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Market Structure and Monetary Non-neutrality

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ABSTRACT

I study a general equilibrium menu cost model with a continuum of sectors, idiosyncratic and aggregate shocks, and the novel feature that each sector consists of strategically engaged firms. Compared to an economy with monopolistically competitive sectors—separately parameterized to match the same microdata on price flexibility—the oligopoly economy features a smaller response of inflation to monetary shocks and output responses that are more than twice as large. Under the same parameters, output responses are five times larger. An oligopoly economy also (i) requires smaller menu costs and idiosyncratic shocks to match the microdata, addressing a significant challenge for mechanisms that generate non-neutrality via strategic complementarities, (ii) implies four times larger welfare losses from same sized nominal rigidities, and (iii) provides a novel rationale for positive menu costs: in an oligopoly firms prefer a degree of rigidity to complete flexibility. Quantitatively, the estimated degree of nominal rigidity is found to be close to optimal, from firms' perspective.

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1 Introduction

Golosov and Lucas (2007) demonstrate that an equilibrium version of the menu cost model of Barro (1972) implies approximate monetary neutrality when confronted with microdata on price changes (Bils and Klenow, 2004). The framework has since been adapted to generate a smaller elasticity of inflation to real marginal cost, yet responses remain much larger than employed in quantitative New Keynesian models that do not match microdata on price adjustment (Christiano, Eichenbaum, and Trabandt, 2016; Violante, Kaplan, and Moll, 2018). Realistic and complementary mechanisms are required in order to generate realistic monetary non-neutrality.[1]

This paper presents a mechanism based on an ignored feature of microdata: goods markets are concentrated. I accommodate this fact through an oligopolistic market structure as in Atkeson and Burstein (2008) but in which—due to pricing frictions—firms compete dynamically. At the macro level, monetary business cycles are two and a half times larger than under monopolistic competition, flattening the implied Phillips curve by a factor of four. Table A1 puts this in the context of the literature. At the micro level, productivity shocks cause idiosyncratic price changes that are large and almost identically distributed for both market structures and under similar parameters. The framework therefore avoids a well known road block to incorporating pricing complementarities in menu cost models: the inability to generate large price changes (Klenow and Willis, 2016; Burstein and Hellwig, 2007).[2] Moreover, strategic complementarities in prices mean the model is outside the "broad class of models" for which results in Alvarez and Lippi (2014) apply: kurtosis, frequency, and size of price adjustment are identical across market structures, but aggregate dynamics vary.

Studying the quantitative macroeconomic implications of dynamic oligopolistic firms is new. Many markets are concentrated, but most existing macroeconomic models aggregate behavior of non-strategic agents. Figure 1 documents market concentration for a range of narrowly defined goods markets: a product category (e.g., ketchup) within a state.[3] The median market has many

[1]Table A1 contributes a meta-study of existing results. The implied slope of the Phillips curve λ in extensions of the menu cost model are significantly smaller than the baseline Golosov and Lucas (2007) model, but still significantly larger than required to generate monetary business cycles of the magnitude of estimated New Keynesian models.

[2]The issues raised in these papers are discussed extensively by Nakamura and Steinsson (2010), Gopinath and Itskhoki (2011) and Midrigan (2011a).

[3]IRI data are used to construct measures of wholesale-firm-level revenue, which are then used to construct measures of concentration. The IRI data are weekly good-level data for the universe of goods in a panel of over 5,000 supermarkets in the US from 2001 to 2011. Wholesale firms, such as Kraft in the market for ketchup, are identified from the first six digits of a barcode. For a detailed description of how these measures are constructed, see Appendix B.

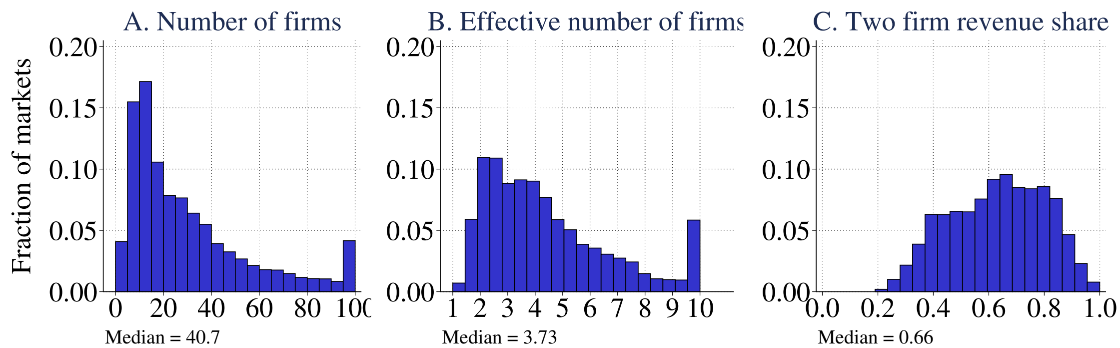

Figure 1: Market concentration in the IRI data

Notes: (i) A market is defined as an IRI product category p within state s in quarter t giving 191,833 observations. (ii) A firm i is defined within a pst market by the first 6 digits of a product's barcode, which identifies the wholesale firm. (iii) Revenue r_{ipst} is the sum over the revenue from all products of firm i in market pst. See Appendix B for more details on the data. (iv) Medians reported in the figure are revenue weighted. Unweighted medians are A. 21, B. 3.88, C. 0.64. (v) Each histogram has 20 bins. **Panel A**: Number of firms is the total number of firms with positive sales in market pst. **Panel B**: Effective number of firms is given by the inverse Herfindahl index h_{pst}^{-1}, where the Herfindahl index is the revenue-share-weighted average revenue share of all firms in the market, $h_{pst} = \sum_{i \in \{pst\}} (r_{ipst}/r_{pst})^2$. **Panel C**: Two-firm revenue share is the share of total revenue in market pst accruing to the two firms with the highest revenue.

firms, but sales accrue to a few. The revenue share of the top two firms is two thirds.[4] If firms with market power set prices strategically, what are the implications for macroeconomic dynamics?

To answer this question I develop a new quantitative framework: a general equilibrium menu cost model of price adjustment featuring a duopoly within each of a continuum of sectors. Firms' goods are imperfectly substitutable such that prices are complements, face large and persistent idiosyncratic shocks, persistent aggregate shocks to money growth, choose when to pay a cost to change their price, and compete strategically under a Markov perfect equilibrium (MPE) concept. In consequent work, Werning and Wang (2020) derive highly informative analytical results for a continuous time dynamic oligopoly without idiosyncratic shocks, where changes in the money stock are permanent and unanticipated, and firms are chosen at random to change their price (i.e. Calvo). The main exercise compares the dynamic oligopoly economy to an economy with the standard market structure used throughout monetary economics: a (monopolistically) competitive market with a continuum of non-strategic firms. The oligopoly model introduces no free parameters, so both models can be transparently calibrated to the same microdata on price changes and markups. In these two economies with different market structures but identical measures of idiosyncratic price flexibility, the aggregate price level is less flexible under oligopoly. As a consequence, output fluctuations in response to monetary shocks are around two and half times as

[4] Recent studies have documented similar concentration in labor markets (Azar, Marinescu, Steinbaum, and Taska, 2020; Benmelech, Bergman, and Kim, 2020; Rinz, 2020; Berger, Mongey, and Herkenhoff, 2019). Like the new facts documented in Figure 1 the average effective number of firms in a market (commuting-zone, NAICS3 combination) is around 5, much less than the average number of firms around 40 (Berger, Mongey, and Herkenhoff, 2019, Table 1).

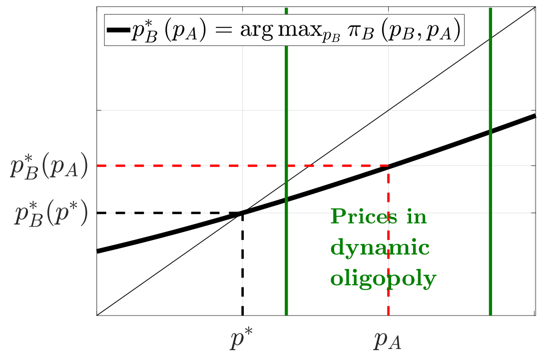

Figure 2: Static complementarity

Notes: The x-axis plots the price of Firm A, the y-axis plots the static best response of Firm B: $p_B^*(p_A)$. This is the price that maximizes π_B given p_A. The black solid line is the 45-degree line. The price p^* is the *frictionless* Nash equilibrium price satisfying $p^*(p^*) = p^*$.

large.[5]

The model delivers additional macroeconomic results regarding output losses due to nominal rigidity, aggregate dynamics under alternative price-setting technologies (i.e. Calvo), and quantifies a novel feature of the model, that firms have a preference *for* nominal rigidity. First, I briefly describe the mechanism underlying this main result.

Mechanism. The mechanism has two ingredients, (i) dynamic complementarity in prices which is new to the menu cost literature, (ii) the interaction of (i) with a monetary shock.

A natural property of oligopoly with imperfectly substitutable goods is that firms' prices are complements: off-equilibrium, my best response is increasing in my competitor's price. The theoretical literature discussed below refers to this as *static complementarity* in prices. In a single period model with frictionless adjustment, each firm reasons through these *off equilibrium* best responses in which each firm undercuts its competitor. Iterating on best responses yields a frictionless Nash equilibrium p^*.[6] Figure 2 plots such a best response function and static equilibrium between two symmetric firms.

Including menu costs and dynamics shifts the equilibrium set of prices and policies. Menu costs make future price cuts expensive. So when menu costs are positive, a higher price at one firm yields a response of a higher price from its competitor *in equilibrium* rather than just off-

[5]The real effects of monetary shocks are measured as the time series standard deviation of output in an economy with only monetary shocks. To match evidence on the distribution of price changes, I assume that menu costs are random in both models. I show that output fluctuations in the duopoly model with random menu costs are five times larger than the monopolistically competitive model with fixed menu costs.

[6]In the Online Appendix (at the end of this document) I derive these properties of the best response function under a profit function $\pi^i(p_1, p_2)$ with general complementarity ($\pi_{12} > 0$). I also derive a number of useful results for a static game which provides intuition for the mechanism discussed.

equilibrium. This *dynamic complementarity* has been studied in the theoretical literature (Maskin and Tirole, 1988b; Lapham and Ware, 1994; Jun and Vives, 2004).

I show that dynamic complementarity curtails the well documented *selection effect* that drives monetary non-neutrality in a monopolistically competitive menu-cost economy.[7] When changing prices is costly, an increase in the money supply causes firms with the most out-dated prices—firms with low markups—to change their prices first. The aggregate price index, therefore, quickly incorporates the deviation of these firms' prices from their optimal price. Matching microdata on price adjustment demands that these deviations and therefore price increase are large, such that a monetary shock causes a spike in inflation with little output response.

In the dynamic oligopoly model the *selection effect* is dampened. In sectors where past idiosyncratic shocks have forced the firms' markups apart, the falling markup of the high markup firm puts a brake on the response of the low markup firm. As prices are *dynamic complements*, the falling markup at the high markup firm reduces the low markup firm's (i) optimal reset price, and (ii) value of a price increase as its residual demand declines. Low markup firms adjust less, and by smaller amounts, dampening the selection effect. I extend existing decompositions of aggregate inflation (Caballero and Engel, 2007) to characterize how this mechanism dampens inflation.

I show that holding the amount of *static complementarity* fixed (i.e. preferences and technology), the amount of *dynamic complementarity* behind this result is endogenous. To make this point, I compare the oligopoly model under menu cost and Calvo pricing. Under Calvo, future price changes occur at random, so adjusting firms care less about the falling markup of their competitor. In the monopolistically competitive model Calvo frictions shut down the selection effect and generate 3 times larger output fluctuations. In the oligopoly model, Calvo frictions weaken the dynamic complementarity in prices and generate only 1.3 times larger output fluctuations. In existing models complementarity is controlled by a free parameter, while here it responds to changes in technology and policy.

Other results. Introducing dynamic strategic interaction into a menu cost model leads to other implications for key areas of monetary economics.

First, small menu costs cause large first order output distortions. In a monopolistically competitive market, the average markup is similar with or without menu costs. In an oligopolistic market, menu costs support higher equilibrium markups by making price cuts costly. I find that

[7] For an excellent discussion of the selection effect see Midrigan (2011b).

menu costs cause first order output losses due to higher markups that are three times larger than the second order losses due to price dispersion.[8]

Second, the model rationalizes positive menu costs. On the one hand, larger menu costs allow for larger deviations from the frictionless Nash equilibrium price. On the other hand, larger menu costs make it costly to adjust to large idiosyncratic shocks. From the firms' perspective, these trade-offs yield a positive value-maximizing menu cost.

Third, strategic behavior generates some endogenous price stickiness. The oligopoly model matches the same moments as the monopolistically competitive model under smaller menu costs as a fraction of revenue, and slightly smaller idiosyncratic shocks. If evaluated under the *same parameters*, the oligopoly model would generate half as much idiosyncratic price flexibility and five times larger output fluctuations.

Literature. Complementarity in firm-level pricing is well understood to be an important source of monetary non-neutrality (Woodford, 2003, chap. 3), but has hit road blocks in state-dependent pricing models that match facts on price adjustment (Bils and Klenow, 2004).[9] Existing complementarities make deviations of firm prices from the aggregate price costly, so if the aggregate price does not fully adjust to a monetary shock, firms will stagger their price increases. Klenow and Willis (2016) introduce complementarity through Kimball (1995) preferences. Burstein and Hellwig (2007) introduce complementarity through a decreasing returns to scale production technology. Both yield *negative results*. More complementarity dampens price adjustment following aggregate shocks but causes firms to adjust aggressively to idiosyncratic shocks that would force them away from the aggregate price. Large price changes become impossible to rationalize without implausibly large menu costs and idiosyncratic shocks.[10]

The failure of these approaches is an unresolved *"serious challenge to monetary economics"* (Naka-

[8] Optimal policy in the monopolistically competitive New Keynesian model that resolves the effect of sticky prices on *markup dispersion* (Gali, 2008, chap. 4), would therefore neglect the *markup level*, which is the largest source of output losses due to sticky prices in the oligopoly economy.

[9] Woodford (2003, chap. 3) compares the effect of many such sources of complementarity in the New Keynesian literature (time-dependent adjustment and no idiosyncratic shocks). These include micro-complementarities such as non-CES preferences in Kimball (1995) and firm-level decreasing returns to scale in Sbordone (2002), and macro-complementarities such as roundabout production in Basu (1995).

[10] Beck and Lein (2020) provide an exhaustive study that extends Klenow and Willis (2016). Even under the small departures from CES that they estimate in micro-data, *monthly* productivity shocks of around *22 percent* are needed to generate the size of price changes in microdata. Burstein and Hellwig (2007) require menu costs equivalent to three percent of output to reduce price adjustment to its empirical frequency. Like Burstein and Hellwig (2007), the model of Gertler and Leahy (2008) also features complementarity via increasing marginal costs, but modeled through an increasing sectoral labor supply curve. In my model monthly shocks are around 4 percent, and menu costs are around 0.07 percent of output.

mura and Steinsson, 2010) that is simply addressed in the oligopoly model.[11] Complementarity here is between a firm's price and its competitor's price, with no complementarity with aggregates. Since both firms' prices respond to large idiosyncratic shocks there is nothing preventing large price changes in the model. With almost identical parameters, the model generates larger output responses than the monopolistically competitive model but matches the same data on price adjustment.

These types of complementarity that modify the microeconomic structure of the model and stagger firm adjustment have been categorized as *micro-complementarities*. An alternative type, *macro-complementarities*, aim to stagger the adjustment of aggregate nominal marginal cost. In Nakamura and Steinsson (2010) this is achieved via a roundabout production structure as in Basu (1995). In Burstein and Hellwig (2007) and Klenow and Willis (2016) this is achieved via nominal wage rigidity. None of these approaches quantitatively yield the empirical response of output to a monetary shock, but nonetheless substantially flatten the Phillips curve (see Table A1), so it is important that the mechanism studied here is complementary. I therefore hold the macroeconomic structure of the economy and its parameters constant when comparing microeconomic market structures. An alternative macroeconomic structure—e.g. wage rigidity—would alter the process for aggregate nominal marginal cost, and still lead to less inflation when pushed through an oligopolistic rather than monopolistic market structure. To the best of my knowledge, this is the first paper that studies a *micro-complementarity* that generates amplification without curtailing idiosyncratic adjustment.

An alternative to complementarity in generating less inflation and large output responses to a monetary shock is presented by Midrigan (2011b), Bhattarai and Schoenle (2014), and Alvarez and Lippi (2014). These papers incorporate small price changes which weaken the selection effect.[12] My results do not operate through this channel. I show that the distribution of price changes under both market structures are nearly identical in terms of standard deviation, skewness and kurtosis.[13] Moreover, I use random menu-costs in both models, which yields a smooth bimodal

[11] See also the extensive discussion in Gopinath and Itskhoki (2011) and Midrigan (2011a).

[12] Small price changes are due to multi-product firms with economies of scope in price changes. Midrigan (2011b) shows that the mechanism used to account for small price changes is inconsequential: a single-product model with random menu costs also generates small price changes and generates larger output fluctuations. To this multi-product firm framework Karadi and Reiff (2019) add stochastic volatility to idiosyncratic marginal cost, which further reduces monetary non-neutrality.

[13] Both market structures therefore share the same sufficient statistics for the output response to a monetary shock derived by Alvarez, Le Bihan, and Lippi (2016). That the alternative market structures generate different output responses emphasises a subtlety in their paper: models with complementarity in price setting do not fall into the class of models for which their results apply.

distribution of price changes that matches recent data from Cavallo (2018).

At the microeconomic level, the most closely related papers are Nakamura and Zerom (2010) and Neiman (2011). Both study single sector, partial equilibrium, oligopolistic models of price setting under menu costs. The former studies three firms subject to a sectoral shock to the cost of inputs, but no idiosyncratic shocks. I show that large idiosyncratic shocks are important for creating the within sector price dispersion that is key for dampening inflation. The latter studies two firms subject to idiosyncratic shocks, but focuses on how one firm responds to a shock to its competitor. I study the important question for macroeconomics: how do both firms respond to a common shock?

More generally, this paper demonstrates that the strategic interaction of firms, more commonly studied in the industrial organization literature, can be quantitatively important for the cyclicality and level of output when firms face standard adjustment frictions. The model even shows that the adjustment frictions commonly used in macroeconomic models may be desirable to firms when they interact dynamically and strategically. These points should be of broad interest given recently documented concentration in many sectors of the US economy, which empirical work has associated with numerous trends.[14] This paper contributes a dynamic stochastic general equilibrium framework with heterogeneous agents that may be applied to such issues.

