrium where all  $\alpha_{XY}$ ’s are equal to 1.*
- (b) *If  $U_{GG}^S(0) \geq \hat{U}^E$  then there is a dynamically consistent path with  $\alpha_{GG} = 0$  (the ‘good’ consistent path), and if  $U_{GG}^S(0) > \hat{U}^E$ , then there also exists a mixed strategy path, which is dynamically consistent, and on which  $0 < \alpha_{GG} < 1$ , and  $U_{GG}^S = U_{GG}^E = \hat{U}^E$ . There may exist at most one equilibrium such that  $0 < \alpha_{GG} < 1$ .*
- (c) *For any given power transition probabilities  $p_{XY}$ , good (no repression) dynamically consistent paths and good equilibria exist for a wider range of parameters when  $T$  is high and  $\mu$  is high.*

While Proposition 4 looks technical, it is both important and intuitive. Part (a) simply states that it is always an equilibrium when every leader chooses repression after winning the power struggle. Indeed, if the current decision-maker expects to be repressed once out of power regardless of his reputation, choosing repression today does not have negative implications. Part (b) describes conditions, under which there exists a no-repression equilibrium; if such an equilibrium exists, there exist “intermediate equilibria”, in which a string of no-repression periods might end with repression.

The intuition behind the comparative statics, part (c), is as follows. First, a higher  $\mu$  (the lower effectiveness of repression) means that it is less profitable to repress, and high  $T$  means that gaining bad reputation is more dangerous. Both effects cause no-repression dynamically consistent paths to exist for a wider range of other parameters.

In general, there are two basic channels through which the pay-off of the loser affects the incentives the decision-maker has. First, if the disutility of being removed from power decreases, then for the leaders that makes the decision at the moment, the costs associated with repressing the opposition and gaining bad reputation as a result decreases as well. Second, the current decision maker takes into account thoughts of the next decision maker (the next successful opponent), the one who will be deciding his fate once he loses. Since the next leader also faces lower costs of repressing the opposition, the reputation becomes less valuable for the current one. The impact of the discount factor is straightforward. A decrease in  $\beta$  makes the absence of opponents, which is not achievable if the opposition is not repressed, more valuable.

## 4.2 Equilibrium Paths and Competence

Though equilibria of the game may lead to a variety of different paths, as there are random shocks that influence them, we may delineate three substantially different paths, which correspond to different equilibria. In this paper, we specifically focus on the “degenerate autocracy”, which corresponds to a set of equilibria that generates a specific path. In what follows, we show that these paths that feature frequent, even if not necessarily every-period, repression are robust. Then, we demonstrate that these paths are the worst in terms of the quality of information-gathering and decision-making. (See Subsection 4.3 for a simple exercise simulating the equilibria paths.)

The first group of paths are ‘stable autocracies’. When a leader with no-repression reputation wins the power struggle, he does not need to repress the opposition. When he eventually loses power, he is not repressed and has an opportunity return to power. Thus, he faces relatively low stakes in the future power struggles, which allows him to appoint competent subordinates, make good policy decisions with a higher probability, and have, as a result, a high probability of surviving the next struggle.<sup>7</sup> New leaders that appear because we allow for an exogenous death with a positive probability, play the same strategies. On this path, if we add a possibility of democratization, it does not result in behavioral change of the dictators prior to democratization.

Another group of stable paths are “consecutive degenerate autocracies”. Every time a power struggle occurs, it is followed by repressions. Though some people might die peacefully, this does not lead to the escape from repression trap, in which the new-coming leader organizes repressions against the former regime as he fears their return to power. For our purposes, the most interesting dynamics is the complementarity between the repressiveness of the regime and the low quality of decision-making along this group of stable paths. A vulnerable dictator represses opposition to reduce the probability of a challenge in the next period. This raises the stakes for the leader as, after the repression, the probability that he will be repressed if (when) he loses power, increases. In response to the increasing stakes, the leader has to appoint more loyal, that is, less competent subordinates which results in a higher probability of a bad policy choice.

Without a refinement, our dynamic game has, naturally, more equilibria. The rest of stable paths are “mixed”: the dictator and the opposition may swap their positions several times, or be replaced by newcomers, if they die by chance, but eventually the winner of the power struggle chooses repression rather than no repression. This is followed by a sequence of “repression only” periods: a winner with a history of repression represses a loser with no history of repression, and vice versa. Still, if either dictator or opponent with a reputation of repression dies, the chain of

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<sup>7</sup> Authoritarian regimes, in which the losers of political struggles are able to stage a comeback – that is, regimes that follow our “good path”, are not exceptional. Of 54 leaders of Mexico in the 19th century, 17 have held this position more than one time, and 7 came back to power at least two times. General de Santa Anna, “the Napoleon of Mexico”, came back at least 5 times; most of power changes were military coups. In Chile, General Ramon Freire came back 5 times. In Venezuela, among 56 changes in leadership, elected, military, and provisional in 1830-1910, there were 14 comebacks by 10 leaders who had been leaders before. Post-1945, there was a number of examples when politicians who have fallen from grace, were arrested, tried, jailed, or exiled, have subsequently returned to power. Poland’s Władysław Gomułka, Hungary’s Janos Kadar, and China’s Deng Xiaoping are perhaps most prominent examples.

repression may end, as now both the dictator and opposition do not have history of repressions. Then, the story repeats itself.

On the “mixed” equilibrium paths the autocracy does not “stay degenerate”. It is degenerate – repressive and incompetent – from time to time, probabilistically. Very much in line with the existing anecdotal evidence, the dictatorships become repressive and incompetent at later, rather than earlier, stages of the dictator’s political life.

Our next Proposition 5 states formally the core result about the quality of governance on the equilibrium paths. It is straightforward to observe that on any equilibrium path where dictators always repress the opposition, the competence of lieutenants is the same  $\theta_{BG} = \theta_{GB} = \theta_{GG}^E$ . The interesting case, the one that illustrates our “degenerate autocracy” theory, requires comparing  $\theta_{GG}^E$  and  $\theta_{GG}^S$  in the case where both equilibria exist. As we shall see, the quality of governance – the competence of subordinates and, consequently, the probability of a wrong policy choice – is lower on those equilibrium paths that feature more repression.

**Proposition 5.** *In any equilibrium, the competence of subordinates satisfies the following comparative statics results.<sup>8</sup>*

- (a) *If the power struggle features at least one politician with history of repressions, the dictator’s choice of lieutenant’s competence is the same regardless of the equilibrium played.*
- (b) *If the power struggle features two politicians with no history of repressions, the dictator’s choice of lieutenant’s competence in the good equilibrium is (weakly) higher than the choice of the lieutenant’s competence in any mixed equilibrium, which is in turn (weakly) higher than the lieutenant’s competence in any “consecutive degenerate autocracy” equilibrium.*
- (c) *The probability that repression is ineffective,  $\mu$ , has no effect on the competence of lieutenant in the good equilibrium. In any other equilibrium, the higher effectiveness of repression leads to less competent (and endogenously more loyal) lieutenants.*

The intuition behind the comparison of government competence and, correspondingly, the probability of a wrong policy choice along different equilibrium paths was discussed above. The

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<sup>8</sup>The same is true for dynamically consistent paths. In Appendix, we will prove this (and use this fact to prove the Proposition 5 itself).

Notation	Description	Value
$\mu \in (0, 1)$	Probability of dictator facing an opponent next period if repression is chosen	0.5
$q \in (0, 1)$	Probability of dictator being vulnerable	0.3
$C > 0$	Cost of the strong policy	30
$w$	Lieutenant's wage if he did not betray	1
$-\pi$	Lieutenant's punishment for betrayal	-10
$\beta < 1$	Discount factor	0.95
$H$	Utility for the winner of the power struggle	35
$-T < 0$	Utility for loser if executed	-50
$\delta \in (0, 1)$	Probability of player dying naturally in a given period	0.05
$-K$	Utility for a natural death. $K < T$ , $\delta K < H$	-15
$\lambda$	Rate parameter in exponential distribution for $R$ , the affinity between a lieutenant and the challenger	0.1

Table 1: Parameter Space for Simulations

higher is the probability of repression, the higher are stakes for the dictator in the struggle. This forces him to look for more security in at the expense of spending additional resources, and the subordinates they hire are less competent and more loyal. Therefore, cruel and insecure dictators are more likely to choose poor policymakers in equilibrium.

### 4.3 Simulation Exercise

To illustrate the different paths that we discussed above, we solve numerically for equilibria of the game and then simulate multiple instances of the game for each equilibrium type. The core of the solving algorithm is fixed point iteration. (The algorithm itself is given in the Appendix.) Relying on the logic of Lemma A3 in the Appendix, we know that under the given norm, the set of equilibrium conditions (1)-(5), repeatedly applied, will converge towards a fixed point which is the solution. We incorporate (6) by using the constraints on  $\alpha$  as initial guesses for our iterations, and (7) by using the  $\theta$  that solves (5) to compute  $p_{XY}(\theta)$  as defined in Section 3.2. (To simplify calculations, we further assume that  $\Pr(\hat{s} = v | s = v) = 1$ , while  $\Pr(\hat{s} = n | s = n) = \theta$ .)