Outline. Section 2 presents the model. Section 3 describes the main mechanism. Section 4 presents the calibration. Section 5 presents the main results on the dynamics of output and quantifies the mechanism. Section 6 discusses robustness and relation to the literature. Section 7 describes how nominal rigidities distort the level of output and how positive rigidities are valued by firms.[15]

2 Model

Time is discrete. There are two types of agents: households and firms. A unit measure of identical households consume goods, supply labor, buy state-contingent nominal bonds, and own equal

[14] Autor, Dorn, Katz, Patterson, and Van Reenen (2017) show that across sectors, declines in the labor share are correlated with increases in concentration. Gutierrez and Philippon (2017) show that the decline in the predictive power of Tobin's Q for aggregate investment is due to sectors that have experienced large increases in concentration. de Loecker, Eeckhout, and Unger (2020) provide evidence for increasing average markups.

[15] An appendix contains additional figures and tables, computational and data details and further discussion of model assumptions. An Online Appendix (at the end of this document) provides theoretical results for one- and two-period duopoly price-setting games under menu costs that the reader may find useful throughout.

shares in all firms. Firms are organized in a continuum of sectors indexed $j \in [0,1]$. Each sector contains two firms indexed $i \in \{1,2\}$. Goods are differentiated first across then within sectors. Good ij is produced by a single firm operating a technology with constant returns to scale in labor. Aggregate uncertainty arises from shocks to the growth rate g_t of the money supply M_t, and idiosyncratic uncertainty arises from shocks to preferences for each good z_{ijt}. Each period every firm draws a menu cost $\xi_{ijt} \sim H(\xi)$ and may change their price p_{ijt} by paying ξ_{ijt}.

I write agents' problems recursively, such that the time subscript t is redundant. The aggregate state is denoted $\mathbf{S} \in \mathcal{S}$. The sectoral state is denoted $s \in S$. The measure of sectors with state s is given by $\lambda(s, \mathbf{S})$. When integrating over sectors, I integrate s over $\lambda(s, \mathbf{S})$ rather than j over $U[0,1]$.

2.1 Household

Given prices for all goods in all sectors $p_i(s, \mathbf{S})$, wage $W(\mathbf{S})$, prices of state-contingent nominal bonds $Q(\mathbf{S}, \mathbf{S}')$, aggregate dividends $\Pi(\mathbf{S})$, the distribution of sectors $\lambda(s, \mathbf{S})$, and law of motion for the aggregate state $\mathbf{S}' \sim \Gamma(\mathbf{S}'|\mathbf{S})$, households' policies for consumption demand for each good in each sector $c_i(s, \mathbf{S})$, labor supply $N(\mathbf{S})$, and demand for bonds $B'(\mathbf{S})$ solve

$$\begin{aligned}
\mathbf{W}(\mathbf{S}, B) &= \max_{c_i(s), N, B'(\mathbf{S}')} \log C - N + \beta \mathbb{E}\Big[\mathbf{W}(\mathbf{S}', B(\mathbf{S}'))\Big], \\
\text{where} \quad C &= \left[\int_S \mathbf{c}(s)^{\frac{\theta-1}{\theta}} d\lambda(s, \mathbf{S})\right]^{\frac{\theta}{\theta-1}}, \\
\mathbf{c}(s) &= \left[\Big(z_1(s)c_1(s)\Big)^{\frac{\eta-1}{\eta}} + \Big(z_2(s)c_2(s)\Big)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},
\end{aligned}$$

subject to the nominal budget constraint

$$\int_S \Big[p_1(s, \mathbf{S})c_1(s) + p_2(s, \mathbf{S})c_2(s)\Big] d\lambda(s, \mathbf{S}) + \int_{\mathcal{S}} Q(\mathbf{S}, \mathbf{S}')B'(\mathbf{S}')d\mathbf{S}' \quad \leq \quad W(\mathbf{S})N + B(\mathbf{S}) + \Pi(\mathbf{S})$$

Households discount the future at rate β, have time-separable utility, and derive period utility from consumption adjusted for the disutility of work, which is linear in labor.[16] Utility from consumption is logarithmic in a CES aggregator of consumption utility from the continuum of sectors. The cross-sector elasticity of demand is denoted $\theta > 1$. As in Atkeson and Burstein (2008), utility from sector j goods is given by a CES utility function over the two firms' goods. The within-sector elasticity of demand is denoted $\eta > 1$. These elasticities are ranked $\eta > \theta$. The household is more willing to substitute goods within a sector (Pepsi vs. Coke) than across sectors (soda vs. laundry detergent). Finally, household preference for each good is subject to a shifter $z_i(s)$ that evolves according to a random walk,

[16] A parameter controlling the utility cost of labor can be normalized to one, so is not included.

$$\log z_i'(s') = \log z_i(s) + \sigma_z \varepsilon_i', \quad \varepsilon_i \sim \mathcal{N}(0,1). \tag{1}$$

The shock ε_i' is independent over firms, sectors, and time.

The solution to the household problem consists of demand functions for each firm's output $c_i(s,\mathbf{S})$, a labor supply function $N(\mathbf{S})$, and an equilibrium share price $\Omega(\mathbf{S})$, which will be used to price nominal firm payoffs. Demand functions are given by

$$\begin{aligned} c_i(s,\mathbf{S}) &= z_i(s)^{\eta-1}\left(\frac{p_i(s,\mathbf{S})}{\mathbf{p}(s,\mathbf{S})}\right)^{-\eta}\left(\frac{\mathbf{p}(s,\mathbf{S})}{P(\mathbf{S})}\right)^{-\theta} C(\mathbf{S}), \qquad (2) \\ \text{where} \quad \mathbf{p}(s,\mathbf{S}) &= \left[\left(\frac{p_1(s,\mathbf{S})}{z_1(s)}\right)^{1-\eta} + \left(\frac{p_2(s,\mathbf{S})}{z_2(s)}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}, \\ P(\mathbf{S}) &= \left[\int_S \mathbf{p}(s,\mathbf{S})^{1-\theta} d\lambda(s,\mathbf{S})\right]^{\frac{1}{1-\theta}}. \end{aligned}$$

Aggregate real consumption is $C(\mathbf{S})$. The allocation of $C(\mathbf{S})$ to sector s depends on the level of the sectoral price $\mathbf{p}(s,\mathbf{S})$ relative to the aggregate price $P(\mathbf{S})$. The allocation of expenditure to firm i is then determined by $z_i(s)$ and the level of firm i's price relative to $\mathbf{p}(s,\mathbf{S})$, which is the sectoral price index satisfying $\mathbf{p}(s,\mathbf{S})\mathbf{c}(s) = \sum_{i=1}^2 p_i(s,\mathbf{S})c_1(s,\mathbf{S})$, and is increasing in firm prices.

The aggregate price index satisfies $P(\mathbf{S})C(\mathbf{S}) = \int_S [p_1(s,\mathbf{S})c_1(s,\mathbf{S}) + p_2(s,\mathbf{S})c_2(s,\mathbf{S})]\, d\lambda(s,\mathbf{S})$, such that $P(\mathbf{S})C(\mathbf{S})$ is equal to aggregate nominal consumption. I assume that aggregate nominal consumption must be paid for using money $M(\mathbf{S})$. Money demand is therefore $M^d(\mathbf{S}) = P(\mathbf{S})C(\mathbf{S})$.[17] Money supply is exogenous. Its growth rate $g' = M'/M$ evolves as follows:

$$\log g'(\mathbf{S}') = (1-\rho_g)\log \overline{g} + \rho_g \log g(\mathbf{S}) + \sigma_g \varepsilon_g', \quad \varepsilon_g' \sim \mathcal{N}(0,1). \tag{3}$$

Hence, the nominal economy is trend stationary around $\overline{g}$. An intratemporal condition determines labor supply and Euler equation prices nominal bonds with discount factor $Q(\mathbf{S},\mathbf{S}')$:

$$W(\mathbf{S}) = P(\mathbf{S})C(\mathbf{S}), \qquad Q(\mathbf{S},\mathbf{S}') = \beta\frac{P(\mathbf{S})C(\mathbf{S})}{P(\mathbf{S}')C(\mathbf{S}')}. \tag{4}$$

Pricing all nominal payoffs, $Q(\mathbf{S},\mathbf{S}')$ also discounts nominal firm profits. In equilibrium, (4) implies that $Q(\mathbf{S},\mathbf{S}') = \beta W(\mathbf{S})/W(\mathbf{S}')$.

[17] An alternative assumption is that money enters the utility function as in Golosov and Lucas (2007). As noted in that paper, if utility is separable, the disutility of labor is linear, and the utility of money is logarithmic, one obtains the same equilibrium conditions studied here.

2.2 Firms

I consider the problem for firm i, denoting its direct competitor $-i$. The sectoral state vector s consists of previous prices p_i, p_{-i} and current preferences z_i, z_{-i}.

Within a period, information and timing are as follows. After these states are revealed, both firms independently draw a menu cost for the period ξ_{ij} from the known distribution $H(\xi)$. I make the additional assumption, discussed below, that these draws are private information. At the same time as its competitor, firm i then chooses whether to adjust its price, $\phi_i \in \{0, 1\}$, and if changing its price, changes it to p_i^*. Prices are then revealed, firms produce the quantity demanded by households, and preference shocks evolve (z_i, z_{-i}) to (z_i', z_{-i}'). *Within* a period, *all* moves are simultaneous, such that firms do not respond to each other's new price: $p_i' = \phi_i p_i^* + (1-\phi_i)p_i$. When determining its actions, firm i takes as given the policies of its direct competitor. These consist of its decision to change price $\phi_{-i}(s, \mathbf{S}, \xi_{-i})$ and its optimal price $p_{-i}^*(s, \mathbf{S})$. Since menu costs are sunk, $p_{-i}^*(s, \mathbf{S})$ is independent of ξ_{-i}.

My description of the environment has explicitly restricted firm policies to depend only on payoff relevant information $(s, \mathbf{S})$, that is, they are *Markov strategies*. A richer dependency of policies on the history of firm behavior is beyond the scope of this paper.[18]

Let $V_i(s, \mathbf{S}, \xi_i)$ denote the present discounted expected value of nominal profits of firm i after the realization of the sectoral and aggregate states $(s, \mathbf{S})$ and its menu cost ξ_i. Then $V_i(s, \mathbf{S}, \xi_i)$ satisfies the following recursion, which I unpack below:

$$
\begin{aligned}
V_i(s, \mathbf{S}, \xi_i) &= \max_{\phi_i \in \{0,1\}} \phi_i \left[V_i^{adj}(s, \mathbf{S}) - W(\mathbf{S})\xi_i \right] + (1-\phi_i) V_i^{stay}(s, \mathbf{S}), \\
V_i^{adj}(s, \mathbf{S}) &= \max_{p_i^*} \int \Bigg[\phi_{-i}(s, \mathbf{S}, \xi_{-i}) \left\{ \pi_i\left(p_i^*, p_{-i}^*(s, \mathbf{S}), s, \mathbf{S}\right) + \mathbb{E}\left[Q(\mathbf{S}, \mathbf{S}') V_i\left(s_{adj}', \mathbf{S}', \xi_i'\right) \right] \right\} \\
&\quad + \left(1 - \phi_{-i}(s, \mathbf{S}, \xi_{-i})\right) \left\{ \pi_i\left(p_i^*, p_{-i}(s), s, \mathbf{S}\right) + \mathbb{E}\left[Q(\mathbf{S}, \mathbf{S}') V_i\left(s_{adj}', \mathbf{S}', \xi_i'\right) \right] \right\} \Bigg] dH(\xi_{-i}), \\
\pi_i\left(p_i, p_{-i}, s, \mathbf{S}\right) &= d_i\left(p_i, p_{-i}, s, \mathbf{S}\right)\left(p_i - z_i(s) W(\mathbf{S})\right), \\
s_{adj}' &= \phi_{-i}(s, \mathbf{S}, \xi_{-i}) \times \left(p_i^*, p_{-i}^*(s, \mathbf{S}), z_i', z_{-i}'\right) + \left(1 - \phi_{-i}(s, \mathbf{S}, \xi_{-i})\right) \times \left(p_i^*, p_{-i}, z_i', z_{-i}'\right) \\
\mathbf{S}' &\sim \Gamma(\mathbf{S}' | \mathbf{S}).
\end{aligned}
\tag{5}
$$

[18] Quoting the progenitors of the term Maskin and Tirole (1988a), *"Markov strategies...depend on as little as possible, while still being consistent with rationality."* Rotemberg and Woodford (1992) study an oligopoly with arbitrary history dependence of policies but no nominal rigidity or idiosyncratic shocks. The implicit collusion accommodated by trigger strategies leads to countercyclical markups: the value of deviating from collusion increases when demand is high, reducing the level of the markup that can be sustained.

The first line states the extensive margin problem, where adjustment requires a payment of menu cost ξ_i in units of labor. The value of adjustment is independent of the menu cost and requires choosing a new price p_i^*. The firm integrates out the unobserved state of its competitor—the menu cost ξ_{-i}—and takes as given the effect of its competitor's pricing decisions $\left(\phi_{-i}, p_{-i}^*\right)$ on (i) current payoffs π_i, and (ii) future states s'_{adj}. The term in braces in the second (third) line gives the flow nominal profits plus continuation value of the firm if its competitor does (does not) adjust its price. The value of non-adjustment $V_i^{stay}(s, \mathbf{S})$ is equal to the value of adjustment under $p_i^*(s, \mathbf{S}) = p_i(s)$.

The above flow payoff introduces a role for $z_i(s)$ in costs. As in Midrigan (2011b) and Alvarez and Lippi (2014), I assume that $z_i(s)$—which increases demand for the good with an elasticity of $(\eta - 1)$—also increases total costs with a unit elasticity. This assumption, discussed below, allows me to reduce the state space of the firm's problem, a crucial step to maintain computational tractability of the dynamic oligopoly model.[19]

The household's nominal discount factor $Q(\mathbf{S}, \mathbf{S}')$ is used to discount future nominal profits, and expectations are taken with respect to both the equilibrium transition density $\Gamma(\mathbf{S}'|\mathbf{S})$ and firm-level shocks (z'_i, z'_{-i}). Through the household's demand functions $d_i(p_i, p_{-i}, s, \mathbf{S})$, nominal profit depends on aggregate consumption $C(\mathbf{S})$ and the aggregate price index $P(\mathbf{S})$, which the firm takes as given since it is atomistic with respect to the aggregate economy.

That menu costs are sunk and *iid* allows for two simplifications. First, as the menu cost is sunk, p_{-i}^* is independent of ξ_{-i}, hence it is sufficient for firm i to know only the probability that its competitor changes its price: $\gamma_{-i}(s, \mathbf{S}) = \int \phi_{-i}(s, \mathbf{S}, \xi_{-i}) dH(\xi_{-i})$. Second, being an *iid* draw, ξ_i can be integrated out of firm i's Bellman equation. These observations imply that ξ_i is not a state:

$$\begin{aligned} V_i(s, \mathbf{S}) &= \int \max \left\{ V_i^{adj}(s, \mathbf{S}) - W(\mathbf{S})\xi_i \,,\; V_i^{stay}(s, \mathbf{S}) \right\} dH(\xi_i), \qquad (6) \\ V_i^{adj}(s, \mathbf{S}) &= \max_{p_i^*} \quad \gamma_{-i}(s, \mathbf{S}) \left\{ \pi_i\left(p_i^*, p_{-i}^*(s, \mathbf{S}), s, \mathbf{S}\right) + \mathbb{E}\left[Q(\mathbf{S}, \mathbf{S}') V_i\left(s'_{adj}, \mathbf{S}'\right) \right] \right\} \\ &\quad + \left(1 - \gamma_{-i}(s, \mathbf{S})\right) \left\{ \pi_i\left(p_i^*, p_{-i}(s), s, \mathbf{S}\right) + \mathbb{E}\left[Q(\mathbf{S}, \mathbf{S}') V_i\left(s'_{adj}, \mathbf{S}'\right) \right] \right\}. \end{aligned}$$

Given aggregates $Q(\mathbf{S}, \mathbf{S}')$, $P(\mathbf{S})$, and $\mathbf{C}(\mathbf{S})$, this suggests the following fixed point to obtain a Markov Perfect Equilibrium. Given $p_{-i}^*(s, \mathbf{S})$ and $\gamma_{-i}(s, \mathbf{S})$, one may solve (6) to obtain firm i's optimal price $p_i^*(s, \mathbf{S})$ and probability of price adjustment $\gamma_i(s, \mathbf{S}) = H[(V_{adj}^i(s, \mathbf{S}) - V_{stay}^i(s, \mathbf{S}))/W(\mathbf{S})]$. Using (p_i^*, γ_i) we can similarly obtain (p_{-i}^*, γ_{-i}) and so on.

[19]This assumption does not change the underlying economics of the problem. In reality, idiosyncratic demand or productivity shocks may lead firms to change prices. Under constant returns to scale and homothetic preferences, the two enter symmetrically. Hence this assumption preserves the fundamental idea that a firm increases (decreases) its price when its price is too low (high) relative to some benchmark holds under either demand or productivity shocks. Importantly, the *aggregate* shock is only to nominal demand.

2.3 Equilibrium

Given the above, the aggregate state vector $\mathbf{S}$ must contain the level of nominal demand M, its growth rate g, and distribution of sectors over sectoral state variables λ. A *recursive equilibrium* is

(i) Household demand functions $d_i(p_i, p_{-i}, s, \mathbf{S})$

(ii) Functions of the aggregate state: $W(\mathbf{S})$, $N(\mathbf{S})$, $P(\mathbf{S})$, $C(\mathbf{S})$, $Q(\mathbf{S}, \mathbf{S}')$

(iii) Law of motion $\Gamma(\mathbf{S}, \mathbf{S}')$ for the aggregate state $\mathbf{S} = (g, M, \lambda)$

(iv) Firm value functions $V_i(s, \mathbf{S})$ and policies $p_i^*(s, \mathbf{S})$, $\gamma_i(s, \mathbf{S})$

such that

(a) Demand functions in (i) are consistent with household optimality conditions (2).

(b) The functions in (ii) are consistent with household optimality conditions (4).

(c) Given functions (i), (ii), (iv), and competitor policies, p_i^*, γ_i, and V_i are consistent with firm i optimization and Bellman equation (5).

(d) Aggregate price $P(\mathbf{S})$ equals the household price index under $\lambda(s, \mathbf{S})$, $p_i^*(s, \mathbf{S})$, and $\gamma_i(s, \mathbf{S})$.

(e) Nominal aggregate demand satisfies $P(\mathbf{S})C(\mathbf{S}) = M(\mathbf{S})$.

(f) The price of state contingent nominal bonds is given by (4).

(g) The law of motion for g and the path for M are determined by (3).

(h) The law of motion for λ is consistent with firm policies and (1). Let $X = P_1 \times P_2 \times Z \times Z \in \mathbb{R}^4_+$ and the corresponding set of Borel sigma algebras on X be given by $\mathcal{X} = \mathcal{P}_1 \times \mathcal{P}_2 \times \mathcal{Z}_1 \times \mathcal{Z}_2$. Then $\lambda : \mathcal{X} \to [0, 1]$ and obeys the following law of motion for all subsets of $\mathcal{X}$:[20]

$$\lambda'(\mathcal{X}) = \int_X \mathbb{E}_{\gamma_1(s,\mathbf{S}),\gamma_2(s,\mathbf{S})} \mathbf{1}\left\{(p_1^*(s,\mathbf{S}), p_2^*(s,\mathbf{S})) \in \mathcal{P}_1 \times \mathcal{P}_2\right\} \mathbb{P}\left[z_1' \in \mathcal{Z}_1 | z_1\right] \mathbb{P}\left[z_2' \in \mathcal{Z}_2 | z_2\right] d\lambda(s, \mathbf{S}).$$

This is a new type of recursive competitive equilibrium for an economy with heterogeneous agents and may be extended to many settings: behaviour between agents is competitive when agents are in different sectors of the economy, and strategic when agents inhabit the same sector. Condition (c) requires that this strategic behavior constitutes an MPE.

2.4 Monopolistic competition

The monopolistically competitive economy is identical to the above, but where firm i belongs to a continuum of firms $i \in [0, 1]$ in sector j. The demand system is identical to (2), but where $\mathbf{p}_j(\mathbf{S}) = \left[\int (p(s, \mathbf{S})/z(s))^{1-\eta} d\lambda_j(s, \mathbf{S})\right]^{1/(1-\eta)}$. Crucially, since firms are competitive, they take

[20] In this definition, $\mathbb{E}_{\gamma_1(s,\mathbf{S}),\gamma_2(s,\mathbf{S})}\left[f(s, \mathbf{S})\right]$ is the expectation of f under the sector s probabilities of price adjustment.

$\mathbf{p}_j(\mathbf{S})$ as given. The idiosyncratic state of the firm is therefore its own z_i and past price p_i. Since sectors are homogeneous in parameters, and the law of large numbers applies for each sector, then the distribution of firms λ_j is the same in all sectors. Therefore $\mathbf{p}_j(\mathbf{S}) = \mathbf{p}_k(\mathbf{S})$ for all sectors j and k, which implies that $P(\mathbf{S}) = \mathbf{p}_j(\mathbf{S})$. The cross-sector elasticity of demand θ is therefore entirely absent from the firm problem and all equilibrium conditions, which feature only η.

2.5 Markups

A sectoral MPE, nested in a macroeconomic equilibrium, is computationally infeasible with four continuous state variables at the sector level. Under the standard assumptions regarding idiosyncratic shocks that I also employ here, it may be restated in terms of markups: the ratio of nominal price to nominal marginal cost $\mu_{ij} := p_{ij}/(z_{ij}W)$. Similarly, I define the sectoral markup $\mu_j := \mathbf{p}_j/W$ and aggregate markup $\mu := P/W$. Applying these definitions to (2) gives $\mu_j = [\mu_{1j}^{1-\eta} + \mu_{2j}^{1-\eta}]^{1/(1-\eta)}$, and $\mu = [\int_0^1 \mu_j^{1-\theta} dj]^{1/(1-\theta)}$. Derivations are contained in Appendix C.

Expressed in markups and normalized by the wage, the profit of the firm is

$$\frac{\pi_i\left(\mu_i, \mu_{-i}, \mathbf{S}\right)}{W(\mathbf{S})} = \widetilde{\pi}_i\left(\mu_i, \mu_{-i}\right)\mu(\mathbf{S})^{\theta-1} \quad , \quad \widetilde{\pi}_i\left(\mu_i, \mu_{-i}\right) = \mu_i^{-\eta}\mu_j\left(\mu_i, \mu_{-i}\right)^{\eta-\theta}\left(\mu_i - 1\right). \tag{7}$$

This implies that within a sector markups are strategic complements in the static sense. A higher μ_{-i} increases μ_j, which increases the static profit maximizing μ_i.[21] Meanwhile, the level of aggregate $\mu(\mathbf{S})$ has no impact on the profit-maximizing μ_i. Firm and aggregate markups are not strategic complements as was the case in prior models of strategic complementarity in monopolistically competitive environments (Burstein and Hellwig, 2007; Klenow and Willis, 2016).

Value functions can also be normalized. Let $v_i(s, \mathbf{S}) = V_i(s, \mathbf{S})/W(\mathbf{S})$, then

$$\begin{aligned} v_i\left(\mu_i, \mu_{-i}, \mathbf{S}\right) &= \int \max\left\{v_i^{adj}\left(\mu_i, \mu_{-i}, \mathbf{S}\right) - \xi_i,\ v_i^{stay}\left(s, \mathbf{S}\right)\right\} dH(\xi_i), \\ v_i^{adj}\left(\mu_i, \mu_{-i}, \mathbf{S}\right) &= \max_{\mu_i^*} \gamma_{-i}(\mu_i, \mu_{-i}, \mathbf{S})\left\{\widetilde{\pi}_i\left(\mu_i^*, \mu_{-i}^*(\mu_i, \mu_{-i}, \mathbf{S})\right)\mu\left(\mathbf{S}\right)^{\theta-1} + \beta\mathbb{E}\left[v_i\left(\frac{\mu_i^*}{g'e^{\varepsilon_i'}}, \frac{\mu_{-i}^*(\mu_i, \mu_{-i}, \mathbf{S})}{g'e^{\varepsilon_{-i}'}}, \mathbf{S}'\right)\right]\right\} \\ &\quad + \left(1 - \gamma_{-i}(\mu_i, \mu_{-i}, \mathbf{S})\right)\left\{\widetilde{\pi}_i\left(\mu_i^*, \mu_{-i}\right)\mu\left(\mathbf{S}\right)^{\theta-1} + \beta\mathbb{E}\left[v_i\left(\frac{\mu_i^*}{g'e^{\varepsilon_i'}}, \frac{\mu_{-i}}{g'e^{\varepsilon_{-i}'}}, \mathbf{S}'\right)\right]\right\}. \end{aligned} \tag{8}$$

Despite random walk shocks, this normalization renders the firm problem stationary in markups

[21] Since $\eta > \theta$, then $\widetilde{\pi}_i$ is increasing in μ_j. By definition, complementarity requires that $\mu_j(\mu_i, \mu_{-i})$ has a positive cross-partial derivative. This is the case here. Observe that under CES demand $\partial\mu_j/\partial\mu_i = (\mu_j/\mu_i)^\eta$, which is increasing in μ_j, which itself is increasing in μ_{-i}. Figure A5 plots the level of the profit function as well as first, second, and cross-partial derivatives for various values of θ and η. Figure A4 plots the elasticity and super-elasticity (elasticity of the elasticity) of demand faced by firm 1 as μ_1 and μ_2 vary.

and clarifies the mechanics of the shocks as follows. First, a random walk idiosyncratic shock ε_i' is a permanent *iid* shock to the markup of firm i should the firm not adjust its price. Second, paying the real menu cost ξ_i allows the firm to 'reset' its markup μ_i to a value μ_i^* that depends on μ_{-i} by changing its price p_i. Third, a single positive innovation to money growth causes equilibrium nominal marginal cost to increase, reducing both firms' markups. As money growth returns to $\overline{g}$ at rate ρ_g, firms' markups continue to decline at a decreasing rate.

In this way, all equilibrium conditions can be stated in markups. Note that aggregate consumption is $C(\mathbf{S}) = M(\mathbf{S})/P(\mathbf{S}) = 1/\mu(\mathbf{S})$. An increase in the money supply causes an equilibrium increase in nominal wages, reducing all firms' markups. If prices do not increase one for one with wages, then real wages increases, labor supply increases, and output increases.

2.6 Approximation

A solution for the equilibrium involves the function $\mu(\mathbf{S})$, requiring the infinite dimensional distribution $\lambda\left(\mu_i, \mu_{-i}\right)$ as a state variable. To make the problem tractable, I follow the lead of Krusell and Smith (1998). Since I must specify a price function for μ, a convenient choice of moment to characterize λ is last period's aggregate markup, μ_{-1}. The following then serves as (i) the pricing function and (ii) the law of motion for the approximate aggregate state:

$$\mu\left(\mu_{-1}, g\right) = \exp\Big(\overline{\mu} + \beta_1(\log\mu_{-1} - \log\overline{\mu}) + \beta_2(\log g - \log\overline{g})\Big). \tag{9}$$

The supposition $\mathbf{S}' = (\mu, g')$ in (8) followed by applying (9), verifies that $\mathbf{S} = (\mu_{-1}, g)$. Appendix C provides more details on the solution of the firm problem and equilibrium.

2.7 Assumptions

Appendix D discusses a number of modeling assumptions: (i) CES preferences, (ii) structure of idiosyncratic shocks, (iii) random menu costs and (iv) their information structure. Following the insight of Doraszelski and Satterthwaite (2010), this last assumption is made to accommodate a solution in pure strategies. A model with fixed costs would yield mixed strategy equilibria, becoming computationally infeasible. In the Online Appendix (at the end of this document), I prove a number of results for a one-period game of price adjustment with a fixed menu cost, some initial prices chosen by nature, and a general profit function with complementarity. I establish that for any positive menu cost there exists a non-degenerate set of initial prices for which multiple equilibria exist (see Figure OA.1).

3 Illustrating the mechanism

To understand the dynamics of markups in the two models of market structure, I consider an exercise that corresponds to the central experiment in GL. Idiosyncratic shocks are present, but inflation and aggregate shocks are zero. I then study the response to a one-time unforeseen increase in money in period τ ($g_\tau > 0$, $\rho_g = 0$). Both models are solved under full idiosyncratic risk and then simulated under particular paths of productivity shocks and menu cost realizations that I assign for illustrative purposes. Parameters of each model are those estimated next (Section 4), and what follows is verified in simulations of the model and decompositions of inflation across the distribution of firms (Section 5).

3.1 Monopolistic competition

Figure 3 describes the behavior of firms in the monopolistically competitive model. Green (red) lines describe a firm that, from period 5 onward, is assigned a string of positive (negative) productivity shocks that steadily increase (decrease) its markup. For $t < 5$, I assign menu costs of zero, and for $t \geq 5$, I assign firms large menu cost draws such that their prices do not adjust.[22] Thin solid lines in panel A plot the evolution of each firm's markup absent the increase in money supply. Dashed lines in panel A describe the optimal reset markup of each firm μ^*_{it}. Since μ_{it} is payoff irrelevant once firm i decides to change its price, the reset markup is constant and the same for both firms. Thin lines in panel B plot the firm's probability of adjustment $\gamma_{it} = \gamma(\mu_{it})$.

The thick lines in Figure 3 describe the response to a permanent increase in the money supply ($\Delta M > 0$) in period 40 which, absent adjustment, reduces both firms' markups. The low-markup firm's probability of adjustment increases as its markup moves away from its reset value. The size of its optimal adjustment *increases by* ΔM, accommodating the entire increase in aggregate nominal cost. The high-markup firm moves closer to its reset value, its probability of adjustment falls, and its size of adjustment decreases by ΔM. The firms' optimal markups are unaffected by the shock.

This behavior sharply curtails the real effects of the monetary expansion. The distribution of

[22] Specifically, I set $\mu_{i0} = \mu_{-i0}$ to some arbitrary initial markup. For $t \leq 5$, I set $\xi_{it} = \xi_{-it} = 0$ and $\varepsilon_{it} = \varepsilon_{-it} = 0$ and use the firms' policies to evolve (μ_{it}, μ_{-it}); this means firms quickly adjust to the markup $\overline{\mu}$ that satisfies $\overline{\mu} = \mu^*_i(\overline{\mu}, \overline{\mu})$ where μ^*_i is the *dynamic best response* from the model solved with *full idiosyncratic risk*. For $t \geq 6$ I set realizations of the menu cost $\xi_{it} = \xi_{-it} = \overline{\overline{\xi}}$ such that prices do not adjust. I set realizations of the idiosyncratic shocks $\varepsilon_{it} = \overline{\varepsilon}$ and $\varepsilon_{-it} = -\overline{\varepsilon}$, such that one firm's markup steadily increases, and the other decreases. I then plot $\mu^*_i(\mu_{it}, \mu_{-it})$ and $\gamma_i(\mu_{it}, \mu_{-it})$. This should make clear that the firms' policies are solved under full idiosyncratic uncertainty, and—for illustrative purposes—I am only choosing the realized path for the simulation.

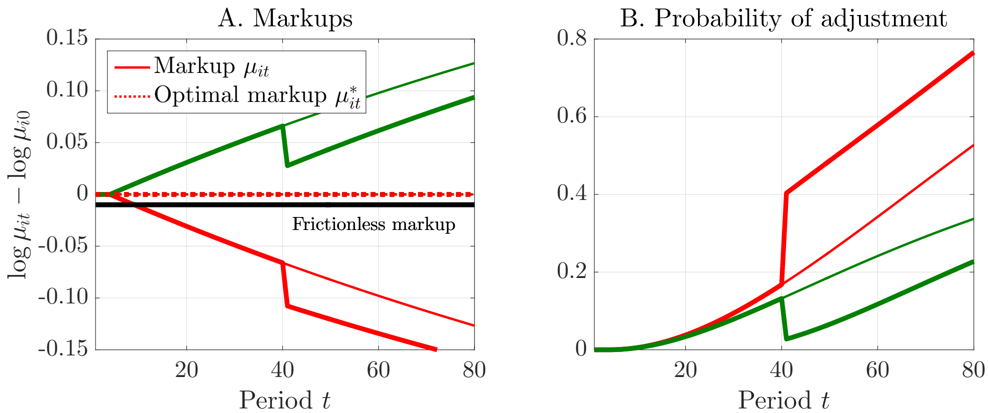

Figure 3: Example of a positive monetary shock in a monopolistically competitive market

Notes: Thin solid lines give exogenous evolution of markups for two firms *within the same sector* absent a monetary shock. Thin dashed lines give corresponding optimal markups conditional on adjustment, where $\mu_1^* = \mu_2^*$. Thick solid lines include a monetary shock in period 40 which decreases both firms' markups. Thick dashed lines—which lie on top of the thin dashed lines before period 40—give the corresponding optimal markups. The model is solved in steady state with respect to aggregate shocks only, and the monetary shock is a one-time unforeseen level increase in money. The parameters of the model are as in Table 1. The y-axis in panel A describes the log deviation of markups from the value chosen when realizations of shocks and menu costs are zero (see footnote 22).

adjusting firms shifts toward those with already low prices. These are firms that are increasing their prices and now by larger amounts. Monetary neutrality owes to the behavior of these firms with low markups and a high probability of adjustment that are *marginal* with respect to the shock.

3.2 Duopoly

I now repeat this exercise in the duopoly model for two firms in the same sector. The firms differ both in (i) their policies absent the shock and (ii) their response to the shock. These differences are due to the interaction of menu costs and pricing complementarity that arise in the duopoly model.

Static complementarity. Prices are *static complements* when the cross-partial derivative of a firm's profit function ($\tilde{\pi}_{12} > 0$) is positive. Economically, this is the case for two reasons: (i) firms are strategic, so they understand how their price affects the sectoral price, and (ii) the household has a lower ability to substitute across sectors than within sectors ($\eta > \theta$). A higher μ_2 therefore means firm 1 sells to the more of the market, lowering firm 1's demand elasticity, which it internalizes, increasing the static optimal markup of firm 1. However, if both firms had identical high markups and no cost of downward adjustment, both would undercut each other and the only equilibrium would be the frictionless Nash equilibrium $\mu_1 = \mu_2 = \mu^*$.[23]

[23]In the Online Appendix, I show that the best-response function in a static, frictionless model under CES preferences with $\eta > \theta$ is upward sloping with a slope less than one. This implies that if μ_{-i} is greater than the frictionless Nash equilibrium markup μ^*, then the static best-response of firm i is to undercut: $\mu_i^*(\mu_{-i}) \in (\mu^*, \mu_j)$. Figure A5 provides comparative statics of the best response function with respect to η and other features of the profit function at the

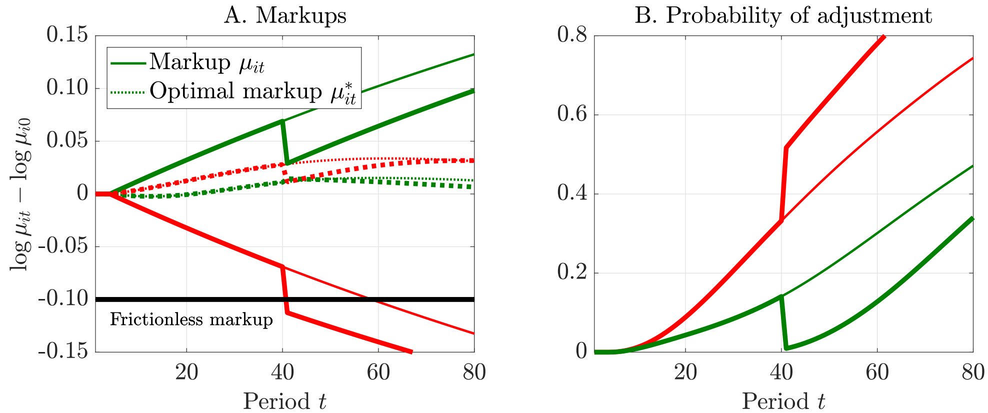

Figure 4: Example of a positive monetary shock in an oligopolistic market

Notes: Thin solid lines give exogenous evolution of markups for two firms *within the same sector* absent a monetary shock. Thin dashed lines give corresponding optimal markups conditional on adjustment $\mu_1^*(\mu_1, \mu_2)$ and $\mu_2^*(\mu_1, \mu_2)$. Thick solid lines include a monetary shock in period 40 which decreases both firms' markups. Thick dashed lines—which lie on top of the thin dashed lines before period 40—give the corresponding optimal markups. The model is solved in steady state with respect to aggregate shocks only, and the monetary shock is a one-time unforeseen level increase in money. The parameters of the model are as in Table 1. The y-axis in panel A describes the log deviation of markups from the value chosen when realizations of shocks and menu costs are zero (see footnote 22).