In all equilibria, our initial guess for the vector  $(U_{XY}, U_{XY}^E, U_{XY}^S, V_{XY}, W_{XY})$  is bounded by the results in Lemma A2. The “good” and “bad” equilibria are the easiest to find, since by Propositions 3 and 4 we know the values of  $\alpha$  for all reputation states. We can thus make initial guesses for  $\alpha_{XY}$  that are already correct, and our fixed point iteration algorithm converges quickly. Since the reputation states are interdependent, we iterate the three states in lockstep.

In the mixed equilibrium, our initial guess for  $\alpha_{GG}$  will be further from the unique interior

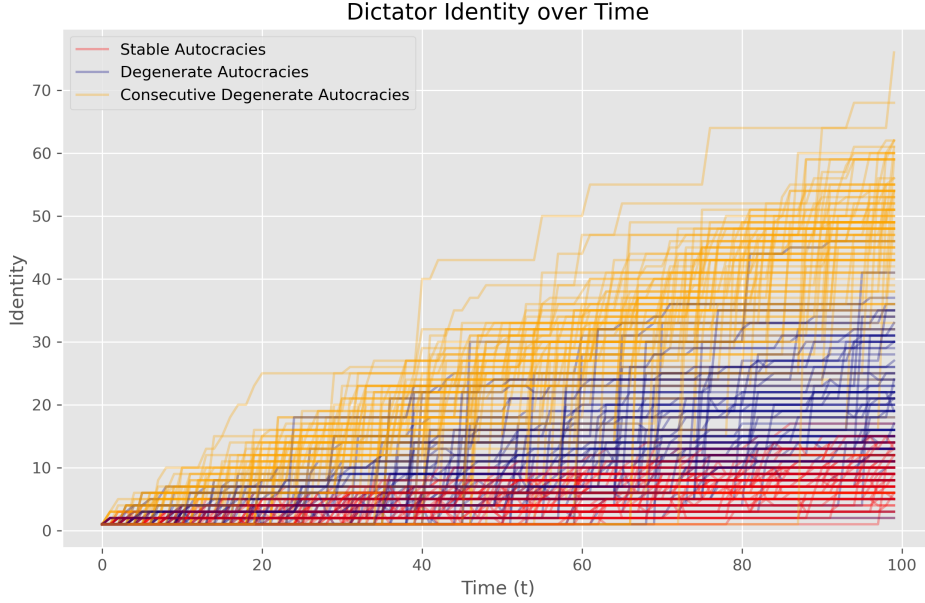


Figure 1: Simulations of Dictator Identity

solution (we still know  $\alpha$  takes the corner solution in  $(G, B)$  and  $(B, G)$  from Proposition 3, with small  $\delta$ ). First, notice we can refine our guess for  $\theta_{GG}$  from  $[0, 1]$  to  $[\theta_{GG}^B, \theta_{GG}^G]$  from Proposition 5(b). Next, we need a new equation that will help us iterate  $\alpha$  towards the solution. Recall that in the case of the mixed equilibrium, (A4) provides a second expression for  $V_{GG}$ . In each iteration, we can solve for  $\alpha$  that equates (4) and (A4). As we iterate the main system of equations toward convergence,  $\alpha$  should also converge (under an absolute convergence criterion). Depending on the initial guess,  $\alpha$  may converge towards the “good” equilibrium with this approach. If so, we restart the algorithm with new initial guesses and repeat until  $\alpha$  converges to an interior solution, which we know will be unique.

The result is a set of three equilibrium solutions, with values of  $(U_{XY}, U_{XY}^E, U_{XY}^S, V_{XY}, W_{XY}, \theta_{XY}, \alpha_{XY})$  for each reputation state. Note that this set contains only three unique values for lieutenant competence  $\{\theta_{GG}^G, \theta_{GG}^B, \theta_{GG}^M\}$ , and three unique values for the probability of repression  $\{0, 1, \alpha_{GG}^M\}$ . We can proceed to simulate the steps of the within-period game tree, with appropriate stochastic elements. We allow each of 100 simulations per equilibrium to run for  $T = 100$  periods, recording the evolution of the choice variables and identities. Figure 1 plots the numeric identity of the dictator over time to establish the general

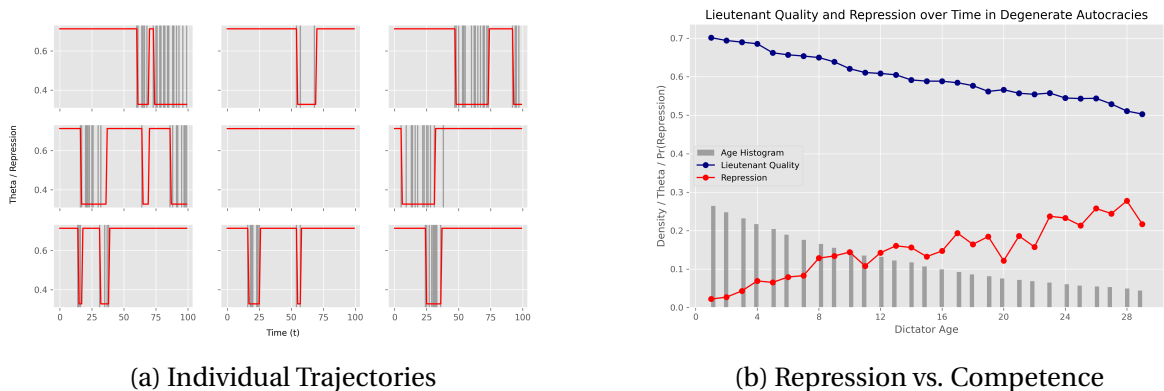


Figure 2: Left: Each picture is 100 simulations: the lieutenant’s quality is in red; period of repressions are shaded. Right: the average probability of repression (red) and quality of the lieutenant (blue) as function of the incumbent’s periods in power.

dynamics of the game, and how the paths compare across equilibria. Mixed and bad equilibrium churn through dictators quicker, since repressions and bad policy decisions are more frequent.

Due to the stochastic elements, the relationship between competence and repression is best viewed as an event study. In the mixed equilibrium, periods of repression, in which repressions are (imperfectly) successive, may begin but the cycle can be broken by exogenous death. Thus, any repression event is preceded and proceeded by other repression events with some probability greater than  $\alpha_{GG}^M$ . By Proposition 5, we expect lieutenant quality to decline during repressive periods.

We then focus on the mixed equilibrium of the game, where repression is possible but does not happen 100% of the time. With parameters set as in Table 1, we simulate the mixed equilibrium 100 times, each simulation lasting 100 periods. Figure Figure 2(a) shows how some of these games evolve individually. Note the correlation between the periods repressions, which are shaded, and the low quality of lieutenants.

Each time repression occurs, we create a window around the repressive event and define event time relative to the period of repression. Thus, each of the 100 simulations potentially yields many windows, one per period of repression. We then plot an event study in Figure A-1 in the Appendix, collapsing each simulation over the event time and taking the mean of the choice variables. Averaging a dummy which records the action  $A_t = E$  gives the probability of repression. Figure A-1(a) shows the probability of repression relative to the 0 moment; Figure A-1(b) shows the evolution

of the quality of lieutenants.

A second useful way to use an event study is to define an event at the moment a player first becomes dictator. This allows us to study behavior over the lifetime of the regime. This view, in [Figure 2\(b\)](#), makes clear the pattern that later in life, even dictators who start out with strong advisors will regress in governance quality and accelerate their repressive activity, despite their vulnerability  $q$  being constant. Few dictators reach old age, as seen in the declining histogram over age. This is partly due to the exogenous death, but is endogenously driven by the trend towards repression, lower quality governance, and eventual fatal mistakes.

## 5 Robustness

There are several modeling choices that require further discussion. First, while reputation plays an important role in the mechanism underlying the degenerate autocracy, our model of reputation, is effectively a shortcut. Second, in our attempt to go beyond the two-state Markovian approach to modeling authoritarian dynamics, we have restricted possible strategies to those that rely on the binary reputation variable. Finally, it may look like that the static model, in which the leader chooses only a single subordinate, whereas realistically there would likely be multiple subordinates, is with loss of generality.

Our choice to make “reputation” a binary variable, instead of using a game of imperfect information with a commitment type following the pioneer work of [Kreps and Wilson \(1982\)](#) and [Milgrom and Roberts \(1982\)](#), has a single goal: to simplify the algebra to be able to focus on the substantive issues. Assuming existence of a particular commitment type, for example, a committed “gentle” leader who never represses the opposition, we would get that any episode of non-repression increases the posterior probability that the dictator is such a type, which improves the odds that he is not repressed. As a result, even “normal” dictators would have an incentive to earn the reputation for being gentle. However, the resulting dynamics will be very similar to the one we study, albeit at a cost of introducing additional notation and cumbersome algebra. In this paper, we opt for simplicity.