Dynamic complementarity. In an MPE with zero menu costs, the presence of static complementarity has no effect on the response to a monetary shock. The best-response function may describe off-equilibrium behavior that is upward sloping but in equilibrium $\mu_1^*(\mu_1, \mu_2) = \mu^*$ is independent of μ_1 and μ_2. An increase in the money supply reduces markups at the start of the period but is completely offset as both firms jump back to μ^*.

In the presence of menu costs this is no longer true. In particular, μ_1^* is not independent of (μ_1, μ_2). As I will show in Figure 4, a higher μ_2 at the start of the period—due to a combination of past actions and shocks—elicits a higher equilibrium response of firm 1 within the period. Just like the static best response, a low-priced firm adjusts to a price that is below but close to its high-priced competitor, but this is no longer an off-equilibrium consideration, as the cost of adjustment dissuades price decreases at its competitor. Prices are *dynamic complements* in that, in equilibrium, increases in the pre-determined state variable of one firm elicit an increasing response from its competitor.[24]

The Online Appendix contributes a complete characterization of behavior in a one-period game with a fixed menu cost in which (μ_1, μ_2) are given. For any menu cost and general profit function with $\pi_{12} > 0$, I show that (i) there exists a set of initial symmetric markups $\mu_1 = \mu_2 > \mu^*$, such that the Nash equilibrium involves no adjustment, and (ii) for initial markups that are very

calibrated values of θ and η.

[24] I take this language from Jun and Vives (2004), who differentiate between static and dynamic complementarity in the MPE of dynamic oligopoly models of Cournot and Bertrand competition with convex costs of adjustment.

high, the only equilibrium involves both firms paying the menu cost and choosing μ^*. Positive menu costs therefore only sustain limited deviations from the equilibrium under no menu costs. This will also be a feature of the dynamic model.

Steady-state policies. Returning to the simulation exercise, Figure 4 confirms that unlike the monopolistically competitive model, the reset markups $\mu_i^*(\mu_i, \mu_j)$ are not equal. The low-markup (red) firm sets μ_{it}^* to below, but near, that of its competitor's markup. As markups diverge, the value of a price cut at the high markup firm increases. The low-markup firm's policies—a high reset markup and high probability of adjustment—discourages undercutting. This maintains the green firm's market share in the short run while supporting a high sectoral price in the medium run. The menu costs faced by the green firm rationalize its low probability of a price cut as a best response to the red firm's policy.

In this way, the firms' policies in the non-cooperative MPE sustain markups substantially above the frictionless Bertrand-Nash equilibrium, even in the presence of large idiosyncratic shocks. Note, however, that the magnitude of this wedge is limited. In terms of flow profits, higher initial markups increasingly invite undercutting. In Figure 4A, this is reflected in the flattening out of the low markup firms' reset markup. If it adjusted to an even higher markup, then both firms would begin the next period with high markups and the small menu cost would be insufficient to dissuade price cuts. Section 7 quantifies this wedge in terms of first order output losses due to nominal rigidity.

Key to these policies is that firms know the distribution of adjustment costs faced by their competitor. In a Calvo model, firms adjust at random. As I show below, the MPE of a Calvo model with the same profit functions, and so the same *static complementarity*, features far less *dynamic complementarity*. When future price adjustments of the green firm are at random, the red firm's optimal markup is less swayed by the green firm's current markup. I show that this weakening of complementarity when firms do not choose their price changes undoes nearly all the usual reduction in monetary neutrality that one observes when replacing menu-cost with Calvo pricing frictions.

Response to monetary shock. Dynamic complementarity leads the duopoly model to respond differently to the monopolistically competitive model following a monetary shock. The desired price increase at the low-markup firm still jumps to cover the increase in aggregate nominal cost, but this is tempered by the decline in its competitor's markup. With a lower markup at its competitor, the increase in the value of adjustment is also dampened since the firms' residual demand curve has shifted in and become more elastic.[25,26] In Figure 4, the probability and size of price adjustment at the marginal firm increase by half as much as in Figure 3.

Monetary non-neutrality occurs because price adjustment at marginal firms is weakened by the falling relative price at inframarginal firms. Figure 4 provides a stark example, describing a sector with initially dispersed markups when the monetary shock hits. The decomposition exercise in Section 5 reveals these to be exactly the sectors that drive the slow response of inflation. Figure A3 repeats the above experiment for sectors with *two low markups*. Duopolists now *over-respond* relative to two monopolistically competitive firms with low prices. With both firms' probability of adjustment increasing, prices increases by more than ΔM. This reduces short term profits, but encourages larger price increases from their competitor in the future.

A complete accounting of the real effects of monetary shocks in a model with oligopolistic sectors therefore requires two key features present here: (i) many sectors in order to aggregate these sectoral differences, and (ii) idiosyncratic shocks which generate within-sector markup dispersion.

4 Calibration

External. Both models are calibrated at a monthly frequency with $\beta = 0.95^{1/12}$. I follow the same procedure as Midrigan (2011b) for calibrating the persistence and size of shocks to the growth rate of money: $\rho_g = 0.61$, $\sigma_g = 0.0019$.[27] I set $\log \overline{g} = 0.0021$ to replicate 2.5 percent average inflation in the US from 1985 to 2016. The final parameter set externally is the cross-sector elasticity θ, which I set to 1.5. This is consistent with Nechio and Hobijn (2019), one of the few studies to provide empirical estimates of upper-level demand elasticities.[28]

[25] Figure A4 panel A plots the elasticity of demand faced by firm one as a function of (μ_1, μ_2), and describes how a decrease in μ_2 increases this elasticity. The super-elasticity of demand is plotted in panel B, and gives the elasticity of the elasticity of demand with respect to μ_2.

[26] For completeness, consider the symmetric case of a negative money supply shock. The nominal wage falls and—conditional on non-adjustment—markups increase. The marginal firm now has the high markup and considers decreasing its markup, while the shock has increased the markup of its competitor. The increasing markup at its competitor shifts the marginal firm's demand curve out and lowers its elasticity, reducing the value and optimal size of a price decrease.

[27] Specifically, I take monthly time series for $M1$ and regress $\Delta \log M1_t$ on current and 24 lagged values of the monetary shock series constructed by Romer and Romer (2004). I then estimate an AR(1) process on the predicted values. The coefficient on lagged money growth is $\rho_g = 0.608$, with standard error 0.045. The standard deviation of residuals gives σ_g.

[28] Edmond, Midrigan, and Xu (2015) estimate $\theta = 1.24$ and $\eta = 10.5$ in a static oligopoly model with trade. In their quantitative application, Atkeson and Burstein (2008) choose θ "close to one" and $\eta = 10$. When estimating within-sector elasticities of substitution, it is common practice in industrial organization to assume that $\theta = 1$ such that preferences are Cobb-Douglas across sectors (for an example, see Hottman, Redding, and Weinstein (2014)). Mean-

Internal. The models have been constructed deliberately such that both have the same remaining set of parameters: (i) within-sector elasticity of substitution η, (ii) size of idiosyncratic shocks σ_z, and (iii) distribution of menu costs. I assume menu costs are uniformly distributed $\xi_{ijt} \sim U\left[0, \overline{\xi}\right]$ and refer to $\overline{\xi}$ as the menu cost. These parameters are chosen to match the average absolute size and frequency of price change in the IRI data, as well as a measure of the average markup.[29]

Price flexibility. The contribution of GL was to show that matching these first two moments severely constrains the ability of the monopolistically competitive menu cost model to generate sizeable output fluctuations. If the average size of price change is large, then the additional low-markup firms adjusting after a monetary shock will have large, positive price changes. If prices change frequently, then the increase in nominal cost is quickly incorporated into the aggregate price index. The average absolute log size of price change (conditional on price change) is 0.10, and the average frequency of price change is 0.13.[30]

Average markup. The third moment, the average markup, is motivated in two ways. First, note that the duopolist faces an overall elasticity of demand ε_i between θ and η since it does not take the sectoral markup as given. Therefore, if θ and η were the same in both, the lower demand elasticity facing the duopolist would lead to less frequent adjustment, which would be remedied by a significantly lower menu cost. Calibrating to the same average markup means the elasticity of demand faced by firms in both models is approximately the same.

In Figure 7, I show that profits as a function of the firm's price, holding all other prices fixed, have essentially the same profile in both models. The figure contrasts this to the Kimball demand specification in Klenow and Willis (2016) where profits are steeply concave in the firm's price.

Second, if the average markup is the same in both models, then average profits are also the same in both models. A ranking of calibrated menu costs is therefore preserved when transformed into the ratio of menu costs to profits.[31] I therefore interpret a model as endogenously

while, Cobb-Douglas preferences across sectors ($\theta = 1$) are commonly used in trade models, for example, Gaubert and Itskhoki (2021) and references therein.

[29]The parameter η has an overwhelming effect on the average markup. Given a value of η, one can match the size and frequency of price change by changing $\overline{\xi}$ and σ_z. Conditional on η, the argument for identification of σ_z and $\overline{\xi}$ is the same as Vavra (2014), Berger and Vavra (2019), and others. Let $x_{it} = \left|\log\left(\mu^*_{it}/\mu_{it}\right)\right|$. Increasing $\overline{\xi}$ lowers adjustment probabilities for any x_{it}, decreasing the frequency of price changes. The average size of price change increases, since x_{it} will on average be larger by the time the firm adjusts. Increasing σ_z increases the frequency of price change, since any large value of x_{it} now occurs more often. The average size of price change increases, since more frequent adjustment is costly, leading the firm to wait until x_{it} is larger before adjusting. As shown by Barro (1972), this argument leads to exact identification in a continuous time, fixed menu cost model.

[30]Appendix B details the construction of these measures from IRI data, noting here that I exclude sales and small price changes that may be deemed measurement error.

[31]Since markup dispersion will turn out to be very similar in both models, then the same statement will be true with

generating more price stickiness if t model requires a smaller menu cost in order match the data on price adjustment. However, since both models match the same data on price adjustment, there is no role for any such endogenous price stickiness to affect aggregate dynamics. The spirit of the experiment is to control for price flexibility with respect to idiosyncratic shocks, then examine the differential response to aggregate shocks, assessing endogenous price stickiness on the side.

I target an average markup of $\mathbb{E}[\mu_{it}] = 1.30$, which forms the consensus of a range of studies using various techniques. In their estimation of markups across 50 sectors, Christopoulou and Vermeulen (2012) find an average markup in the US of 1.32. For the US auto industry, Berry, Levinsohn, and Pakes (1995) estimate an average markup of 1.31. For retail goods, Hottman (2016) estimates an average markup between 1.29 and 1.33.

Discussion. The choices of θ and $\mathbb{E}[\mu_{it}]$ are designed to be conservative with respect to the degree of complementarity in the model. Macroeconomic models with monopolistic competition are commonly calibrated to a lower average markup around 1.20. Fixing θ, this would require a higher η, implying stronger complementarities and larger output fluctuations.[32] Increasing θ would have the opposite effect, but in multisector models $\theta \approx 1$, which is already less than what is used here.

Results. The first two columns of Table 1 provide baseline calibrations Duo_I and MC_I. The calibration exercise successfully delivers two models that have the same good-level price dynamics. The remaining columns provide alternative calibrations of the monopolistically competitive model, discussed below. In Table 4, discussed below, I show that second, third, and fourth moments of the distributions of price changes in both models are very similar.

Menu costs. Menu costs are lower in the duopoly model. The upper bound $\bar{\xi}$ is lower, and given that average markups are the same and $H(\xi_i)$ is uniform, the average menu cost is also lower as a fraction of profits, which is the economically meaningful measure when considering firm pricing decisions. As a further benchmark, total menu costs paid are 0.105 (0.076) percent of total revenue in MC_I (Duo_I), which are a little more than the 0.04 percent average *Physical cost* as a fraction of revenue reported in Zbaracki et. al. (2004).[33]

respect to menu costs as a fraction of output.

[32] See references in footnote 28.

[33] See their Table 5. When including *Managerial costs* and *Physical costs*, total costs amount to 1.22 percent of revenue. This latter statistic is widely used to benchmark menu cost models. Total menu costs paid are 0.522 (0.455) percent of total profits in MC_I (Duo_I), which are a little less than the *Physical cost* as a percentage of *Net margin* in Zbaracki et. al. (2004) Table 5.

		Baseline		Alternative MC models		
		Duo_I	MC_I	MC_{II}	MC_{III}	MC_{IV}
A. Parameters						
Within-sector elasticity of substitution	η	10.5	4.5	10.5	6	10.5
Upper bound of menu cost distribution	$\xi \sim U[0, \bar{\xi}]$	0.17	0.21	0.17	0.29	0.42
Size of shocks (percent)	σ_z	3.8	4.0	3.8	4.1	4.3
B. Moments						
Markup	$\mathbb{E}[\mu_{it}]$	1.30	1.30	1.12	1.22	1.13
Frequency of price change	$\mathbb{E}[\mathbf{1}\{p_{it} \neq p_{it-1}\}]$	0.13	0.13	0.19	0.13	0.13
Log absolute price change	$\mathbb{E}[\lvert\log(p_{it}/p_{it-1})\rvert]$	0.10	0.10	0.05	0.10	0.10
C. Results						
Std. deviation consumption (percent)	$\sigma(\log C_t)$	0.31	0.13	0.06	0.13	0.13
Average minus frictionless markup	$\mathbb{E}[\mu_{it}] - \mu^*$	0.10	0.02	0.01	0.02	0.02

Table 1: Parameters in the duopoly (Duo) and monopolistically competitive (MC) models

Notes: (i) The table presents three alternative calibrations of the monopolistically competitive model. MC_{II} has the same parameters as the baseline duopoly calibration. MC_{III} has a value of η chosen such that it delivers the same frictionless markup as the duopoly model. MC_{IV} has a value of η equal to the duopoly model. Under MC_{III} and MC_{IV}, the values of $\bar{\xi}$ and σ_z are chosen to match the frequency and size of adjustment. (ii) Given that $\log z_{ij}$ follows a random walk, σ_z measures percentage innovations to z_{ij}. (iii) Average log absolute price change is computed conditional on a non-zero price change.

Benchmarking. To demonstrate the importance of benchmarking both models against the same data on good level price dynamics before comparing their implications for aggregate dynamics I consider the following simple experiment. The model MC_{II} describes the monopolistically competitive model at the calibrated values of parameters from Duo_I. MC_{II} has a higher η, lower $\bar{\xi}$, and lower σ_z, so features more frequent and smaller price adjustments than MC_I. With more flexible prices, output fluctuations—as measured by the standard deviation of log aggregate consumption $\sigma(\log C_t)$—are half as large (0.06 vs. 0.13).[34] My quantitative strategy therefore works toward comparatively less, rather than more, amplification in the duopoly model.

5 Aggregate dynamics

Table 1 delivers the main result of the paper. The second last row shows that fluctuations in output are around 2.38 times larger in the duopoly model (0.31 vs. 0.13).[35] Figure 5A plots the impulse response of aggregate consumption to a one standard deviation shock to money growth,

[34]The standard deviation of log consumption is a common summary statistic for the output effects of monetary shocks in the menu cost models cited in Section 1. Specifically, $\sigma(\log C_t)$ is equal to the standard deviation of HP-filtered deviations of log of consumption from its value in an economy in which $g_t = \bar{g}$.

[35]The monopolistically competitive model under random menu costs generates larger output fluctuations than under a fixed menu cost. Calibrated to the same data, a fixed menu cost model delivers $\sigma(\log C_t) = 0.06$. This difference is for reasons discussed extensively in Midrigan (2011b): random menu costs generate some small price changes, dampening the extensive margin response of inflation—or the 'selection effect'—following a monetary shock.

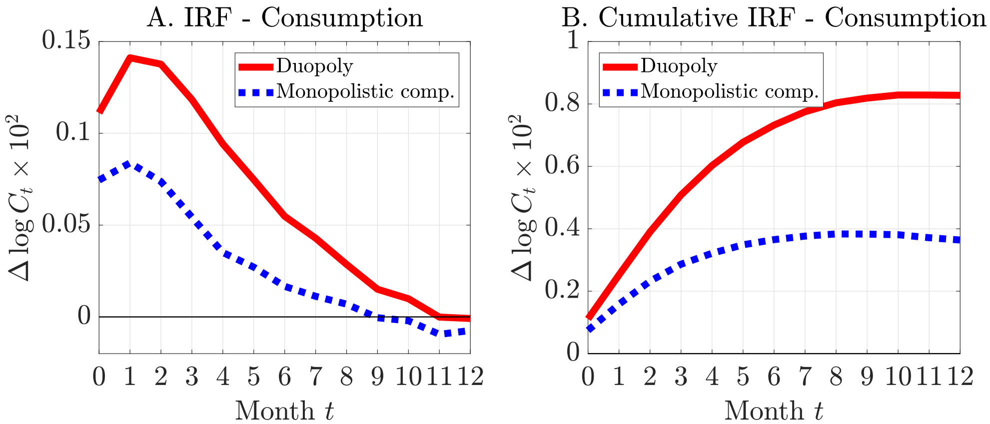

Figure 5: Market structure and monetary non-neutrality

Notes: Parameters for both models are as in Table 1 (Duo_I, MC_I). Impulse response functions are computed by local projection (see footnote 36). The response function plotted IRF_τ for $\Delta \log C_t$ is multiplied by the standard deviation of innovations to money growth $\sigma_g = 0.0019$. This is then multiplied by 100, such that units are log points. The peak response elasticity is therefore $(0.0014/0.0019) = 0.74$.

computed via local projection.[36] Panel B shows that the cumulative response is also more than twice as large in the duopoly model (0.83 vs. 0.36).

This result can be compared with other papers that study the neutrality of money in extensions of the GL model. Output fluctuations are slightly larger than in the multiproduct model of Midrigan (2011b) ($\sigma(\log C_t) = 0.29$). The ratio of $\sigma(\log C_t)$ under duopoly to monopolistic competition is also larger than found by Nakamura and Steinsson (2010) when comparing single and multisector menu cost models (a ratio of 1.82 compared to 2.38 here).[37]

I therefore have (i) added a new and realistic feature to the class of models with menu costs and idiosyncratic shocks—markets are concentrated—and (ii) moved the model toward the large real effects of monetary shocks found in the data, (iii) without deviating away from estimates of the empirical size of menu costs or idiosyncratic shocks. I return to (iii) in detail in the next section.

[36]Impulse response functions in this section are computed as follows, an approach that is econometrically equivalent to the approach used by Jorda (2005). The economy is simulated for 5,000 periods with aggregate and idiosyncratic shocks. Given the known time series of aggregate shocks to money growth ε_t^g, the horizon τ IRF is $IRF_\tau = \sum_{s=0}^{\tau} \widehat{\beta}_\tau$, where $\widehat{\beta}_\tau$ is estimated from OLS on $\Delta \log C_t = \alpha + \beta_\tau \varepsilon_{t-\tau}^g + \eta_t$. To analyse the effect of a one standard deviation shock to ε_t^g, plotted responses give $\sigma_g \times IRF_\tau$. The benefits of computing the IRF in this manner are (i) it is exactly what one would compute in the data if the realized path of monetary shocks was known, which is consistent with the approach that uses identified monetary shocks from either a narrative or high-frequency approach (Gertler and Karadi, 2015); (ii) it avoids the time-consuming approach of simulating the model many times, as is usually done in heterogeneous agents models with aggregate shocks; and (iii) it averages out any state dependence that might bias the results if computing an IRF from a specific state, as well as any non-linearity in the size of the response following positive/negative and small/large shocks; (iv) Berger, Caballero, and Engel (2021) extensively assess the benefits of this approach in accurately capturing the persistence of aggregate dynamics in lumpy adjustment models. To the best of my knowledge, this is the first paper to consider this as the baseline computational approach for the impulse response.

[37]See their Table VI (first row, first two columns). Across different specifications, this ranges from a ratio of 1.63 to 2.00. In general, the *Calvo+* framework increases output responses but does not further amplify the effect of the macro-complementarity.

Importantly, the duopoly mechanism does not exclude existing approaches such as those cited above. So while no existing approach alone generates the real effects of monetary shocks observed in the data, various approaches may be combined in ways that could. For example, the *macro-complementarity* studied in Nakamura and Steinsson (2010)—which slows the pass-through of monetary shocks to aggregate marginal cost—would operate independently of the *micro-complementarity* studied here. To reiterate, the macroeconomic structure of both economies is identical. See Table A1 for further comparisons to the literature and the model's implied slope of the Phillips curve.

5.1 Verifying the mechanism I: Price adjustment at low and high markup firms

To check whether the intuition from Section 3 holds in the full model, I study the response of the average absolute size and frequency of price change for low- and high-markup firms following a positive monetary shock. Figure 6 shows that the broad dynamics of both models are the same, consistent with the standard selection effect. Low-markup firms—in red here and in Figures 3 and 4—adjust more (panel A), and the size of their price change increases (panel B). High-markup firms adjust less, and the size of their price change falls. However, both the frequency and size of price change of low-markup firms respond by less in the duopoly model. For low markup firms, concurrent idiosyncratic shocks may increase or decrease their competitor's markup, but the aggregate shock implies that *on average* their competitor's markup falls, which due to the dynamic strategic complementarity in prices *on average* reduces the value of a price increase and the optimal price conditional on adjustment.[38]

5.2 Verifying the mechanism II: Decomposing inflation

The response of inflation can be more formally decomposed into an extensive and intensive margin response, and these margins compared across sectors of the economy. I follow the spirit of the theoretical decomposition in Caballero and Engel (2007), which can be applied to a wide class of lumpy adjustment models.[39]

[38]The average size of price changes at high-markup firms falls by less in the duopoly model. The increase in probability of upward adjustment at their low-markup competitor reduces the incentive for high-markup firms to decrease their price. This would be a force toward a *larger inflation response* in the duopoly model. However, the falling probability of adjustment for high-markup firms implies that the reduction in the size of optimal downward adjustment is rarely incorporated into the aggregate price index.

[39]See Figure A1 for a diagrammatic representation of this decomposition in a monopolistically competitive model with fixed menu costs.

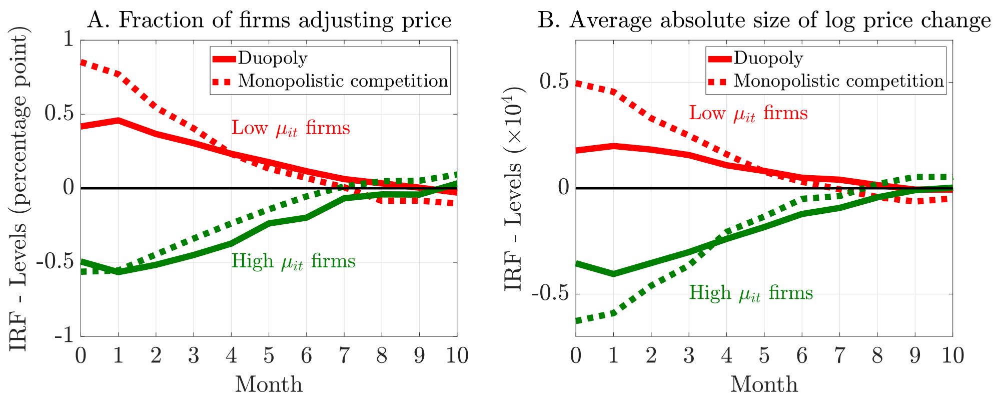

Figure 6: Impulse responses of frequency and size of adjustment to a positive monetary shock

Notes: Impulse response functions are computed by local projection (see footnote 36). For panel A, the dependent variable is the change in the fraction of firms adjusting price. For panel B, the dependent variable is the change in the average absolute size of log price changes. To isolate the effect of a positive monetary shock, only positive innovations to money growth $\varepsilon_t^g > 0$ are included in the regressions. Black (grey) lines correspond to low (high) markup firms. In the duopoly model, firms are assigned to the low-markup group if, within their sector, they have the lowest markup. In the monopolistically competitive model, pairs of firms are drawn at random and assigned to the low-markup group if their markup is the lowest in the pair.

Consider two simulations of the model, where the model has been *solved* in the presence of aggregate shocks. In one simulation, aggregate shocks are set to zero, leaving only trend inflation. A second simulation features identical draws of idiosyncratic shocks but includes a single shock to the money growth rate at date t. Denote by $\Delta\overline{p}_t$ the log change in the aggregate price index in the first simulation and by $\Delta\widehat{p}_t$ the same statistic in the simulation with the shock. Inflation due to the shock is $\pi_t = \Delta\widehat{p}_t - \Delta\overline{p}_t$. Let $x_{it} = \log p_{it}^* - \log p_{it-1}$ denote the optimal log price change of firm i if it were to adjust its price and γ_{it} the probability of price change. Then $\Delta p_t \approx N^{-1}\sum_{i=1}^N \gamma_{it}x_{it}$. This implies the following decomposition of inflation:

$$\pi_t \approx N^{-1}\sum_{i=1}^{N} \underbrace{\overline{\gamma}_{it}\left(\widehat{x}_{it} - \overline{x}_{it}\right)}_{\text{1. Intensive}} + \underbrace{\overline{x}_{it}\left(\widehat{\gamma}_{it} - \overline{\gamma}_{it}\right)}_{\text{2. Extensive}} + \underbrace{\left(\widehat{\gamma}_{it} - \overline{\gamma}_{it}\right)\left(\widehat{x}_{it} - \overline{x}_{it}\right)}_{\text{3. Covariance}}. \tag{10}$$

Panel A of Table 2 provides this decomposition for each of the two models. The first two rows show that in both models, inflation is generated roughly equally by adjustment on the intensive and extensive margins. The main result from the previous section was that inflation responds by more in the monopolistically competitive model, producing smaller output effects. Panel B shows that the difference in inflation is roughly equally accounted for by decreases in all margins of adjustment. This is consistent with Figure 4: low markup firms optimal size of adjustment declines and their probability of adjustment declines.

Panel C accounts for these differences across the distribution of sectors. For example, the bot-

		1. Intensive	2. Extensive	3. Covariance
A. Fraction of inflation accounted for by each margin				
Monopolistic competition	π_t^{mc}	0.40	0.55	0.05
Duopoly	π_t^{d}	0.41	0.58	0.01
B. Fraction of the difference in inflation accounted for by each margin				
Monopolistic competition minus duopoly	$(\pi_t^{mc} - \pi_t^{d})$	0.36	0.45	0.19
C. Fraction of the difference in each margin accounted for by regions of the distribution of markups				
One below, one above the median	$\left(\mu_i^L, \mu_j^H\right)$	1.81	1.65	1.05
Both markups below the median	$\left(\mu_i^L, \mu_j^L\right)$	-0.90	-0.73	-0.50
Both markups above the median	$\left(\mu_i^H, \mu_j^H\right)$	0.09	0.08	0.45

Table 2: Market structure and the composition of monetary non-neutrality

tom left entry states that 9 percent of the difference in the intensive margin of adjustment can be accounted for by sectors in which both firms have markups above the median markup.[40] Panel C quantifies the earlier claim that sectors with dispersed markups due to accumulated large idiosyncratic shocks account for the difference between the two models. This result motivated the simulations studied in Section 3, Figure 4. By extension, this implies that the presence of independent idiosyncratic shocks—a key feature of the menu cost literature in monetary economics—is important.

Panel C also shows that sectors with low markups contribute substantially toward greater aggregate price flexibility (recall the discussion of Figure A3). In the oligopoly model with menu costs, the presence of static complementarity does not uniformly imply more aggregate price stickiness across all sectors. In these sectors, the probability of adjustment on the *extensive margin* increases for *both* firms, and so the firms adjust more on the *intensive margin*. Quantitatively, however, dispersed markup sectors determine aggregate inflation for two reasons. First, there are simply twice as many sectors with low and high markups than with two low markups. Second, sectors with two high markups are inframarginal and respond similarly in both models.

[40] In these experiments, the realizations of random numbers used to generate the simulations are the same across models. Two firms in one sector in the duopoly model therefore have two corresponding, but unrelated, firms in the monopolistically competitive model. The different parameters of each model map random numbers into different idiosyncratic shocks and menu costs, but the underlying random numbers are the same for each of these pairs. In each model, these pairs of firms are then assigned to quadrants of the distribution of markups according to their markups relative to the median markup.

Price setting →	A. Fixed menu cost	B. Random menu cost		C. Calvo	
Market structure →	**MC**	Duo.	**MC**	Duo	**MC**
Output response, $\sigma(\log(C_t)) \times 100$	**0.08**	0.31	**0.13**	0.41	**0.38**
Multiplier: Mon.Comp vs. Duo | Price setting		2.38	-	1.07	-
Multiplier: Menu cost vs. Calvo | Market structure		-	-	1.32	**2.92**

Table 3: Market structure, monetary non-neutrality, and price-setting technologies

Notes: (i) Panel B Menu cost model results are from Table 1: Duo_I and MC_I. (ii) Panel C Calvo model frequency of price change $\alpha = 0.13$ and size of shocks $\sigma_z = 0.05$ are chosen to match the same frequency and average size of absolute price change as the menu cost model. (iii) Note, the same σ_z is able to be used in both the duopoly and monopolistically competitive Calvo models due to the lack of pricing complementarity under Calvo. (iv) The last two lines compute the relative $\sigma(\log(C_t))$ for the relevant comparison.

6 Discussion of main results

This section presents additional experiments and robustness. First, relative to menu costs, Calvo pricing does not have the same dampening effect on inflation that one might be used to observing under monopolistic competition. Second, delivering more market power to firms in the monopolistically competitive model by lowering η does not deliver larger output responses. Third, I distinguish the model from the previous literature and explain how strategic complementarity is not a barrier to large firm level price adjustment. Finally, higher order moments of the distribution of price changes are the same across market structures MC_I and Duo_I.

6.1 Calvo price setting weakens the duopoly mechanism

GL show that a monopolistically competitive menu cost model with idiosyncratic shocks exhibits far greater neutrality under menu costs than Calvo. The same is true under random menu costs. Table 3 shows that a monopolistically competitive model under a Calvo price-setting technology—where the frequency of price change α and size of shocks σ_z are recalibrated to match the empirical size and frequency of adjustment—generates 300 hundred percent larger output fluctuations (0.38 vs. 0.13).

The key result of this section is that this amplification is severely dampened under duopoly. Under duopoly, output fluctuations are only 30 percent larger under Calvo (0.41 vs. 0.31).[41] What drives this result? Monetary non-neutrality in the duopoly model is due to the amount of dynamic complementarity that the pricing friction generates. When a firm knows prices will change at random—as under Calvo—the incentive of a low-priced firm to reprice close to its competi-

[41] A feature of the literature has been to ask whether state-dependent models can deliver output fluctuations as large as time-dependent models. For example, in Midrigan (2011b), a GL model delivers $\sigma(C_t) = 0.07$, a Calvo model $\sigma(C_t) = 0.35$, and the author's benchmark multiproduct model $\sigma(C_t) = 0.29$. The headline statistic is that the multiproduct model generates real effects of monetary shocks that are 78 percent ($= 0.29/0.35$) as large as a Calvo model. Here, that statistic is 82 percent ($= 0.31/0.38$).

tor is weakened. Hence the declining markup of inframarginal firms have less of an impact on marginal firms' adjustment (recall Figure 4). The large attenuation of the intensive margin response observed in Table 2B is therefore weaker under Calvo.

This has three significant implications. First, adding Calvo-like elements may not reduce monetary non-neutrality when firms behave strategically.[42] Second, changing market structures *within* the Calvo model will have negligible effects (0.41 vs. 0.38), which may be important for understanding Werning and Wang (2020) which is a Calvo model without idiosyncratic shocks, despite large effects in a menu cost model with idiosyncratic shocks. Third, the amount of complementarity in equilibrium is not invariant to changes in policy or technologies. This is potentially of interest given recent evidence that the responsiveness of firms to shocks is (i) countercyclical (Berger and Vavra, 2019) (ii) declining over time (Decker, Haltiwanger, Jarmin, and Miranda, 2020).

6.2 Giving monopolistically competitive firms more market power

An alternative strategy for calibrating η might ensure that markups in frictionless versions of the two economies coincide.[43] The Online Appendix derives the closed form expressions for markups in each model under frictionless price setting:

$$\mu^*_{Duo} = \frac{\frac{1}{2}\left(\eta_{Duo} + \theta\right)}{\frac{1}{2}\left(\eta_{Duo} + \theta\right) - 1} \quad , \quad \mu^*_{MC} = \frac{\eta_{MC}}{\eta_{MC} - 1}.$$

The baseline calibration of $(\theta, \eta_{Duo_I}) = (1.5, 10.5)$ implies $\mu^*_{Duo_I} = 1.20$. To obtain $\mu^*_{MC} = \mu^*_{Duo_I}$ requires $\eta_{MC} = 6$. Calibration MC_{III} in Table 1 uses $\eta_{MC_{III}} = 6$ and a higher value of the menu cost in order to match the same moments. The key result is that the real effects of monetary shocks are unchanged. Calibration MC_{IV} takes the extreme case of $\eta_{MC_{IV}} = \eta_{Duo_I} = 10.5$. Again, after matching the microdata, $\sigma(\log C_t)$ is unaffected.

Figure A2 shows this result holds for $\eta_{MC} \in [2, 10]$ ($\mu^*_{MC} \in [1.11, 2.00]$). Solid lines describe the monopolistically competitive model under different values of η_{MC}, each time recalibrating the menu cost (panel A) to best match the data (panel B).[44] Dashed lines describe the same economies

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[42] E.g. A common resolution of monetary neutrality in menu cost models has been to allow a small, random fraction of firms to adjust for free each period. Nakamura and Steinsson (2010) show that this *Calvo-plus* model replicates well the distribution of price changes, however when firms behave strategically it will also weaken the dynamic complementarity in prices.

[43] Such an approach is appealing. Benchmarking models in the absence of nominal rigidity is better situated to answer the question, *"How do the effects of nominal rigidity depend on market structure?"* This is the spirit of Maskin and Tirole (1988b), Lapham and Ware (1994), and Jun and Vives (2004), who ask how *introducing* price stickiness may affect the pricing of oligopolists.

[44] This is imperfect since for simplicity Figure A2 leaves σ_z fixed at its value under MC_I.

but with the menu cost fixed at $\overline{\xi}_{MC_{III}} = 0.29$. In all cases, $\sigma(\log C_t) \approx 0.13$. Larger output fluctuations cannot be obtained by giving *more market power to monopolistically competitive firms*.[45]

6.3 Alternative sources of non-neutrality in quantitative menu cost models

Previous extensions of GL reduce monetary neutrality by (A) changing the *macroeconomic* environment to introduce complementarities between aggregate nominal cost and the aggregate price level, (B) changing the *microeconomic* environment to introduce complementarities between the firm's price and the aggregate price level, a category this paper fits into, and (C) increasing the kurtosis in the distribution of desired price changes. First, I discuss why (A) is a complement to, not substitute for, the duopoly mechanism. Second, I summarize why (B) has have been unsuccessful and so now abandoned, but not an issue here. Finally, I verify that the model does not simply increase kurtosis of price changes and so is not (C) in disguise.

6.3.A Models of macro-complementarity

The macroeconomic environment of the duopoly and monopolistically competitive model are identical: pass-through of M_t to aggregate nominal cost W_t is immediate in both cases. Since this is the case, I do not compare the model to those that reduce aggregate price flexibility by altering the *macroeconomics* of the model in order to slow the pass-through of M_t to nominal marginal cost. Nominal wage rigidity (Burstein and Hellwig, 2007; Klenow and Willis, 2016) or sticky prices of intermediate goods (Nakamura and Steinsson, 2010) could be added and lead to larger output responses in both the monopolistically competitive and duopoly models. Quantitatively, such *macroeconomic complementarities* by themselves have been shown to significantly reduce monetary neutrality, but for reasonable calibrations still imply a steep Phillips curve.[46]

The finding of significant monetary non-neutrality from the duopoly model therefore serves as a complement to, rather than a substitute for, these approaches. At the same time, the results overturn the decade long position that *macro-complementarities* are the only type of complementar-

[45] Alvarez, Le Bihan, and Lippi (2016) prove that to a second order approximation, the real effects of small monetary shocks in monopolistically competitive menu cost models will be equal, provided they match the same frequency, average absolute size, and kurtosis of price changes. Changing the elasticity of demand while recalibrating the model ensures that these statistics are the same. Figure A2 demonstrates that their theorems hold in a model without any such approximations and under the empirical size of monetary shocks.