The two-state Markov models have become a staple of studies in authoritarian dynamics,

thanks to the combination of tractability and richness of potential strategic interactions (Tornell and Lane, 1999; Acemoglu and Robinson, 2001; Bueno de Mesquita et al., 2003; Acemoglu, 2003; Gallego and Pitchik, 2004; Acemoglu and Robinson, 2005; Jack and Lagunoff, 2006; McGillivray and Smith, 2006; Padró i Miquel, 2007; Robinson and Torvik, 2009; Svobik, 2009; Herrera and Martinelli, 2013; Leventoğlu, 2014). For questions like ours, however, using the same approach creates a substantive problem: we do want to study the endogenous dynamics, in which changes in the previous periods lead to different choices in the subsequent ones. As a result, we opted for, essentially, a game of imperfect public monitoring: players' strategies depend not on whole histories but only on binary variables that summarize the acquired reputation of the winner (the new dictator) and the loser (the new opposition) of a power struggle.<sup>9</sup>

Egorov and Sonin (2015) allowed the winner's strategy to depend on the total number of periods in which he, the winner, and the loser have chosen repression in the past. As a result, there are equilibria, in which the probability that the winner of the power struggle who has already repressed in the past is growing monotonically for a finite number of periods. In fact, for a range of parameters, one might construct an equilibrium in which this probability strictly increases for any fixed number of periods. With power struggle lottery probabilities fixed, this is an extension of Proposition 4. However, the complexity of that model makes it difficult to study the core issue of the current paper, the dynamic relationship between repressiveness and quality of government. Unlike the current model, Egorov and Sonin (2015) is a model in which the power struggle probabilities are exogenous.<sup>10</sup> Still, its results serve as a robustness check for the inter-period part of our story.

Another major simplification in our baseline model is the choice of an institution that determines which information the dictator's decisions are based on. There, the leader appoints a

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<sup>9</sup>It needs to be noted that, formally, nearly any game can be made Markov as defined in Fudenberg and Tirole (1991), with a proper labeling of histories as states. In our case, if the reputation pairs are considered states, our equilibrium is Markov perfect with respect to this state partition, and assuming some minimal change in cost of repression for the first and subsequent times (e.g., because of learning by doing) would make these states payoff relevant. Still, we prefer to make a distinction between models in which the possible state include, e.g., the state of the economy as in Acemoglu and Robinson (2001) – the dictator's "vulnerability" is such a state in our model, and models like ours in which we allow strategies to rely on a pair of binary variables summarizing reputation.

<sup>10</sup>Egorov and Sonin (2011) considered a static model, in which the probability of the dictator losing power depended, stochastically, on both the type and actions of the opponent. The current model does not have this complication. This allows, instead, to make the competence-loyalty model a construction block in a dynamic model, without which it is not possible to have a theory of degenerate autocracy.

single subordinate that gathers information for the leader to use in the power struggle. In fact, the logic naturally extends to a situation when the leader appoints a council of lieutenants, with lieutenants of, potentially, heterogeneous information-processing capacity. With a council instead of a single lieutenant, the added complication is that they need to coordinate their actions of betraying the leader. This makes the “institutional” part of our model more realistic: [Gehlbach and Keefer \(2011\)](#) considered the Chinese Communist party as an information-gathering device, a far more complicated institution than a single lieutenant. (See [Francois, Trebbi and Xiao, 2023](#), for a deep investigation of the role of factions in the authoritarian politics of China; see XXX for a survey of recent research.)

Appointing a council instead of a single lieutenant, the dictator enjoys two kinds of benefits. First, with a council, more information can be aggregated, especially if members’ signals are uncorrelated or only partially correlated. Second, the members of the council face a coordination problem with respect to treason, one which dictators usually took advantage of. While the latter advantage was already well-understood by Machiavelli, the former is much less so.

Assume that the dictator has to hire two lieutenants of types  $\theta_1$  and  $\theta_2$ .<sup>11</sup> We consider the following modification of our basic setup. Suppose that each agent receives the same (correct!) information about potential reward  $R$ , but the signals about the leader’s vulnerability,  $\widehat{s}_1, \widehat{s}_2 \in \{v, n\}$  are independent conditional on  $s \in \{v, n\}$ . (Thus, information can indeed be aggregated.) As in the baseline model, each lieutenant chooses either a weak or a strong policy; they choose their actions simultaneously and independently. The total cost of strong policy is  $C$ . Again, the dictator survives if he is not vulnerable or *one* agent chooses the strong policy. Lieutenants are rewarded or punished independently, i.e. participation of one of them in a coup does not undermine the potential benefits of the other. Finally, we assume that dictator’s orders to lieutenants is  $d(\widehat{s}_j = v) = \overline{P}$ ,  $d(\widehat{s}_j = n) = \underline{P}$ ,  $j \in \{1, 2\}$ .

As before, we proceed with backward induction. Again, we observe that if agent  $i$  receives signal  $\widehat{s}_i = n$ , then  $i$  obeys the dictator (and chooses  $\underline{P}$ ). If  $\widehat{s}_i = v$ , the Bayes’ formula yields

$$\mathbf{P}(s = v \mid \widehat{s}_i = v) = \frac{q\theta_i}{q\theta_i + (1 - \theta_i)(1 - q)}.$$

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<sup>11</sup>Alternatively, we could consider the case where agents may have distinct types  $\theta_1$  and  $\theta_2$ . However, this would at some point lead to unnecessary complications that would obscure the coordination effect we want to depict.

Now, the probability that the other lieutenant  $-i$  received signal  $\widehat{s}_{-i} = v$  conditional on  $s = v$  is  $\theta_{-i}$ . Thus,

$$\mathbf{P}(\widehat{s}_{-i} = v, s = v \mid \widehat{s}_i = v) = \theta_{-i} \frac{q\theta_i}{q\theta_i + (1 - \theta_i)(1 - q)}.$$

Let  $R_j^* = R_j^*(\theta_j)$  be the threshold such that if  $R > R_j^*$ , lieutenant  $j$  chooses weak policy when  $\widehat{s}_j = v$ ,  $j \in \{1, 2\}$ . The thresholds  $R_i^*$  are solutions for the following system of equations:

$$R_i^* \frac{q\theta_i\theta_{-i}}{q\theta_i + (1 - \theta_i)(1 - q)} (1 - F(R_{-i}^*)) - \pi \left( 1 - \frac{q\theta_i\theta_{-i}}{q\theta_i + (1 - \theta_i)(1 - q)} (1 - F(R_{-i}^*)) \right) = w, \quad i \in \{1, 2\}.$$

In a symmetric equilibrium,  $R_1^*(\theta) = R_2^*(\theta)$ . Compared to the case of one lieutenant, the multi-agent threshold  $R_1^*(\theta) > R^*(\theta)$  for every  $\theta$ . This is intuitive as probability that the challenger wins, conditional on the lieutenant's signal that the dictator is vulnerable, is lower in the case of two lieutenants than in the case when there is a single lieutenant. As a result, the dictator could choose lieutenants of a higher quality when they form a committee. However, the main comparative statics result remains the same: when the leader has higher stakes, the equilibrium choice of the lieutenants' competence is lower.

## 6 Conclusion

Many modern dictatorships have ended up making disastrous decisions, such as Galtieri's attack on the Falklands in 1982 or Putin's invasion of Ukraine in 2022. Even when these decisions are not ultimately fatal for the regime, such as Mao's Great Leap Forward in China or Pol Pot's collectivization drive in Cambodia, they typically involve both a monumental miscalculation and an institutional environment in which subordinates have no chance to prevent the decision from being made and implemented.

In this paper, we develop a theory of degenerate autocracy, a stage in the life cycle of an authoritarian regime that is characterized by increased repressiveness and, simultaneously, a deteriorating quality of decision-making. We show that these two tendencies reinforce each other. Repression against political opponents increases the stakes for the dictator, which in turn shifts his priorities from competence to loyalty. Our theory sheds light on the governing mechanisms in repressive, inefficient authoritarian regimes.

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# Appendix

## A1 Summary of Notation

Notation	Description
$i = 1, 2, \dots \infty$	Players
$t = 1, 2, \dots \infty$	Discrete time
$D_t \in \mathbb{N}$	The identity of incumbent dictator in period $t$
$O_t \in \mathbb{N}$	The identity of opponent in period $t$ (if one exists, else $O_t = 0$ )
$N_t \in \mathbb{N} > 2$	The identity of “next” challenger who has not joined the game by period $t$
$W_t, L_t \in \mathbb{N}$	The identity of the winner and the loser in period $t$
$A_t \in \{E, S\}$	Action of the winner in period $t$ to either repress (execute, $E$ ) or not repress (spare, $S$ )
$\mu \in (0, 1)$	Probability of dictator facing an opponent in next period following repression
$q \in (0, 1)$	Probability of dictator being vulnerable
$d \in \{\bar{P}, \underline{P}\}$	Policy of dictator, either strong ( $\bar{P}$ ) or weak ( $\underline{P}$ )
$C > 0$	Cost of the strong policy
$v$	State of the world in which dictator is vulnerable
$n$	State of the world in which dictator is secure
$\theta \in [0, 1]$	Competence of the lieutenant chosen by the dictator
$Z(\theta)$	Dictator’s objective function in choosing a lieutenant
$s \in \{n, v\}$	Lieutenant’s signal about the state of the world
$w$	Lieutenant’s wage if he did not betray
$-\pi$	Lieutenant’s punishment for betrayal
$R \in [0, +\infty)$	Lieutenant’s reward if the opponent supplants the dictator
$F(x), f(x)$	CDF and PDF of the random variable $R$ ; $f(x) > 0$ and $f'(x) < 0$ for $x > 0$
$U_t(i)$	Instantaneous utility received by player $i$ in period $t$
$\beta < 1$	Discount factor
$H$	Utility for the winner of the power struggle
$-T < 0$	Utility for loser if executed
$\delta \in (0, 1)$	Probability of player dying naturally in a given period
$-K$	Utility for a natural death. $K < T$ , $\delta K < H$
$G, B$	Reputation of a player, either good or bad
$(X, Y)$	A tuple of the reputations of the winner and loser
$\Lambda$	Set of all combinations of winner’s and loser’s reputations
$p \in (0, 1)$	Probability of winning the power struggle for the dictator
$r \in (0, 1)$	Probability of the lieutenant recommending the strong policy
$U$	Expected utility for the dictator of winning the power struggle
$V$	Expected utility for the dictator of losing the power struggle
$U_{XY}^E$	Continuation value of the winner when the choice is to repress (execute) the opponent for a given pair of winner-loser reputations
$U_{XY}^S$	Continuation value of the winner when the choice is to not repress (spare) the opponent for a given pair of winner-loser reputations
$U_{XY}$	Continuation value of winner following an optimal choice over whether to repress or not the opponent for a given pair of winner-loser reputations
$V_{XY}$	Continuation value of loser of the power struggle for a given pair of winner-loser reputations
$W_{XY}$	Continuation value of the dictator following an optimal choice of lieutenants’ competence for a given pair of winner-loser reputations
$\alpha_{XY} \in (0, 1)$	Probability of repressions for a given pair of winner-loser reputations
$p_{XY} \in (0, 1)$	Probability of the dictator winning the power struggle for a given pair of winner-loser reputations
$\hat{p} \in (0, 1)$	Equilibrium probability of the dictator winning the power struggle in the case of reputations $(B, G)$ or $(G, B)$
$\hat{U}^E$	Equilibrium continuation value of the winner when the choice is to repress in the case of reputations $(B, G)$ , $(G, B)$ or $(G, G)$

## A2 Detailed Game Description

**Formal Setup.** There is an infinite sequence of identical players  $i = 1, 2, \dots \infty$  who join the active part of the game in this sequence. In each period  $t$ , one player, denoted by  $D_t \in \mathbb{N}$ , is the incumbent dictator in this period. There may also be or not be an opponent. We write  $O_t \in \mathbb{N}$  to denote the opponent's number in the sequence if there is one, and  $O_t = 0$  if there is none. Initially,  $D_1 = 1$  and  $C_1 = 2$ .

For each period  $t$ , denote the least of the identities of players who have not joined the active part of the game yet by  $N_t$  (for instance,  $N_1 = 3$ ). Let  $W_t$  and  $L_t$  denote the winner and the loser, respectively.

Denote the instantaneous utility player  $i$  receives in period  $t$  by  $U_t(i)$ . We assume that if  $i \neq D_t$  and  $i \neq O_t$ , then  $U_t(i) = 0$ . In other words, only actively participating players can get a non-trivial utility in the current period. At each period  $t$ , player  $i$  (actually, only player  $W_t$ ) maximizes his utility  $U(i) = \sum_{\tau=1}^{\infty} \beta^\tau U_\tau(i)$ , where  $\beta < 1$  is the discount factor, common for all agents. In each period  $t$ , the sequence of actions and events is as follows.

1. The dictator appoints a lieutenant of competence  $\theta_t$  who lives for one period.
2. There is an opponent  $O_t$  with probability  $\mu(1 - \delta)$  if the opponent was repressed last period ( $A_{t-1} = E$ ), and an opponent with probability  $(1 - \delta)$  otherwise. If  $O_t \neq 0$ , then the opponent attempts to become the dictator in a power struggle. If  $O_t = 0$ , then  $W_t = D_t$ ,  $L_t = O_t = 0$ ,  $O_{t+1} = N_t$ , and  $N_{t+1} = N_t + 1$ , and in this case steps 3 – 7 are skipped.
3. The dictator's lieutenant learns the realization of the signal about the dictators's vulnerability,  $\hat{s}_t \in \{n, v\}$  (based on state of the world  $s_t \in \{n, v\}$ ) and of his affinity with the opponent  $R_t \in [0, +\infty)$ . After that, the lieutenant recommends policy  $d_t \in \{\bar{P}, \underline{P}\}$ .
4. The opponent wins with probability  $0 < p_t < 1$ , which is determined endogenously. In other words,  $\Pr(L_t = D_t) = \Pr(W_t = O_t) = p_t$ , and  $\Pr(W_t = D_t) = \Pr(L_t = O_t) = 1 - p_t$ .
5.  $W_t$  decides on his action  $A_t$ , whether to execute ( $A_t = E$ ) or spare ( $A_t = S$ ) the loser  $L_t$ .
6. If  $A_t = E$ , then  $U_t(L_t) = -T$ , and with probability  $\mu < 1$  there is still an opponent in the

next period ( $O_{t+1} = N_t$  and  $N_{t+1} = N_t + 1$ ), and with probability  $1 - \mu$  there is no opponent ( $O_{t+1} = 0$ ). If  $A_t = S$ , then  $U_t(L_t) = 0$ , and  $O_{t+1} = L_t$ .

7. The winner gets  $U_t(W_t) = H$ , and becomes the next dictator, i.e.  $D_{t+1} = W_t$ .
8.  $D_{t+1}$  and  $O_{t+1}$  (if there is one) die independently with probability  $\delta$ . If only the opponent dies, he is replaced by the next one in the line. If the dictator dies, then the opponent (if any) becomes the dictator, and the next player becomes the opponent. If both die, or the dictator with no opponent dies, then a pair of players become the dictator and the opponent for the next period.

Players get one-period utility  $H$  if they are the winners of the period and the chosen policy was cost-less (see below), and  $H - C > 0$  if they are the winners, but the policy was strong. They get  $-T < 0$  if they are executed in this period. When they die peacefully,  $K < T$  is subtracted from their utility. In all other circumstances, their one-period utility equals 0.

The lieutenants' actions may depend on his signal  $s$  and value  $R$  only. We say a player has a good reputation ( $G$ ) if he has never killed before, and bad reputation ( $B$ ) if he ever has killed. all possible combinations of the winner's and the loser's types belong to the set  $\Lambda = \{(B, G), (G, B), (G, G)\}$ . If the dictator repressed before, then the opponent is a "novice", and therefore  $(B, B)$  is impossible for any history, on or off the equilibrium path.

**Equilibrium Definition.** Now we formally define the equilibrium concept of the game. We restrict to Markovian strategies (depending on the state of the world only), though the game is not truly Markovian, since the identity of the decision-maker (player) may change over time without being included in the state variables. Moreover, in the definition, we confine ourselves to one-shot deviations only (this is without loss of generality, [Fudenberg and Tirole, 1991](#)).

**Definition 3.** *A strategy profile consists of*

(a) *winner's loser's-fate function  $\alpha^* : \Lambda \rightarrow [0, 1]$ , where  $\alpha_t^*(X, Y)$  is the probability with which the winner of reputation  $X$  represses the loser with reputation  $Y$  in period  $t$ .*

(b) *Dictator's choice-of-lieutenant function  $\theta^* : \Lambda \rightarrow [0, 1]$ , where  $\theta^*(X, Y)$  is the competence*

of the lieutenant chosen by the dictator of reputation  $X$  that expects to meet an opponent with reputation  $Y$ .

(c) lieutenant's decision function  $d^* : [0, 1] \times \{n, v\} \times [0, +\infty) \rightarrow \{\bar{P}, \underline{P}\}$  such that for any  $\theta \in [0, 1]$ ,  $d^*(\theta, \cdot, \cdot)$  is lieutenant's decision on a recommended policy, if his competence is  $\theta$ .

We define equilibrium in 'Markovian' strategies in the following way.

**Definition 4.** A strategy profile  $(\alpha^*, \theta^*, d^*)$  is called an equilibrium if and only if

(a) In any decision node where a winner of reputation  $X$  determines the fate of a loser with reputation, executing with probability  $\alpha^*(X, Y)$  weakly dominates other options;

(b) In any decision node where a dictator of reputation  $X$  chooses a lieutenant to counter an opponent with reputation  $Y$ , choosing lieutenants with competence  $\theta^*(X, Y)$  is weakly dominant;

(c) In any decision node where a lieutenant of competence  $\theta$  suggests the policy choice, having received signal  $\hat{s}$  and expecting reward  $R$ , he finds choosing  $d^*(\theta, \hat{s}, R)$  weakly optimal, and if he is indifferent,  $d^*(\theta, \hat{s}, R) = \bar{P}$ ;

(d) For a dictator that appointed a lieutenant of competence  $\theta^*$ , it is optimal to follow the lieutenant's recommendation about the policy.