[46] For example, Nakamura and Steinsson (2010) find that the integration of the Basu (1995) roundabout production model into a GL framework yields a $\sigma(\log C_t)$ that is 1.8 times larger than a model without intermediate inputs. Burstein and Hellwig (2007) find that reduced-form wage rigidity in the form $W_t = Y_t^{\gamma} M_t$, with $\gamma = 0.8$, can double the size of the output response to a monetary shock. These and others, including Gertler and Leahy (2008) who study complementarity in a model without idiosyncratic shocks, are included in the meta-study presented in Table A1.

ity able to both (i) slow macro responses of the price level to monetary shocks, and (ii) generate large micro responses of prices to idiosyncratic shocks.

6.3.B Models of micro-complementarity

As noted by Nakamura and Steinsson (2010), *"Monetary economists have long relied heavily on complementarity in price setting to amplify monetary non-neutrality generated by nominal rigidities."* This is the same conclusion of the comprehensive Chapter 3 of Woodford (2003). In a monopolistically competitive menu cost model, however, the only place to put such complementarity is between the firm's price and the aggregate price. The standard approaches that achieve this have turned up negative results, *"render[ing] the [menu cost] model unable to match the average size of micro-level price changes for plausible parameter values"*, which *"cast[s] doubt on [micro-]complementarity as a source of amplification."*[47]

The duopoly model introduces complementarity to the GL model, significantly reduces the response of inflation, but also matches the micro-data under similar parameters. To understand the simple way that the model avoids these issues, I first summarize existing approaches, how they slow inflation, why they require "implausibly" large menu costs and shocks, then the simple way my model differs.

Features. Microcomplementarities have been introduced by modifications to preferences and technology. First, Kimball (1995) preferences introduced by Klenow and Willis (2016) and Beck and Lein (2020), generate variable *marginal revenue*. When quantity sold decreases, the elasticity of demand increases, as captured by the following reduced form for demand:

$$y_i = \left(\frac{\mu_i}{\mu}\right)^{-\varepsilon_i} Y, \quad \varepsilon_i = \eta \exp\left(-\chi \frac{y_i}{Y}\right) \quad , \quad \chi > 0 \tag{11}$$

Second, a decreasing returns to scale technology (DRS) introduced by Burstein and Hellwig (2007), generates variable *marginal cost*. When quantity sold decreases, marginal cost decreases, as captured by the following reduced form for supply:[48]

$$mc_i \propto \frac{y_i^{\chi}}{z_i W} \quad , \quad \chi \geq 0. \tag{12}$$

[47] For a similar discussion and summary see Gopinath and Itskhoki (2011) p.270, who refer to these complementarities as *real rigidities*: *"[These studies] conclude that the levels of real rigidity sufficient to generate significant monetary non-neutrality have implausible implications for the required size of menu costs and idiosyncratic productivity shocks."* To this we may add the findings of Beck and Lein (2020) and Dossche, Heylen, and den Poel (2010) that—as I paraphrase from Klenow and Willis (2016)—estimates from retail data imply that the elasticity of demand is decreasing in a firm's relative quantity ($\chi \approx 1$), but not as much as assumed by the macro literature (cf: Smets and Wouters (2007), Eichenbaum and Fisher (2007) ($\chi \approx 10$)).

[48] If $y_i = z_i^{\alpha} n_i^{\alpha}$, then $\chi = (1-\alpha)/\alpha$.

In both cases, a parameter χ determines the degree of complementarity. Crucially, lower output (y_i/Y) decreases the firm's desired price via a more elastic demand or lower marginal cost.

Amplification. Consider a firm with a relatively low markup of μ_i and desired markup $\mu_i^* > \mu_i$. Since prices are sticky, an increase in the money supply leads to a decline in the aggregate markup: μ falls to $\mu' < \mu$. Is μ_i^* still the firm's optimal markup? With $(\mu_i^*/\mu') > (\mu_i^*/\mu)$, the relative quantity sold at μ_i^* falls. Under Kimball (DRS) ε_i increases (mc_i decreases) at μ_i^*, implying a lower optimal markup $\mu_i^{*\prime} < \mu_i^*$. As we ramp up χ, low-priced firms reduce their desired markup following a monetary shock, slowing inflation.

Issue. A by-product of increasing χ is that firms become overly responsive to idiosyncratic shocks. Consider the same firm's response to a decrease in productivity to $z_i' < z_i$. If the firm leaves its price fixed then its markup falls as marginal cost increases $(\mu_i'/\mu) < (\mu_i/\mu)$, so the quantity sold at μ_i' would *increase*. The complementarity now works in reverse. Under Kimball (DRS) ε_i decreases (mc_i increases) at μ_i' if the firm does not adjust, increasing the value of a price change. As we ramp up χ, low-priced firms become more responsive to negative idiosyncratic shocks, which are exactly the shocks that lead them to change their price.

Increasing the sensitivity of prices to large idiosyncratic shocks therefore poses a quantitative issue. The key insight of GL is that the Bils and Klenow (2004) facts suggest most price changes are due to idiosyncratic shocks, which are large, not aggregate shocks, which are small. Klenow and Willis (2016) and Burstein and Hellwig (2007) find that values of χ that reduce monetary neutrality cause such excess responsiveness to idiosyncratic shocks that "implausibly" large menu costs and idiosyncratic shocks are required to get the model to generate the large price changes observed in the data.[49] Hence the quote from Nakamura and Steinsson (2010).

Solution. In the duopoly model amplification occurs due to complementarity at similar $\bar{\xi}$ and σ_z as the monopolistically competitive model. How are the above issues avoided? First, in my model the relevant relative price that determines profits is not (μ_i/μ), but (μ_i/μ_{-i}). Second, as opposed to the small shocks to μ, shocks to μ_{-i} are of the same magnitude as those to μ_i. A large negative shock to μ_i now no longer necessarily causes (μ_i/μ_{-i}) to fall.

In this simple way, the model avoids the key issues that hamper the ability of other models of micro-complementarity to accommodate large idiosyncratic price adjustments. As opposed to

[49] Klenow and Willis (2016) find that the standard deviation of shocks at a monthly frequency would need to be 28 percent to accommodate $\chi = 10$, which delivers amplification similar to my main result. In an exhaustive study of the menu cost model under Kimball preferences, Beck and Lein (2020) reach the same conclusion for even smaller values of χ. Burstein and Hellwig (2007) conclude that with DRS, matching the observed magnitude of price changes *"requires menu costs that are much higher than existing estimates"*, in their case around three percent of revenue.

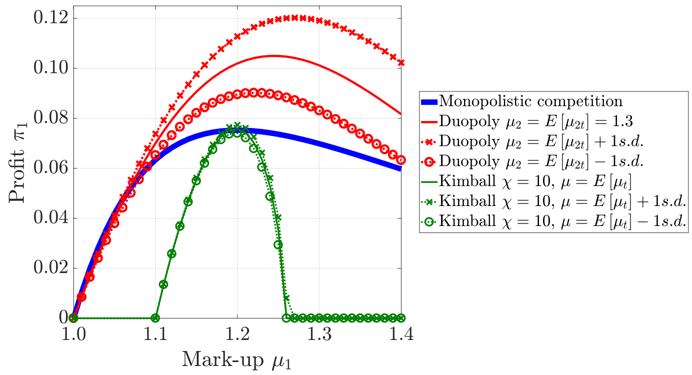

Figure 7: Market structure and the profit function of a firm

a complementarity tieing the firm to an average of infinitely many prices that moves very little, the complementarity here ties the firm to an idiosyncratic price that moves a lot. Despite large idiosyncratic movements, *on average* markups at high-markup firms decrease slightly following an increase in the money supply which will *on average* lead to lower adjustment from their low markup competitors.

Figure 7 emphasizes this point. The profit function with Kimball demand and $\chi = 10$ as in Klenow and Willis (2016) is sharply concave—which causes the firm to quickly correct mispricing—and changes little as aggregate μ fluctuates by plus and minus one standard deviation.[50] For a fixed μ_{-i} the duopoly profit function is only slightly more concave than under monopolistic competition—which accommodates large degrees of mispricing—while is curvature and level change substantially as μ_{-i} changes by large amounts.[51]

6.3.C Models that affect the distribution of price changes

Holding the average size of price changes fixed, the size of the extensive margin response in the GL model is determined by the new mass of firms increasing their prices following a positive monetary shock (see Figure A1). This is determined by the gradient of the distribution of *desired price changes* at the adjustment thresholds. More kurtosis reduces the gradient, leading to larger output responses, and more kurtosis in the distribution of *realized price changes*.

In Midrigan (2011b) and Alvarez and Lippi (2014), kurtosis arises from multiproduct firms

[50] The lines marked with crosses (circles) plot profit functions following one standard positive (negative) deviation shocks to aggregate markup.

[51] Figure A4 panel B, shows that the implied super-elasticity of demand in the duopoly model is also much lower than in Klenow and Willis (2016), around 3.5. In a menu-cost model with Kimball demand, Berger and Vavra (2019) match the observed pass-through of exchange rate shocks to import prices with an elasticity (ε) and super-elasticity of demand (χ) consistent with a desired markup elasticity of $\Gamma = \chi/(\varepsilon - 1)$ equal to 0.60. Panel C shows that in the region of the average markup in the duopoly model, this is around 0.70.

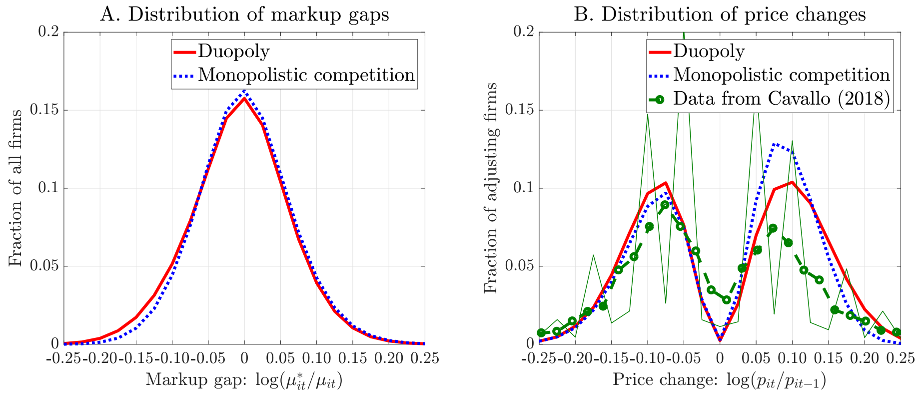

Figure 8: Distributions of markup gaps and price changes

Notes: The markup gap $\log\left(\mu^*_{it}/\overline{\mu}_{it}\right)$ is defined with respect to the markup that would occur if the firm does not change its price, and the optimal markup μ^*_{it}. The distribution of desired price changes is equal to the distribution of markup gaps $\log\left(p^*_{it}/p_{it-1}\right) = \log\left(\mu^*_{it}/\overline{\mu}_{it}\right)$. For a derivation see Appendix C.1. Firms are binned in 0.025 intervals of the values on the x-axis. Raw data used to construct the empirical distribution in Panel B are from Cavallo (2018). The data provides prices p_{it} and p_{it-1} conditional on price change, from which I compute $\log\left(p_{it}/p_{it-1}\right)$. Thin line gives the raw histogram of price changes binned in 0.025 intervals. Thick dashed line with circle markers gives kernel density plot with bandwidth of 0.025.

Market structure		Std.	Skew.	Kurt.	$p10$	$p25$	$p75$	$p90$	Increases
Monopolistic competition	MC_I	0.11	-0.26	1.88	-0.14	-0.10	0.09	0.12	0.58
Duopoly	Duo_I	0.11	-0.06	1.79	-0.15	-0.10	0.10	0.14	0.54

Table 4: Moments of the distribution of price changes: $x_{it} = \log\left(p_{it}/p_{it-1}\right)$

Notes: (i) All moments are from the distribution of price changes. (ii) pX gives the X^{th} percentile of the price change distribution. (ii) *Increases* gives the fraction of prices changes that are positive.

with economy of scope in price changes. In Gertler and Leahy (2008), infrequent large shocks throw the firm's markup conditional on non-adjustment far beyond the adjustment threshold so that the firm adjusts while close to its reset value. Alvarez, Le Bihan, and Lippi (2016) (hereafter, ABL) show that—*within this class of models*—the frequency and kurtosis of realized price changes are sufficient statistics for the real effects of small monetary shocks.

Figure 8A shows that the distribution of *desired markup changes* is almost identical in both models.[52] Figure 8B and Table 4 show that the distribution of *realized price changes* is also almost identical in both models. Under MC, a larger fraction of price increases and additional right skewness arises due to the asymmetry of the CES profit function, as observed in Figure 7.[53]

The duopoly model has the same frequency, standard deviation, and kurtosis of price changes as the monopolistically competitive model, and no thicker tails to the price change distribution. That larger output effects occur under duopoly only confirms that—owing to the presence of com-

[52] Appendix C shows that this distribution is equivalent to the distribution of *desired price changes*.

[53] These third order properties imply a relatively sharper decline in profits at low prices, leading to a slightly higher frequency of price increases.

plementarity in prices—it does not belong to the class of models for which the sufficient statistics of ABL apply.[54]

Finally, the uniform random menu cost model produces a price change distribution that is an impressive fit relative to recent evidence. A smooth bimodal distribution is found in recent data compiled by Cavallo (2018), which I reproduce in Figure 8B.[55] Cavallo and Rigobon (2012) test for bimodality in the distribution of price changes at 30 retailers across 15 countries. They reject the null hypothesis of a unimodal distribution in over 80 percent of retailers (see their Table 2).

7 Welfare implications of nominal rigidity

7.1 Nominal rigidities cause first order welfare losses under oligopoly

The oligopoly model has novel implications for the welfare costs of frictions, here nominal rigidity. Studying these implications is important, especially when we recall that optimal policy in the benchmark New-Keynesian model produces the same macroeconomic dynamics as an economy without nominal rigidity. As summarized in Gali (2008, chap. 4), the distortions in the New-Keynesian model separate neatly into those due to (a) the presence of market power in goods markets, which affect the *average markup*, and (b) the presence of sticky prices, which affect the *dispersion of markups*. The distortion due to market power under monopolistic competition is unrelated to the presence of sticky prices. Oligopoly breaks this neat separation. The market power distortion is amplified by the presence of nominal rigidity.

Table 1 showed that in the presence of menu costs, strategic firms are able to sustain markups that are higher than the frictionless markup: $\mathbb{E}[\mu_{it}] = 1.30 > 1.20 = \mu^*_{Duo_I}$. Similar to the stylized model of Maskin and Tirole (1988b) or models with convex adjustment costs such as Jun and Vives (2004), price frictions bestow dynamic commitment to high prices, which may be leveraged when prices are static complements.

My contribution is to quantify this wedge and its output consequences in a model that matches

[54] The derivations of ABL require that—to a first order—a firm's profit function is independent of all other prices. This rules out complementarity. In the duopoly model (Kimball, DRS), a competitor's price enters the first-order conditions of the firm, breaking the application of these sufficient statistics. In the monopolistically competitive model with Kimball or DRS, the aggregate price enters the first-order conditions of the firm, breaking the application of these sufficient statistics.

[55] The empirical distribution of price changes in Figure 8 is computed using data available from the companion website for Cavallo (2018): http://www.mit.edu/ afc/data/data-page-scraped.html. The exact data used in Figure 8B exclude price changes due to sales and are from an unspecified US retailer. Cavallo (2018) studies five retailers, I use data from *USA5*.

			Mon. Comp. MC_I	Duopoly Duo_I
(1)	Output under flexible prices ($\mu_{it} = \mu^*$)	Y^*	0.78	0.83
(2)	Output under no dispersion ($\mu_{it} = \mathbb{E}[\mu_i]$)	$\overline{Y}$	0.77	0.77
(3)	Output in full model	Y	0.76	0.75
(1)-(3)	Total output loss due to nominal rigidity	$(Y^* - Y)/Y^*$	2.6%	9.6%
(1)-(2)	Fraction due to level of markups (percent)	$(Y^* - \overline{Y})/(Y^* - Y)$	49.8%	77.3%
(2)-(3)	Fraction due to dispersion in markups (percent)	$(\overline{Y} - Y)/(Y^* - Y)$	51.2%	22.7%

Table 5: Market structure and output losses due to nominal rigidity

Notes: (i) Calibration of both models is as in Table 1, MC_I and Duo_I. Recall that these calibrations are such that $\mathbb{E}[\mu_{it}]$ is the same in both models, hence $\overline{Y} = 1/\mathbb{E}[\mu_{it}]$ is the same in both models. (ii) When the markups of all firms are equal $\mu_t = \mu_{it}$, so under $P_tY_t = M_t$, then $Y_t = M_t/P_t = 1/\mu_t$. This is used to simply compute output under the counterfactuals in rows (1) and (2). Row (3) takes average output from simulations of the model with aggregate shocks.

the salient features of good-level data: large, frequent adjustment. Under flexible prices $\mu_{it} = \mu^*$, which depends only on η and θ, and output is $Y^* = 1/\mu^*$. Let $\overline{Y}$ be output if all markups μ_{it} are equal to the average markup $\mathbb{E}[\mu_{it}]$. Finally, let Y be the actual measure of output in the economy. The gap between $Y > \overline{Y}$ reflects markup dispersion, and the gap between $\overline{Y} > Y^*$ reflects the distortion in the level of markups due to sticky prices.

Table 5 shows that output losses due to nominal rigidity are 9.6 percent in the duopoly model but only 2.6 percent under monopolistic competition. Given that markup dispersion is almost identical in both models (Figure 8), the output loss between lines 2 and 3 due to markup dispersion are close, and small. The large difference is due to the 77.3 percent of the output loss due to the level of the markup. Sticky prices interact with market power to exacerbate the market power distortion and lower output. Note, however, that if firms could perfectly coordinate, markups would depend on θ with $\mu^*_\theta = \theta/(\theta - 1) = 3$, and output would be $Y_\theta = 1/\mu_\theta = 0.33$, which is significantly lower. Menu costs are small, idiosyncratic shocks are big and η is large, which all constrain the ability of firms to leverage pricing frictions to maintain higher prices.[56]

7.2 From the firm's perspective the optimal degree of frictions is positive

Figure 9 quantifies a related result: in a duopoly, firms value pricing frictions. The value of the firm is hump-shaped in the size of the friction. On the one hand, greater frictions accommodate higher markups which increase firm value. On the other hand, greater frictions reduce price flexibility, reducing firm value. The resulting non-monotonic relationship is clear in both the menu cost

[56] As an analogy, consider two firms writing their prices on billboards. Firms would tear down their billboards and erect new ones if (i) having a slightly lower price than its competitor delivers a firm a lot of the market (high η), (ii) the cost of tearing down a billboard is very low (low $\overline{\xi}$), (iii) firms idiosyncratic costs change by a lot (high σ).