**Definition 5.** The tuple  $(\alpha_{BG}^*, \alpha_{GB}^*, \alpha_{GG}^*, \theta_{BG}^*, \theta_{GB}^*, \theta_{GG}^*, d^*(\theta, s, R))$ , where  $\alpha_{XY}^*$  and  $\theta_{XY}^*$  are numbers on  $[0, 1]$  for all  $(X, Y) \in \Lambda$ , and  $d^*$  is a mapping from tuple  $(\theta, s, R)$  of lieutenant's competence  $\theta \in [0, 1]$ , signal  $\hat{s} \in \{n, v\}$  and stochastic part of lieutenant's reward  $R$  into the recommended policy  $d \in \{\bar{P}, \underline{P}\}$ , is called equilibrium if and only if the following conditions are satisfied:

(i) When making his decision on the recommended policy, a lieutenant of competence  $\theta$  chooses  $d = d^*(\theta, \hat{s}, \hat{s}R)$  after having received signal  $\hat{s}$  and learned the stochastic part of reward  $R$ <sup>12</sup>;

(ii) Given the strategies of all other people, when a dictator with reputation  $X$  faces opponent with reputation  $Y$ , he opts to hire an lieutenant of competence  $\theta_{XY}^*$  (for all  $(X, Y) \in \Lambda$ );

(iii) Given the strategies of all other people, when winner with reputation  $X$  determines the fate of the loser with reputation  $Y$ , he represses him with probability  $\alpha_{XY}^*$  (for all  $(X, Y) \in \Lambda$ ), i.e. no other strategy yields higher expected utility.

<sup>12</sup>Since every lieutenant lives for one period only, once he becomes the decision-maker, he does not care about the strategies of players and other agents. For this reason, we do not require that he acts being aware of the equilibrium strategy of other people.

### A3 Propositions and Proofs

**Proposition A1.** (i)  $U_{GG}^S(\alpha_{GG})$  is a continuous strictly decreasing function of  $\alpha_{GG}$ , and  $U_{GG}^S(1) < \hat{U}^E$ . If  $U_{GG}^S(0) \geq \hat{U}^E$  then there is a dynamically consistent path with  $\alpha_{GG} = 0$  (a 'good' path), and if  $U_{GG}^S(0) > \hat{U}^E$ , then there also exists a mixed strategy path, which is dynamically consistent, and on which  $0 < \alpha_{GG} < 1$ , and  $U_{GG}^S = U_{GG}^E = \hat{U}^E$ . These paths exist for a wider range for other parameters when  $T$  is higher and  $\mu$  is higher. Moreover, competence  $\theta_{GG}$  on the good path is at least as high as in the mixed one, which is in turn at least as high as in the bad one.

(ii) Good equilibria exist for a wider range of parameters if the same conditions as for dynamically consistent paths (i.e.  $T$  is high and  $\mu$  is higher) hold. There may exist at most one equilibrium such that  $0 < \alpha_{GG} < 1$ . The equilibrium competence in the mixed equilibrium is at least as high as in the bad one, and the competence in a good equilibrium is at least as high as in mixed and bad ones.

#### Proof of Proposition A1.

Proofs for (i) and (ii) are similar and we will not repeat the algebra separately for dynamically consistent paths and equilibria. As demonstrated in Section 3, if the dictator expects to get  $U$  after winning and  $V$  after losing, then his maximization problem may be written as

$$(1 - p(\theta))U + p(\theta)V - r(\theta)C \rightarrow \max_{\theta}.$$

Leaving aside terms that do not include  $\theta$ , it has a unique solution  $\theta^*(U, V)$  if and only if the following function has a unique maximum on  $\theta \in [\frac{1}{2}, 1]$ .

$$Z(\theta) = (q\theta(U - V) - (q\theta + (1 - \theta)(1 - q))C)F(R^*(\theta)).$$

For brevity, denote  $M \equiv q(U - V - C) + (1 - q)C$ ,  $N = \frac{1-q}{q}(w + \pi)$ , and  $L = -(1 - q)C$ . Now,

$$\frac{\partial Z}{\partial \theta} = MF - (M\theta + L)\frac{N}{\theta^2}f, \quad (\text{A1})$$

$$\frac{\partial^2 Z}{\partial \theta^2} = \frac{2N}{\theta^3}f + (M\theta + L)\frac{N^2}{\theta^4}f'. \quad (\text{A2})$$

As  $N < 0$  and  $f' < 0$ , we conclude that  $\frac{\partial^2 Z}{\partial \theta^2} \leq 0$ , which guarantees either an interior maximum  $\theta^* \in [\frac{1}{2}, 1]$  or  $\theta^* = 1$ .

To determine how the optimal choice of the lieutenant competence  $\theta^*$  depends on other parameters, we compute the signs of cross-derivatives of  $Z(\theta)$  and use the standard monotone

comparative statics results in an appropriate neighborhood of the optimum. First, observe that

$$\frac{\partial^2 Z}{\partial \theta \partial U} = -\frac{\partial^2 Z}{\partial \theta \partial V} = F - \frac{N}{\theta} f. \text{ At } \theta^*, \text{ we have}$$

$$\frac{\partial Z}{\partial \theta}(\theta^*) = MF - (M\theta + L) \frac{N}{\theta^2} f = 0,$$

which implies

$$F - \frac{N}{\theta} f = \frac{1}{M} L \frac{N}{\theta^2} f.$$

As  $L < 0$ , we have  $\frac{\partial^2 Z}{\partial \theta \partial U} = F - \frac{N}{\theta} f < 0$ . As this is true in an open neighborhood of  $\theta^*(U)$  for any  $U$ , and interval  $[\frac{1}{2}, 1]$  is a compact set, we have  $\frac{\partial^2 Z}{\partial \theta \partial U} < 0$  across the whole interval. This gives us the necessary comparative statics:  $\theta^*$  (weakly) increases with  $U - V$ .

The comparative statics with respect to  $C$  is straightforward: as  $r'(\theta) < 0$ ,  $\theta^*$  is an increasing function of  $C$ . ■

**Lemma A2.** *Let  $X$  be any of the values  $U_{XY}, U_{XY}^E, U_{XY}^S, V_{XY}, W_{XY}$ . Then  $-T \leq X < \frac{H - \delta K}{1 - (1 - \delta)\beta}$ .*

**Proof of Lemma A2.** At each period (including the current one) a player may not expect to get more than  $H - \delta K$ . The probability that a player will survive till the next period is not greater than  $1 - \delta$ , and the discount factor is  $\beta$ . Therefore, the sum of discounted expected utilities does not exceed  $\frac{H - \delta K}{1 - (1 - \delta)\beta}$ , and since there is a non-trivial chance that the player will eventually be away from office, the inequality is strict.

Observe that utility in each period, utility may not be less than  $-T$ , and it may be negative only if the player dies or is repressed (another source of negative utility is expenditures on policy  $C$ , but they are always compensated by utility from winning  $Y$  enjoyed in that period). Therefore, a player may get negative utility in only one period (probably current one), and so expected utility cannot be less than  $-T$ . ■

**Lemma A3.** *For any  $\alpha_{XY}$  and  $p_{XY}$  in  $[0, 1]$ , equations (1) – (5) have a unique solution  $(U_{XY}, U_{XY}^E, U_{XY}^S, V_{XY}, W_{XY})$ . Moreover, these values continuously depend on parameters  $\alpha_{XY}, p_{XY}, \beta, \delta, H, C, K, T$ .*

**Proof of Lemma A3.** The mapping from 15-dimensional space of  $(U_{XY}, U_{XY}^E, U_{XY}^S, V_{XY}, W_{XY})$  to itself, set by equations (1) – (5), is contracting in metrics induced by norm

$\|U_{XY}, U_{XY}^E, U_{XY}^S, V_{XY}, W_{XY}\| = \max_{XY \in \{BG, GB, GG\}} \max \{U_{XY}, \beta^{-\frac{1}{2}} U_{XY}^E, \beta^{-\frac{1}{2}} U_{XY}^S, V_{XY}, W_{XY}\}$ , and the proof is straightforward. This proves the first part of the statement. To prove the latter part, we cannot use the implicit function theorem directly, for equations are not differentiable (because of max in the first equation). We proceed as follows. Denote any set of parameters  $(\alpha_{XY}, p_{XY}, \beta, \delta, H, C, K, D)$  by  $\lambda$  and consider a sequence  $\lambda_1, \lambda_2, \dots$  converging to  $\lambda_0$  (in Euclidean metric). For any  $i$ , let  $U_i$  be the stable point for the set of parameters  $\lambda_i$ . Let us prove that sequence  $U_1, U_2 \dots$  converges to  $U_0$ . If it is not the case, there exists  $\varepsilon > 0$  and subsequence  $U_{k_1}, U_{k_2} \dots$ , the elements of which do not lie in  $\varepsilon$ -proximity of  $U_0$ . By Proposition A2, all utilities  $U_k$  (starting from a certain  $k$ ) lie between  $-T$  and  $\frac{H-\delta K}{1-(1-\delta)\beta}$ , and therefore there is a subsequence  $U_{m_1}, U_{m_2} \dots$  converging to some finite point  $U \neq U_0$ . Since the right-hand side of (1) – (5) is continuous with respect to all variables, its value at  $(\lambda_0, U)$  equals the double limit of values at  $m_1, m_2 \dots$  and  $U_{m_1}, U_{m_2} \dots$ , and in particular equals the limit of values at  $(m_i, U_{m_i})$ . These values equal  $U_{m_i}$ , respectively, and tend to  $U$ , which means that  $U$  is a stable point of the mapping, and this contradicts  $U \neq U_0$ . This contradiction completes the proof. ■

**Bad Always Executes Good.** Here we establish that  $B$  always represses  $G$ . In particular,  $\alpha_{BG} = 1$ .

Assume the contrary. Then  $U_{BG}^E \leq U_{BG}^S$ . From (2) and (3) it follows that

$$(1 - \mu) (H + (1 - \delta) \beta W_{BG} - \delta K) + \mu W_{BG} \leq W_{BG}.$$

Rearranging and dividing by  $1 - \mu$ , we get

$$H + (1 - \delta) \beta W_{BG} - \delta K \leq W_{BG},$$

which implies

$$W_{BG} \geq \frac{H - \delta K}{1 - (1 - \delta) \beta}.$$

This contradiction completes the proof. ■

**Good Executes Bad If Delta Is Low.** By Proposition 3,  $\alpha_{BG} = 1$ , and thus  $V_{BG} = -T$ . By Proposition A2,  $V_{BG} \geq -T$ . We prove that for sufficiently small  $\delta$ ,  $U_{GB}^E > U_{GB}^S$ . To do that, we prove this strict inequality for  $\delta = 0$ , and then use continuity stated in Proposition A3.