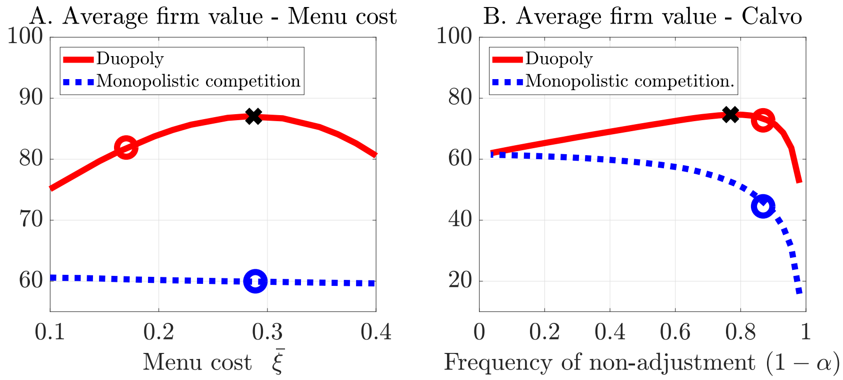

Figure 9: Comparative statics: Markups and firm value

Notes: (i) Figures plot comparative statics of the average firm value—in real terms—given by Bellman equation (8), with respect to changes in the size of nominal rigidity in the menu cost model (panel A) and Calvo model (panel B). (ii) The models are calibrated according to Table 1, Duo_I and MC_{III}; they therefore have the same markup and same real average value under no pricing frictions (see notes for Table 1), the circle mark gives the calibrated value, (iii) The cross mark gives the size of the friction that maximizes firm value in the duopoly model. (iv) Note that the scale of the y-axis differs. This is because the menu cost and Calvo models are not comparable in terms of firm value *in the presence of pricing frictions*. For a given frequency of price change, firm value is larger in the menu cost model due to the ability to time price changes.

and Calvo models. While monopolistically competitive firms always prefer smaller frictions and more adjustment, for duopolists, their value is maximized under $\bar{\xi}^* > 0$. At $\bar{\xi}^*_{Duo_I} = 0.29$, prices change 25 percent less frequently than in the baseline, but the real value of the firm is 9 percent larger. However, in the Calvo model, where complementarities are weaker, smaller frictions are preferred.

Four further observations may be made. First, the model rationalizes why firms appear to engage in investments that increase the cost of price changes.[57] Second, a high inflation policy that forces more frequent adjustment may have first-order output effects.[58] Third, markup estimates from models of static oligopoly are systematically biased downward. For any unbiased estimates of preference parameters a static model predicts μ^*_{Duo} which is less than $\mathbb{E}[\mu_{it}]$. Finally, these results flip the standard intuition for the macroeconomic implications of adjustment frictions . The standard intuition holds in the monopolistically competitive model: firms and households both dislike frictions. In an oligopoly, frictions may be redistributive, causing profits to increase and real wages to fall.

[57] For example, firms print brochures with prices fixed for some period of time.

[58] This is certainly true in the limit. High trend inflation would cause firms to reset their prices every period, yielding the frictionless Nash equilibrium. This would eliminate the first-order welfare losses of nominal rigidity but also eliminate any stimulative role for monetary policy, presenting a trade-off for policy.

8 Conclusion

This paper establishes that the competitive structure of markets can be quantitatively important for the transmission of macroeconomic shocks. In particular, in a menu cost model of firm-level price setting—which aggregates to a monetary business cycle model—a monopolistically competitive market structure and a duopoly market structure generate different levels of monetary non-neutrality. Even when calibrated to match the same salient features of price flexibility in the data, the duopoly model generates larger output responses. Following a monetary expansion, the incentive for low-priced firms to respond to the shock increases less sharply as a lower sectoral price reduces the incentive to adjust. Idiosyncratic shocks—which create within-sector markup dispersion—and state-dependent frictions—which make repricing predictable—are shown to be key for this mechanism.

The duopoly model does not exclude other mechanisms that have been found to be successful in generating monetary non-neutrality in a menu cost model while also being consistent with the microdata. Trying to understand how combining these may generate empirically plausible monetary business cycles is a practical topic for future research.

More broadly, this paper contributes a framework that expands the set of quantitative heterogeneous agent general equilibrium macroeconomic models that may be used to interpret microdata. A pervasive feature of microdata on firm assets, employment, sales, bank-deposits and so on, are fat-tailed size distributions, even within narrow industries or geographies. A pervasive feature of heterogeneous agent macroeconomic models used to rationalize data on inaction and slow adjustment are frictions. Having quantitative models that might accommodate strategic interaction between large agents that face frictions is therefore important for future research.

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APPENDIX - NOT FOR PUBLICATION

This Appendix is organized as follows. Section A provides additional tables and figures referenced in the main text. I also derive properties of the firm's frictionless best response function and profit functions under general complementarity in pricing and for CES preferences, and study a monetary shock in this environment. Section B describes the IRI data and their treatment in the paper. Section C describes the computational methods used to solve the model in Section 2. Section D discusses model assumptions. An Online Appendix (at the end of this document) proves the results for a static game with menu costs and exogenously specified initial markups.

A Additional tables and figures

Table A1: A guide to monetary non-neutrality in a selection of existing studies

Paper	Model	Ref.	Peak IRF $\varepsilon_{Y,M} = \frac{\Delta \log \widehat{Y}_t}{\Delta \log \widehat{M}_t}$	P.C. slope $\lambda = \frac{\partial \pi_t}{\partial \widehat{mc}_t}$	Freq. α	Dur. $1/\alpha$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A. - Menu cost models						
Golosov-Lucas	Menu cost	Fig. 4a	**0.42**	1.36	0.67	1.49
Nakamura-Steinsson	14-sector	Fig. VIII	**0.50**	1.00	0.62	1.62
	+ Round-a-bout*	Fig. IX	**0.80**	0.25	0.39	2.56
Gertler-Leahy	Baseline	Fig. 2	**0.45**	1.22	0.65	1.53
	+ sectoral labor**	Fig. 3	**0.75**	0.33	0.43	2.30
Burstein-Hellwig	Baseline	Fig. 5	**0.34**	1.94	0.73	1.37
	+ DRS**	Fig. 5	**0.56**	0.79	0.58	1.73
	+ Wage rigidity*	Fig. 5	**0.70**	0.43	0.47	2.11
Klenow-Willis	Baseline	Fig. 4	-	**0.51**	0.50	1.99
	+ Kimball**	Fig. 4	-	**0.40**	0.46	2.16
This paper	Monopolistic comp.	Fig. 5	**0.42**	1.38	0.67	1.49
	Duopoly**	Fig. 5	**0.74**	0.36	0.45	2.25
*Forms of pricing complementarity: * = 'macro'-complementarity, ** = 'micro'-complementarity*						
Panel B. - Data and New-Keynesian models						
Calvo model	Bils-Klenow	Tab. 1	0.967	0.167	0.22	**4.55**
	6 month price duration		0.967	0.033	0.17	**6**
	9 month price duration		0.986	0.014	0.11	**9**
	12 month price duration		0.992	0.008	0.08	**12**

Notes: **(A)** Column (4) provides my calculation of the ratio of (i) the peak deviation of output from steady-state $\Delta \log \widehat{Y}_t$ to (ii) the size of the monetary shock: $\Delta \log \widehat{M}_t$. Column (1) is the relevant paper, column (2) the model, and column (3) the figure. E.g. Fig. VIII of Nakamura and Steinsson (2010) shows a peak response of $\Delta \log Y_t = 0.005$, in response to $\Delta \log M_t = 0.010$. **(B)** The remaining columns are constructed as follows for Panel A. Now consider a New Keynesian model with constant returns to scale in production, no idiosyncratic shocks, and Calvo price adjustment as in the benchmark model of Gali (2008). Output is proportional to real marginal cost, so the response of inflation to changes in real marginal cost is given by $\lambda = (1 - \varepsilon_{Y,M})/\varepsilon_{Y,M}$ in column (4). In this model,

$\lambda = (1-\beta\theta)(1-\theta)/\theta$ is the slope of the Phillips curve where $\beta \approx 1$ and $\theta = 1-\alpha$ is the frequency of non-adjustment. Column (6) provides the required α to generate λ by this relationship. Column (7) provides the average duration of prices in months implied by α. This is *"the average duration of prices in a baseline New-Keynesian model that would deliver the same relationship between real marginal cost and inflation following monetary shocks, as that implied by the model in column (2)."* **(C)** For Panel B I proceed backward, starting with an average duration of price change in column (7). Bils and Klenow (2004) compute a median duration of 4.55 months from CPI data. Smets and Wouters (2007) estimate 6 months in a model with significant complementarity and sticky wages. Christiano, Eichenbaum, and Trabandt (2015) estimates imply 12 months. **(D)** The models referred to are as follows. Nakamura and Steinsson (2010): roundabout production technology with an intermediate share of 0.70. Gertler and Leahy (2008): sectoral labor supply with a unit Frisch elasticity of labor supply. Burstein and Hellwig (2007): "+DRS" a decreasing returns production technology (labor share equal to 0.55), "+ Wage rigidity" aggregate nominal wage rigidity: $W_t = Y_t^{-0.8} M_t$. Klenow and Willis (2016): Kimball (1995) preferences and super-elasticity of demand $\chi = 10$.

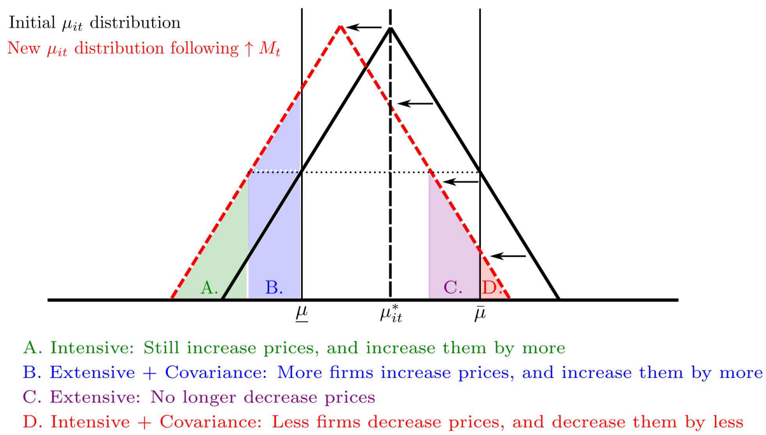

Figure A1: Decomposing markup adjustment in a monopolistically competitive model

Notes: Vertical solid lines give the thresholds for adjustment $\underline{\mu} < \overline{\mu}$. Following an increase in the money supply, all markups decrease by the same amount, as given by the leftward shift in the distribution. For a permanent one-time increase in the money supply, the optimal markup μ_{it} and thresholds for adjustment are not affected by the shock.

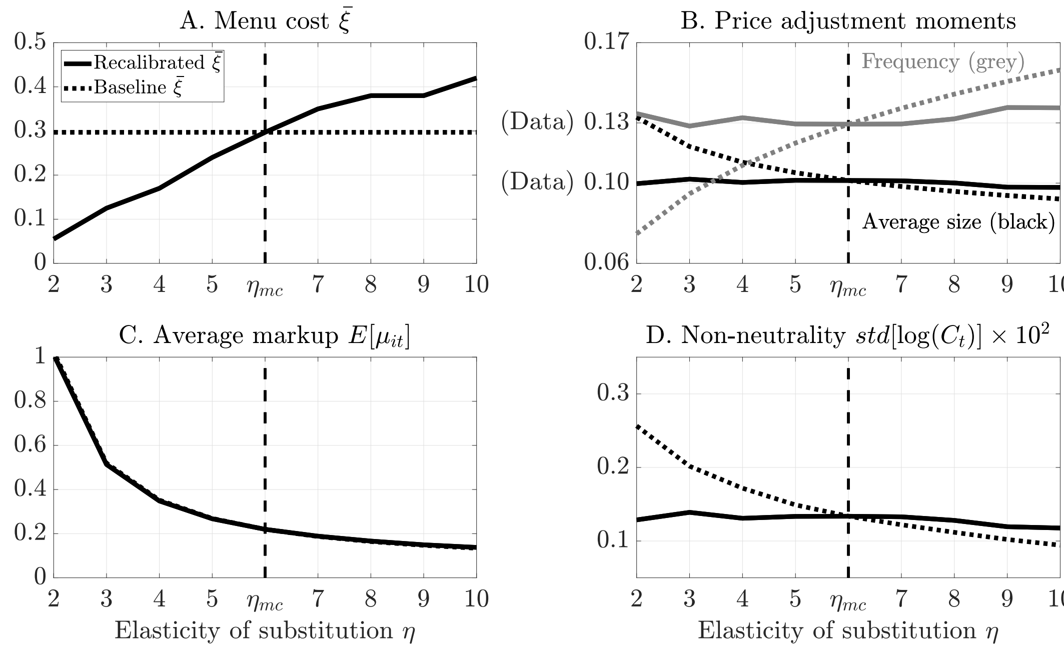

Figure A2: Elasticity of substitution comparative statics and monopolistic competition

Notes: Solid lines denote values for the monopolistically competitive model under $\sigma_z = 0.041$ and the recalibrated values of $\bar{\xi}$ given by the solid line in panel A. These values of $\bar{\xi}$ are chosen to best match data on both the frequency and size of price change (panel B). Dashed lines denote values for the monopolistically competitive model under $\sigma_z = 0.041$, with $\bar{\xi}$ fixed at its value from calibration MC_{III} of Table 1. The vertical black lines mark the value of $\eta_{mc} = 6$ under this calibration.

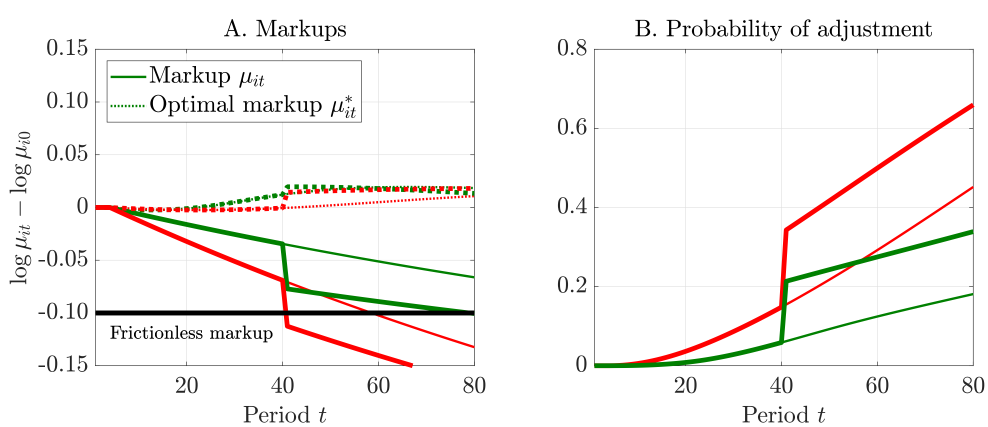

Figure A3: Positive monetary shock with oligopoly: Low-markup firms

Notes: Thin solid lines give exogenous evolution of markups for two firms *within the same sector* absent a monetary shock. Thin dashed lines give corresponding optimal markups conditional on adjustment $\mu_1'(\mu_1, \mu_2)$ and $\mu_2'(\mu_1, \mu_2)$. Thick solid lines include a monetary shock in period 40, which decreases both firms' markups. Thick dashed lines (which lie on top of the thin dashed lines) give the corresponding optimal markups. The model is solved in the absence of *aggregate* shocks only and the monetary shock is a one-time unforeseen level increase in money. The y-axis in panel A describes the log deviation of markups from the value chosen when shocks and menu costs are zero, $\overline{\mu} = 1.30$, which equals the average markup.

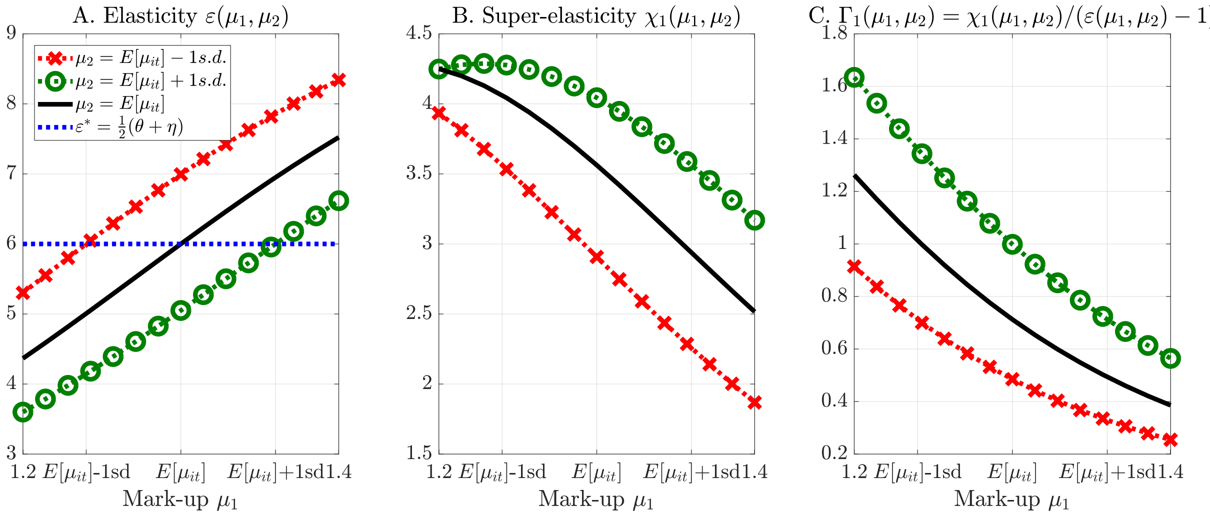

Figure A4: Demand elasticities, the super elasticity of demand and responsiveness

Notes: Panel A plots firm one's elasticity of demand as a function of its markup, under alternative values for μ_2. The values are given by $\mu_2 = \mathbb{E}[\mu_{it}] = 1.30$ and when the markup of firm two is one standard deviation above (circles) and below (crosses) the average markup. (Note that the elasticity of demand is independent of the aggregate markup). The dashed line plots the elasticity faced by firms when $\mu_1 = \mu_2$, in which case revenue shares are both 0.50, and $\varepsilon = (1/2)(\theta + \eta)$. Panel B plots the super-elasticity of demand $\chi_1(\mu_1, \mu_2) := d \log \varepsilon_1(\mu_1, \mu_2) / d \log \mu_1$. Panel C plots the measure of *responsiveness* used in Gopinath and Itskhoki (2011) and Berger and Vavra (2019).