Now assume the contrary, i.e.  $U_{GB}^E \leq U_{GB}^S$ . Then  $U_{GB} = U_{GB}^S$ , while  $U_{BG} \geq U_{BG}^S$ . Evidently,  $V_{BG} = -T$  and  $V_{GB} \geq -T$ .

Let  $\tilde{\theta}$  be such that

$$W_{GB} = (1 - p(\tilde{\theta})) U_{GB} + p(\tilde{\theta}) V_{BG} - r(\tilde{\theta}) C.$$

Then

$$W_{BG} \geq (1 - p(\tilde{\theta})) U_{BG} + p(\tilde{\theta}) V_{GB} - r(\tilde{\theta}) C,$$

because this holds for any  $\tilde{\theta}$ . Now, notice that

$$\begin{aligned} U_{GB}^S &= H + \beta W_{GB} = H + \beta ((1 - p(\tilde{\theta})) U_{GB} + p(\tilde{\theta}) V_{BG} - r(\tilde{\theta}) C) \\ &= H + \beta \left( (1 - p(\tilde{\theta})) U_{GB}^S - p(\tilde{\theta}) D - r(\tilde{\theta}) C \right) \end{aligned}$$

and

$$\begin{aligned} U_{BG}^S &= H + \beta W_{BG} \geq H + \beta ((1 - p(\tilde{\theta})) U_{BG} + p(\tilde{\theta}) V_{GB} - r(\tilde{\theta}) C) \\ &\geq H + \beta \left( (1 - p(\tilde{\theta})) U_{BG}^S - p(\tilde{\theta}) D - r(\tilde{\theta}) C \right). \end{aligned}$$

Therefore,

$$U_{BG} \geq U_{BG}^S \geq \frac{H - \beta (p(\tilde{\theta}) D + r(\tilde{\theta}) C)}{1 - \beta (1 - p(\tilde{\theta}))} \geq U_{GB}^S = U_{GB},$$

which, combined with  $V_{BG} \geq V_{GB}$ , implies that

$$(1 - p(\theta)) U_{BG} + p(\theta) V_{BG} - r(\theta) C \geq (1 - p(\theta)) U_{GB} + p(\theta) V_{GB} - r(\theta) C$$

for any given  $\theta$ . Therefore,  $W_{BG} \geq W_{GB}$ . Hence, from the known inequality for  $W_{BG}$  we find that

$$(1 - \mu) (H + \beta W_{BG}) + \mu W_{BG} < W_{BG} \leq W_{GB}.$$

Therefore, for  $\delta = 0$ ,  $U_{GB}^E > U_{GB}^S$  holds. Therefore, the same holds for  $\delta$  small enough. ■

**Same Competence If One Is Bad.** Proposition 3 implies that  $V_{BG} = V_{GB} = -T$  and  $U_{BG} = U_{BG}^E = U_{GB}^E = U_{GB}$ , since  $U_{BG}^E$  and  $U_{GB}^E$  are equal, as can be seen from (2). This, in turn, implies  $W_{BG} = W_{GB}$ , since problems in (5) become identical. By Proposition 2, the solution to maximization problem (5) is unique, and hence lies on a dynamically consistent path,  $\theta_{BG} = \theta_{GB}$ . To prove that these values do not depend on  $p_{XY}$ 's, note that they are determined uniquely from a subset of equations (1) – (5) for  $XY \in \{BG, GB\}$ . These equations do not include  $p_{GG}$ , and if  $\alpha_{BG} = \alpha_{GB} = 1$ , they do not include  $p_{BG}$  and  $p_{GB}$  either. Hence,  $\theta_{BG}$  and  $\theta_{GB}$  do not depend on  $p_{XY}$ 's. Since these values

are equal to each other and the same for any dynamically consistent path, they are the same in any equilibrium. In an equilibrium, (7) must hold, therefore,  $p_{BG} = p_{GB}$ , and these values are the same for any equilibrium. ■

**Existence Of Bad Equilibrium.** It is evident that  $\alpha_{GG} = 1$ ,  $V_{GG} = -T$ ,  $U_{GG} = U_{GG}^E = U_{GB}$ ,  $U_{GG}^S = U_{GB}^S$ ,  $W_{GG} = W_{GB}$  satisfy all equations (1) – (5) and (6), because equations for  $XY = GG$  become the same as for  $XY = GB$ . Therefore, this forms a dynamically consistent path. If we set  $p_{XY} = p(\theta_{XY})$  for all  $(X, Y) \in \Lambda$ , we will get an equilibrium, since dynamically consistent path does not depend on  $p_{XY}$ 's (follows Proposition 3 and formulae for variables with index  $GG$ ). Conditions  $\alpha_{BG} = \alpha_{GB} = \alpha_{GG} = 1$  determine all utilities, and thus  $\theta_{XY}$ 's, uniquely, because in this case  $p_{XY}$ 's are not included in the system of equations. This implies that  $p_{XY}$ 's are also uniquely determined, and thus only one equilibrium with all  $\alpha_{XY}$ 's equal to 1 exists, which completes the proof. ■

**Existence Of And Competence In Good And Mixed Paths.** If  $U_{GG}^E > U_{GG}^S$ , then in an equilibrium,  $\alpha_{GG}$  should equal 1. It is straightforward to check that it is indeed an equilibrium. Otherwise,  $U_{GG}^E \leq U_{GG}^S$ , or equivalently,  $U_{GG} = U_{GG}^S$ , holds. Now, our goal is to find  $U_{GG}^S$  as a function of  $\alpha_{GG}$  and to compare it with  $U_{GG}^E$  which is known. Obviously,  $\alpha_{GG} = 0$  would form an equilibrium if  $U_{GG}^S \geq U_{GG}^E$ , while  $\alpha > 0$  would if  $U_{GG}^S = U_{GG}^E$ .

Let us prove that  $U_{GG}^S$  is a strictly decreasing function of  $\alpha_{GG}$ . Since for  $\alpha_{GG} < 1$  we have  $U_{GG} = U_{GG}^S$ , then by (3),

$$U_{GG}(W_{GG}) = H + (1 - \delta) \beta W_{GG} - \delta K. \quad (\text{A3})$$

Then, from (4) we get

$$V_{GG}(W_{GG}, \alpha_{GG}) = \frac{(1 - \alpha_{GG}) ((1 - \delta) \beta ((1 - \delta) p_{GG} (H + (1 - \delta) \beta W_{GG} - \delta K) + \delta W_{GG}) - \delta K) - \alpha_{GG} D}{1 - (1 - \alpha_{GG}) (1 - \delta) \beta (1 - \delta) (1 - p_{GG})}. \quad (\text{A4})$$

Therefore,  $W_{GG}$  satisfies equation

$$L(W_{GG}, \alpha_{GG}) \equiv W_{GG} - \max_{\theta} ((1 - p(\theta)) U_{GG}(W_{GG}) + p(\theta) V_{GG}(W_{GG}, \alpha_{GG}) - r(\theta) C) = 0, \quad (\text{A5})$$

where  $U_{GG}(W_{GG})$  and  $V_{GG}(W_{GG}, \alpha_{GG})$  are taken from (A3) and (A4).  $L(W_{GG}, \alpha_{GG})$  is strictly increasing with respect to both  $W_{GG}$  and  $\alpha_{GG}$ . To prove that, we use envelope's theorem, we get

$$\frac{\partial L}{\partial \alpha_{GG}} = -p(\theta) \frac{\partial V_{GG}}{\partial \alpha_{GG}}, \quad (\text{A6})$$

while (omitting  $GG$  subscript for brevity)

$$\frac{\partial V_{GG}}{\partial \alpha_{GG}} = \frac{K\delta - W\beta\delta(1-\delta) - U\beta p(1-\delta)^2 - T(1-\beta(1-\delta)^2(1-p))}{(1-(1-\alpha)(1-\delta)\beta(1-\delta)(1-p))^2}.$$

Since  $K < T$ ,  $U \geq -T$ ,  $w \geq -T$ , the numerator is less than  $-T(1-\beta)(1-\delta) < 0$ . Therefore,

$\frac{\partial Z}{\partial \alpha_{GG}} > 0$ . Now,

$$\frac{\partial L}{\partial W_{GG}} = 1 - (1-p(\theta)) \frac{dU_{GG}}{dW_{GG}} - p(\theta) \frac{\partial V_{GG}}{\partial W_{GG}}. \quad (\text{A7})$$

Evidently,  $\frac{dU_{GG}}{dW_{GG}} = (1-\delta)\beta < 1$ . It is sufficient to demonstrate that  $\frac{\partial V_{GG}}{\partial W_{GG}} < 1$ , but we would later need  $\frac{\partial V_{GG}}{\partial W_{GG}} < (1-\delta)\beta$ , which we establish right away.

$$\frac{\partial V_{GG}}{\partial W_{GG}} = \frac{(1-\alpha)(1-\delta)\beta((1-\delta)p((1-\delta)\beta + \delta))}{1 - (1-\alpha)(1-\delta)\beta(1-\delta)(1-p)}.$$

The denominator is clearly greater than 0. Thus, we need to check that

$$(1-\delta)\beta(1 - (1-\alpha)(1-\delta)\beta(1-\delta)(1-p)) - (1-\alpha)(1-\delta)\beta((1-\delta)p((1-\delta)\beta + \delta)) > 0.$$

This is equivalent to

$$\beta(1-\delta)(1 - (1-\delta)(1-\alpha)(\beta(1-\delta) + p\delta)) > 0.$$

Clearly, this expression lies between 0 and 1. Therefore,  $\frac{\partial V_{GG}}{\partial W_{GG}} < (1-\delta)\beta$ , which implies  $(1-p(\theta)) \frac{dU_{GG}}{dW_{GG}} + p(\theta) \frac{\partial V_{GG}}{\partial W_{GG}} < (1-\delta)\beta < 1$ . Hence,  $\frac{\partial L}{\partial W_{GG}} > 0$ , and therefore, function  $W_{GG}(\alpha_{GG})$ , defined as an implicit function by (A5), satisfies

$$\frac{dW_{GG}}{d\alpha_{GG}} = -\frac{\frac{\partial L}{\partial \alpha_{GG}}}{\frac{\partial L}{\partial W_{GG}}} < 0. \quad (\text{A8})$$

This proves that  $W_{GG}(\alpha_{GG})$  and, by (A3),  $U_{GG}(W_{GG}(\alpha_{GG}))$  are strictly decreasing functions of  $\alpha_{GG}$ .

One can immediately check that  $U_{GG}(W_{GG}(1)) < \hat{U}^E$ . If it were not the case, there would be a dynamically consistent path with  $\alpha_{GG} = 1$  and  $U_{GG} = U_{GG}^S$ , while Proposition 4 states that there is only one dynamically consistent path with  $\alpha_{GG} = 1$  in which, by Proposition 3,  $U_{GG} = U_{BG} > U_{BG}^S = U_{GG}^S$ . This leads us to a contradiction, and we conclude that  $U_{GG}(W_{GG}(1)) < \hat{U}^E$ .

It is obvious that  $\alpha_{GG} < 1$  leads to a dynamically consistent path either if  $\alpha_{GG} = 0$  and  $U_{GG}(W_{GG}(0)) \geq \hat{U}^E$  or if  $U_{GG}(W_{GG}(\alpha_{GG})) = \hat{U}^E$ . Hence, continuity and monotonicity of  $U_{GG}(W_{GG}(\alpha_{GG}))$  implies that if  $U_{GG}(W_{GG}(0)) = \hat{U}^E$ , there is a dynamically consistent path with

$\alpha_{GG} = 0$ , and if  $U_{GG}(W_{GG}(0)) > \hat{U}^E$ , there is one dynamically consistent path with  $\alpha_{GG} = 0$  and one with  $0 < \alpha_{GG} < 1$ .

To proceed, denote utilities that correspond to bad, mixed, and good dynamically consistent paths with upper indices  $(B)$ ,  $(M)$ , and  $(G)$ , respectively. Our goal is to demonstrate that  $U_{GG}^{(B)} - V_{GG}^{(B)} > U_{GG}^{(M)} - V_{GG}^{(M)} > U_{GG}^{(G)} - V_{GG}^{(G)}$ , since then the necessary result would immediately follow from Proposition 2. To prove the first inequality, notice that  $U_{GG}^{(B)} = U_{GG}^{(M)} = \hat{U}^E$ ,  $V_{GG}^{(B)} = -T$ , while  $V_{GG}^{(M)} > -T$  (the latter follows from (4), noting that if  $\alpha_{GG} < 1$ , there is a non-trivial chance of getting  $-\delta K$  which is greater than  $-T$ ). The proof of the latter inequality is a bit trickier. We will prove that  $U_{GG}(W_{GG}(\alpha_{GG})) - V_{GG}(W_{GG}(\alpha_{GG}), \alpha_{GG})$  is an increasing function of  $\alpha_{GG}$ . For brevity, we will omit index  $GG$  in formulae. First,

$$\frac{d(U - V)}{d\alpha} = -\frac{\partial V}{\partial \alpha} + \left( \frac{dU}{dW} - \frac{\partial V}{\partial W} \right) \frac{dW}{d\alpha}.$$

We have already proved that  $\frac{\partial V}{\partial w} < (1 - \delta)\beta$ . Using (A8), (A6), and (A7), we find

$$\frac{dW}{d\alpha} = -\frac{\frac{\partial L}{\partial \alpha}}{\frac{\partial L}{\partial W}} = -\frac{-p(\theta) \frac{\partial V}{\partial \alpha}}{1 - \frac{dU}{dW} + p(\theta) \left( \frac{dU}{dW} - \frac{\partial V}{\partial W} \right)}.$$

Consequently,

$$\frac{d(U - V)}{d\alpha} = -\frac{\partial V}{\partial \alpha} \left( 1 - \frac{p(\theta) \left( \frac{dU}{dW} - \frac{\partial V}{\partial W} \right)}{1 - \frac{dU}{dW} + p(\theta) \left( \frac{dU}{dW} - \frac{\partial V}{\partial W} \right)} \right) = -\frac{\partial V}{\partial \alpha} \frac{1 - \frac{dU}{dW}}{1 - \frac{dU}{dW} + p(\theta) \left( \frac{dU}{dW} - \frac{\partial V}{\partial W} \right)} > 0,$$

because, as we demonstrated before,  $\frac{\partial V}{\partial \alpha} < 0$ ,  $\frac{dU}{dW} = (1 - \delta)\beta < 1$  and  $\frac{\partial V}{\partial W} < (1 - \delta)\beta$ , thus making  $\frac{dU}{dW} - \frac{\partial V}{\partial W} > 0$ . From this, we conclude that  $\theta_{GG}^{(B)} \leq \theta_{GG}^{(M)} \leq \theta_{GG}^{(G)}$ .

To finish the proof, we need to show that  $U_{GG}(W_{GG}(0)) - \hat{U}^E$  is increasing with respect to both  $T$  and  $\mu$ , and decreasing with respect to  $H$ . The first thing to observe is that  $U_{GG}(W_{GG}(0))$  does not depend on  $T$  and  $\mu$ . Now consider derivatives of  $\hat{U}^E$ .

$$\frac{d\hat{U}^E}{dD} = (1 - \delta)\beta \left( (1 - \mu)(1 - \delta)\beta + \mu \right) \left( (1 - p_{BG}) \frac{d\hat{U}^E}{dD} - p_{BG} \right), \quad (\text{A9})$$

because  $V_{BG} = -T$ . Similarly,

$$\frac{d\hat{U}^E}{d\mu} = (1 - \delta)\beta \left( (1 - (1 - \delta)\beta) W_{BG} - (H - \delta K) \right) + (1 - \delta)\beta \left( (1 - \mu)(1 - \delta)\beta + \mu \right) (1 - p_{BG}) \frac{d\hat{U}^E}{d\mu}. \quad (\text{A10})$$

Rearranging (A9) and (A10), we get  $\frac{d\hat{U}^E}{dT} < 0$  and  $\frac{d\hat{U}^E}{d\mu} < 0$  (recalling that  $W_{BG} < \frac{Y - \delta K}{1 - (1 - \delta)\beta}$ ). Therefore, good equilibrium is more likely to exist when  $T$  or  $\mu$  are higher. ■

**Existence And Competence Of Good And Mixed Equilibria.** When studying equilibria, we restrict ourselves to the case where  $p_{BG} = p_{GB} = p(\theta_{BG})$  (recall that the latter does not depend on  $p_{GG}$ ). First, we check that  $U_{GG}$  is increasing with respect to  $p_{GG}$ . By (3), it is sufficient to check that  $W_{GG}(0)$  is increasing with respect to  $p_{GG}$ . We obtain

$$\frac{dL}{dp_{GG}} = -p(\theta_{GG}) \frac{\beta(1-\delta)^2 \delta K + (1 - \beta(1-\delta)^2)H + \beta(1-\delta)^2(1 - \beta(1-\delta))W}{(1 - (1-\delta)\beta(1-\delta)(1-p))^2}.$$

Since  $\alpha_{GG}$ , all utilities with  $GG$  subscript are not less than  $-\delta K$ , which means that the numerator is not less than  $(1 - \beta(1-\delta)^2)Y + \beta^2(1-\delta)^3 \delta K$ , i.e. it is positive (in other words,  $\frac{\partial V}{\partial \alpha} > 0$ ). Since  $\frac{\partial L}{\partial W_{GG}} > 0$ ,  $\frac{dW_{GG}(0)}{dp_{GG}} > 0$ .

Now let us demonstrate that  $U_{GG} - V_{GG}$  is decreasing with respect to  $p_{GG}$ . Evidently (omitting  $GG$  subscript),

$$\frac{d(U - V)}{dp} = -\frac{\partial V}{\partial p} + \left( \frac{dU}{dW} - \frac{\partial V}{\partial W} \right) \frac{dW}{dp}.$$

Since

$$\frac{dW}{dp} = -\frac{\frac{\partial L}{\partial p}}{\frac{\partial L}{\partial W}} = -\frac{-p(\theta) \frac{\partial V}{\partial p}}{1 - \frac{dU}{dW} + p(\theta) \left( \frac{dU}{dW} - \frac{\partial V}{\partial W} \right)},$$

we conclude that

$$\frac{d(U - V)}{dp} = -\frac{\partial V}{\partial p} \frac{1 - \frac{dU}{dW}}{1 - \frac{dU}{dW} + p(\theta) \left( \frac{dU}{dW} - \frac{\partial V}{\partial W} \right)} < 0,$$

because  $\frac{\partial V}{\partial \alpha} > 0$ . This means that  $\theta_{GG}$  is (weakly) increasing with respect to  $p_{GG}$ , and so does  $p(\theta_{GG})$ . We will need this later; for now, notice that when  $p_{GG}$  grows from 0 to 1,  $p(\theta_{GG}(p_{GG}))$  lies strictly between 0 and 1, and it does not depend on  $T$  and  $\mu$ . Therefore, there is a fixed set of values  $p_{GG}$  such that  $p_{GG} = p(\theta_{GG}(p_{GG}))$ , and the question is only whether for this value  $\hat{U}^E \leq U_{GG}(W_{GG}(0))$ . This is more likely if  $T$  is high or  $\mu$  is high.

On a mixed-strategy dynamically consistent path,  $U_{GG} = U_{GG}^S = \hat{U}^E$ , and  $\hat{U}^E$  does not depend on  $p_{GG}$ . Therefore, (3) implies that  $W_{GG}$  is fixed, and (4) in its turn implies that  $V_{GG}$  is fixed (otherwise  $W_{GG}$  would be greater for larger  $V_{GG}$ ). Hence,  $U_{GG} - V_{GG}$  is the same, and therefore  $\theta_{GG}$  and  $p_{GG}$  are fixed. This means that there may exist only one mixed equilibrium, and for this equilibrium

(if it exists)  $p_{GG}$  is found as described. Mixed equilibrium exists if and only if for this  $p_{GG}$  there is a good dynamically consistent path.

Let us prove that competence in bad equilibrium is at least as low as in any other one. Denote competence in the latter equilibrium by  $\theta^*$ , and let  $p^* = p(\theta^*)$ . For  $p_{GG} = p^*$ , there is a bad dynamically consistent path as well, and on this path,  $\theta_{GG}^{(B)} \leq \theta^*$  by (i) in Proposition A1. However,  $\theta_{GG}^{(B)}$  is fixed and equal to  $\theta_{BG}$  by Proposition 3. In bad equilibrium, competence is also equal to  $\theta_{BG}$ , and therefore it is not greater than  $\theta^*$ .

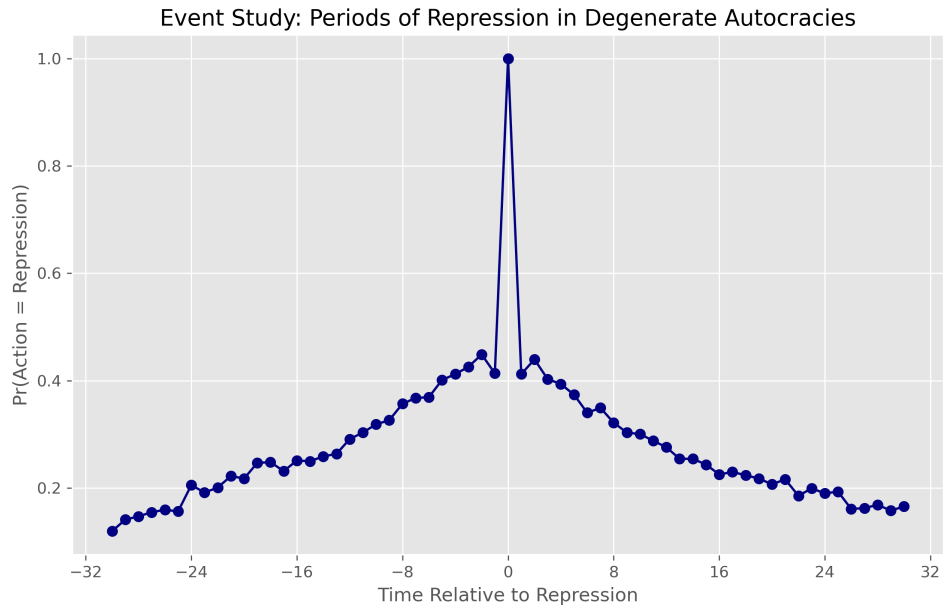
Finally, let us demonstrate that competence in the mixed equilibrium (if there exists one) is not greater than competence in a good equilibrium. Denote competence in the latter one by  $\theta^*$ , and let  $p^* = p(\theta^*)$ . Consider the mixed-strategy dynamically consistent path for  $p_{GG} = p^*$ . On this path, competence  $\theta_{GG}^{(M)}$  is less than  $\theta^*$ . However, on all mixed paths (and thus in the mixed equilibrium) competence is the same, which means that in the mixed equilibrium, competence does not exceed that in a good equilibrium. ■

**Competence On Different Paths.** Competence on the path where dictators execute their enemies coincides with competence in  $GG$  case in the bad equilibrium. Therefore, according to (ii) in Proposition A1, competence of agents is better (not worse) on the good path. Utilities, and thus competence, of the good equilibrium does not depend on  $\mu$ , as it was shown above. On the other hand,  $\frac{d\hat{U}^E}{d\mu} < 0$  and  $\frac{dV_{XY}}{d\mu} = 0$  on the bad path, since  $V_{XY} = -T$ . Consequently, higher  $\mu$  leads to lower difference in utilities and thus to higher competence. ■

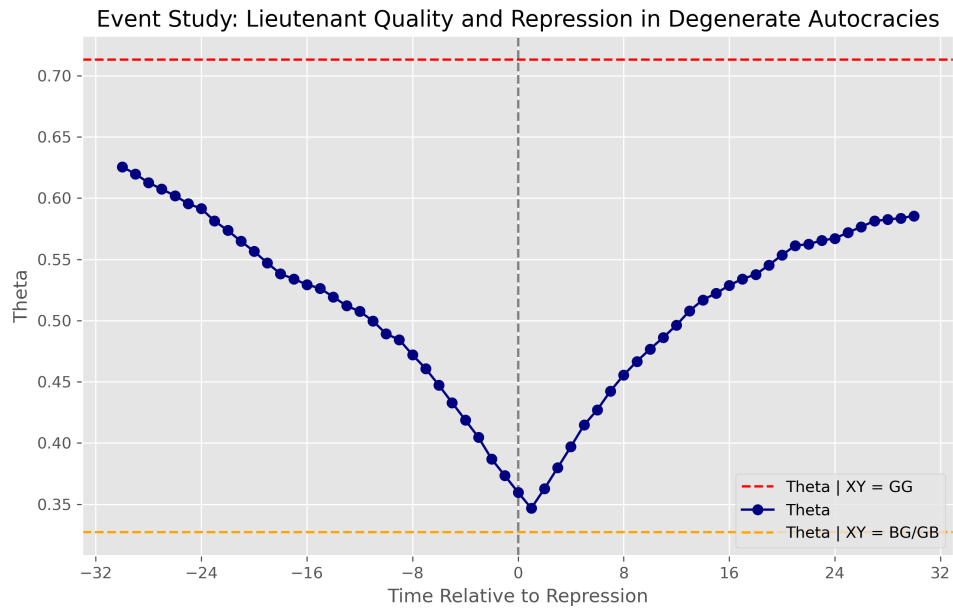
## A4 Supplement for Simulation Exercise in Subsection 4.3

```
1 1. Define system of equations  $g(x)$  from equations 1 - 7
2 2. Define norm  $f(x)$ 
3 3. Input:
4     a. Initial guess  $x_0 = \{U, U^E, U^S, V, W, \theta, \alpha\}$ 
5     b. Tolerable error  $e$ 
6     c. Maximum iteration  $N$ 
7 5. Initialize iteration counter:  $step = 1$ 
8 6. Do
9      $x_1 = g(x_0)$ 
10    If finding mixed equilibrium
11        Solve  $\alpha \Rightarrow V_{GG} == V_{GG}$  from equations 4 and 11
12     $step = step + 1$ 
13    If  $step > N$ 
14        Print "Not convergent"
15        Restart from step 3
16     $x_0 = x_1$ 
17 While  $abs f(x_1) - f(x_0) > e$  and  $abs \alpha_1 - \alpha_0 > e$ 
18 7. Verify solution using proposition 3
19 8. Return equilibrium as  $x_1$ 
```

Listing 1: Solving Algorithm for the Simulation Exercise



(a) Repression



(b) Lieutenant Competence

Figure A-1: Simulations of Repression Event Studies