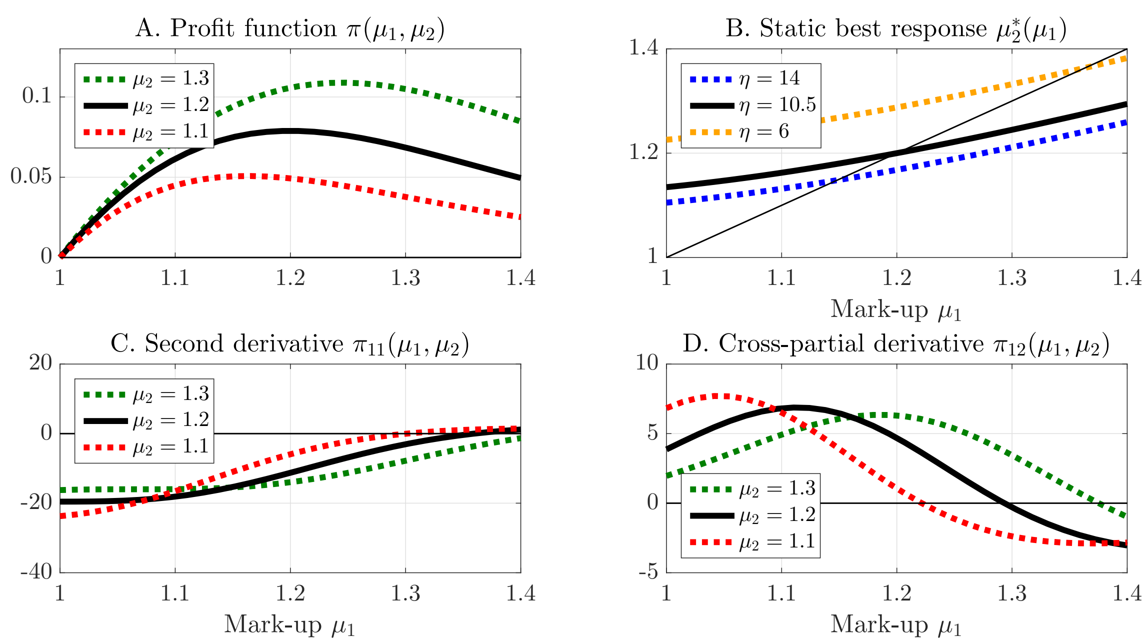

Figure A5: Properties of firm profit functions

Notes: Panels A, C, and D display features of the duopoly profit functions under $\theta = 1.5$, $\eta = 10.5$ as in Table 1 (Duo_I). Given these parameters, the frictionless Nash-Bertrand markup is 1.20 due to an effective elasticity of demand of $\varepsilon = (1/2)(\theta + \eta)$ and a symmetric equilibrium. Panel B plots the static best response function $\mu_i^*(\mu_j)$ under $\theta = 1.5$ and different values of η. Higher values of η reduce the Nash equilibrium markup—given by the intersection of the best response with the 45-degree line—and increase the slope of the best-response function.

B Data

The data used throughout this paper come from the IRI Symphony data. Details can be found in the summary paper by Bronnenberg, Kruger, and Mela (2008).[59] The data are at a weekly frequency from 2001 to 2011 and contain revenue and quantity data at the good level, where a good is defined by a unique overline code number (Universal Product Code—UPC). Data are collected in over 5,000 stores covering 50 metropolitan areas.[60] For each store, data are recorded for all UPCs within each of 31 different product categories. Product categories—for example, toothpaste—are determined by IRI and were designed such that the vendor could sell data, by product category, to interested firms.[61] This provides an economically meaningful way to separate goods categories, since firms presumedly would be interested in purchasing data relevant to their product market. The measures that I construct from these data and use in the paper relate to (i) market concentration, and (ii) price changes. In both cases I define a market by product category p, state s and month t.

Constructing measures of market concentration requires market-level sales for each firm. To identify a firm, I use the first six digits of a good's UPC. This uniquely identifies a company. For example, the five digits 00012 in the overline code 00012100064595 identify Kraft within a market for mayonnaise; 48001 would identify Hellman's. As my measures are constructed within a market pst, I consider Kraft within the mayonnaise market in Ohio as a different firm from Kraft within the margarine market in Ohio. Revenue r_{fpst} for each firm f in market pst is the sum of weekly revenue from all UPCs at all stores within pst. The preferred concentration measure in the paper is the effective number of firms, as measured by the inverse Herfindahl index, which is $h_{pst} = \sum_{f \in pst}(r_{fpst}/r_{pst})^2$.

Computing measures of price changes first requires a measure of price. To obtain weekly prices for each good, I simply divide revenue by quantity. I compute price change statistics monthly and measure prices in the third week of each month. I focus only on regular price changes and deem a price to have been changed between month $t-1$ and t if it (i) changes by more than 0.1 percent, considering price changes smaller than this to be due to rounding error from the construction of the price, and (ii) was on promotion neither in month $t-1$, nor in month t. The IRI data include indicators for whether a good is on promotion, and so I use this information directly rather than using a sales filter. This second requirement means that I exclude both goods that go on promotion and come off promotion. The frequency of price change in market pst is the fraction of goods that change price in market pst between $t-1$ and t. The size of price change in market

[59] Other recent papers to use these data include Stroebel and Vavra (2019) and Coibion, Gorodnichenko, and Hong (2015). See http://www.iriworldwide.com/en-US/solutions/Academic-Data-Set.

[60] Details on the identification of stores are removed from the data and replaced with a unique identifying number. Walmart is not included in the data.

[61] For completeness, the categories are: beer, razor blades, carbonated beverages, cigarettes, coffee, cold foods, deodorant, diapers, facial tissues, frozen dinner entrees, frozen pizza, household cleaning goods, hot dogs, laundry detergent, margarine and butter, mayonnaise, milk, mustard and ketchup, paper towels, peanut butter, photo products, razors, salted snacks, shampoo, soup, pasta sauces, sugar and substitutes, toilet tissue, toothbrushes, toothpaste, and yogurt.

pst is the average absolute log change in prices for all price changes in market pst between $t-1$ and t.

When computing moments for use in the calibration of the model, I first take a simple average over s and t for each product p. I then take a revenue-weighted average across products, where revenue weights are computed using average national revenue for product p: $r_p = T^{-1}\sum_{t=1}^{T}\left(\sum_{s=1}^{S} r_{pst}\right)$.

C Computation

First I show that Bellman equation (5) in prices corresponds to the Bellman equation in markups under the equilibrium conditions of the model (8), as the latter is used in computation. Second, I describe the numerical methods used in computing the equilibrium of the model.

Price indices. Denote the first firm's markup $\mu_{ij} = p_{ij}/z_{ij}W$. Using this, the sectoral price index $\mathbf{p}_j$ can be written as

$$\mathbf{p}_j \quad = \quad \left[\left(\frac{p_{1j}}{z_{1j}}\right)^{1-\eta} + \left(\frac{p_{2j}}{z_{2j}}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} = W\left[{\mu_{1j}}^{1-\eta} + {\mu_{2j}}^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

Define the sectoral markup $\mu_j = \mathbf{p}_j/W$, which implies that $\mu_j = \left[\mu_{1j}^{1-\eta} + \mu_{2j}^{1-\eta}\right]^{1/(1-\eta)}$. Using the sectoral markup, the aggregate price index P can be written

$$P = \left[\int_0^1 \mathbf{p}_j^{1-\theta} dj\right]^{\frac{1}{1-\theta}} = \left[\int_0^1 \mu_j^{1-\theta} dj\right]^{\frac{1}{1-\theta}} W.$$

Define the aggregate markup $\mu = P/W$, which implies that $\mu = \left[\int_0^1 \mu_j^{1-\theta} dj\right]^{1/(1-\theta)}$.

Profits. The expressions for markups can be used to rewrite the firm's profit function. Start with the baseline case

$$\pi_{ij} = z_{ij}^{\eta-1}\left(\frac{p_{ij}}{\mathbf{p}_j}\right)^{-\eta}\left(\frac{\mathbf{p}_j}{P}\right)^{-\theta}\left(p_{ij} - z_{ij}W\right)C.$$

The equilibrium household labor supply condition requires $PC = W$. The definition of the aggregate markup therefore implies that $C = 1/\mu$. This, along with $p_{ij} = \mu_{ij} z_{ij} W$, $p_j = \mu_j W$, and $P = \mu W$, gives

$$\pi_{ij} \quad = \quad \left(\frac{\mu_{ij}}{\mu_j}\right)^{-\eta}\left(\frac{\mu_j}{\mu}\right)^{-\theta}\left(\mu_{ij} - 1\right)\frac{W}{\mu} = \widetilde{\pi}\left(\mu_{ij}, \mu_{-ij}\right)\mu^{\theta-1} W.$$

The function $\widetilde{\pi}$ depends on the aggregate state only indirectly through the policies of each firm within the sector. This makes clear the use of the technical assumption that the demand shifter z_{ij} also increases average cost, allowing profits to be expressed only in markups.

Markup dynamics. Suppose that a firm sells at a markup of μ_{ij} this month. The relevant state next month is the markup that it will sell at if it does not change its price $\mu'_{ij} = p_{ij}/z'_{ij}W'$. Replacing p_{ij} with μ_{ij}, we can write μ'_{ij} in terms of this month's markup, the equilibrium growth of the nominal wage, and the growth rate of idiosyncratic demand:

$$\mu'_{ij} = \mu_{ij}\frac{z_{ij}}{z'_{ij}}\frac{W}{W'} = \mu_{ij}\frac{1}{g'e^{\varepsilon'_{ij}}}.$$

The random walk assumption for z_{ij} implies that $z'_{ij}/z_{ij} = \exp(\varepsilon'_{ij})$. The equilibrium condition on nominal expenditure $PC = M$, combined with the equilibrium household labor supply condition $PC = W$, implies that in equilibrium $W = M$. The stochastic process for money growth then implies that $W'/W = g'$.

Bellman equation. Using these results in the firm's Bellman equation reduces the value of adjustment from (5) to the following (here for clarity I assume that the competitor's markup μ_{-i} is fixed):

$$V_i^{adj}\Big(\mu_i, \mu_{-i}, \mathbf{S}\Big) = \max_{\mu_i^*} \widetilde{\pi}\Big(\mu_i^*, \mu_{-i}\Big)\mu(\mathbf{S})^{\theta-1}W(\mathbf{S}) + \mathbb{E}\left[Q(\mathbf{S},\mathbf{S}')V_i\left(\frac{\mu_i^*}{g'e^{\varepsilon'_i}}, \frac{\mu_{-i}}{g'e^{\varepsilon'_{-i}}}, \mathbf{S}'\right)\right].$$

The equilibrium discount factor is $Q(\mathbf{S},\mathbf{S}') = \beta W(\mathbf{S})/W(\mathbf{S}')$. This implies that all values can be normalized by the wage, where $v_i(\mu_i, \mu_{-i}, \mathbf{S}) = V_i(\mu_i, \mu_{-i}, \mathbf{S})/W(\mathbf{S})$:

$$v_i^{adj}\Big(\mu_i, \mu_{-i}, \mathbf{S}\Big) = \max_{\mu_i^*} \widetilde{\pi}\Big(\mu_i^*, \mu_{-i}\Big)\mu(\mathbf{S})^{\theta-1} + \beta\mathbb{E}\left[v_i\left(\frac{\mu_i^*}{g'e^{\varepsilon'_i}}, \frac{\mu_{-i}}{g'e^{\varepsilon'_{-i}}}, \mathbf{S}'\right)\right].$$

Replacing the aggregate state $\mathbf{S} = (g, \lambda)$ with that used in the approximation $\mathbf{S} = (g, \mu_{-1})$, we have the following:

$$v_i^{adj}\Big(\mu_i, \mu_{-i}, g, \mu_{-1}\Big) = \max_{\mu_i^*} \widetilde{\pi}\Big(\mu_i^*, \mu_{-i}\Big)\widehat{\mu}\Big(g, \mu_{-1}\Big)^{\theta-1} + \beta\mathbb{E}\left[v_i\left(\frac{\mu_i^*}{g'e^{\varepsilon'_i}}, \frac{\mu_{-i}}{g'e^{\varepsilon'_{-i}}}, g', \widehat{\mu}\Big(g, \mu_{-1}\Big)\right)\right],$$

where $\widehat{\mu}$ is given by the assumed log-linear function: $\log\widehat{\mu} = \alpha_0 + \alpha_1 g + \alpha_2 \log\mu_{-1}$.

The equilibrium condition requiring that the price index be consistent with firm prices has also been restated in terms of markups, which implies the entire equilibrium is now restated in terms of markups. To simulate changes in prices, it is sufficient to know a path for markups μ_{ijt}, innovations ε_{ijt}, and money growth g_t. To determine quantities, I need to also simulate paths for M_t and z_{ijt}.

C.1 Price changes.

In the notes for Figure 8 I state that *markup gaps* are equal to *price changes* for firms changing their prices. The markup gap is the gap between the firm's markup that would occur should the firm not change its price ($\overline{\mu}_{ijt}$)—which is its state variable and depends on p_{ijt-1}—and its desired markup (μ^*_{ijt}), which depends on

p^*_{ijt}:

$$\overline{\mu}_{ijt} = \frac{p_{ijt-1}}{z_{ijt}W_t}, \quad \mu^*_{ijt} = \frac{p^*_{ijt}}{z_{ijt}W_t}.$$

Therefore the firm's desired price change and its markup gap are equivalent:

$$\log \frac{p^*_{ijt}}{p_{ijt-1}} = \log \frac{\mu^*_{ijt}}{\overline{\mu}_{ijt}}.$$

C.2 Solving the MPE.

First, for simplicity, suppose that $\theta = 1$ such that $\mu(\mathbf{S})$ does not enter the firm's problem, and so no direct function of the aggregate state enters the firm's problem. Suppose also that shocks to the growth rate of money supply are entirely transitory ($\rho_g = 0$, equivalently the money supply M_t follows a random walk). In this case, the state variables of the firm's problem are only μ_i and μ_{-i}. Since the parameters associated with each firm in each sector are symmetric, I only consider solutions in symmetric policies $\mu(\mu_i, \mu_{-i})$ and $\gamma(\mu_i, \mu_{-i})$. Suppose that these functions are known; then solving the firm's problem amounts to solving a simple Bellman equation. Define the firm's expected value function $v_i^e(\mu_i', \mu_j') = \mathbb{E}\left[v_i\left(\frac{\mu_i'}{g'e^{\varepsilon_i'}}, \frac{\mu_{-i}'}{g'e^{\varepsilon_{-i}'}}\right)\right]$. I can approximate v_i^e with a cubic spline and, given a starting guess, use standard collocation tools to solve the firm's Bellman equation. This requires specifying a grid of collocation nodes for μ_i and μ_{-i}, and then solving for splines with as many coefficients as collocation nodes. Given an approximation of v_i^e and the policies of a firm's competitor, the choices of a firm on these nodes can be solved for, and the values on these nodes used to update the approximation using Newton's method (see Miranda and Fackler (2002)). An alternative approach is to iterate on the Bellman equation.

When solving the MPE, the competitor policies are not initially known. In solving the model, I take a number of approaches, each of which yields the same equilibrium policies. In all cases, I approximate the optimal markup and probability of adjustment policies using cubic splines. The first approach is to consider some large T and assume that from this period onward, prices are perfectly flexible such that the unique frictionless Nash equilibrium is obtained. This determines a starting guess for the policies and value function. Random menu costs imply that each stage game has a unique equilibrium for each point in the state space, which implies that this long subgame perfect Nash equilibrium is unique. One can then iterate backward to $t = 0$, or truncate iterations once the policy functions and values of the firm converge. The second approach is to fix a competitor's policies, solve a firm's Bellman equation, use this to compute new policies, and then continue to iterate in this manner until all objects converge. In practice, both approaches were found to lead to the same policy and value functions. The second approach is faster, since collocation methods can be used to quickly solve the Bellman equation, keeping the competitor policies fixed.

Under $\theta > 1$ and persistent shocks to money growth, then the approximate aggregate state (g, μ_{-1}) also enters the firm's state vector. The solution algorithms for the MPE, however, do not change. I approximate the firm's policies using linear splines in each of these additional dimensions. Policy and value functions are approximated using 25 evenly spaced nodes, and the aggregate states are approximated using 7 evenly spaced nodes.[62] Approximating the expected value function implies that expectations are only taken once in each iterative step while solving the value function, rather than on every step of the solver for the optimal μ_i^*. This, along with the use of a continuous approximation to the value function, allows for a high degree of precision in updating the expected value function. Given an expected value function, an optimal policy can be computed, delivering a new value function, which is then integrated over 100 points in both ε_i' and ε_{-i}' in order to compute a new expected value function.[63]

Issues for high and low menu costs. For a fixed set of collocation nodes, issues arise when trying to solve the model for very low or very high menu costs. For very low menu costs, the adjustment probabilities of the firm take on a steep V-shape, and small deviations in markups lead to a sharp increase in the probability of adjustment. Approximating such functions is difficult with a conservative number of nodes for the approximant of $\gamma(\mu_i, \mu_{-i})$. When menu costs are very large, the adjustment probabilities take on a very shallow U-shape, and markups deviate more widely. This also is hard to approximate with a conservative number of nodes for the approximants.

Figure 9 is symptomatic of this issue. Note that in the Calvo model of adjustment these issues do not arise, since I no longer have to approximate the probability of adjustment function. Therefore the Calvo model can be solved at a very high frequency of adjustment. Figure 9 verifies that as α tends towards one, the value of the firm in the duopoly model smoothly approaches the value of the firm in the monopolistically competitive model, since both models are calibrated to the same frictionless markup.

Krusell-Smith algorithm. I first solve the economy under $\mu_t = \mu^*$, where μ^* is the frictionless Nash equilibrium markup. I then proceed with the Krusell-Smith algorithm, refining the firm's forecast. Solving the model under the initial forecasting rule, I can then simulate the economy. Since firm-level shocks are large, then even for large numbers of simulated sectors, there will be small fluctuations in aggregates. In implementing the Krusell-Smith algorithm I therefore proceed as follows. Let $\{E_t\}_{t=0}^T$ be a sequence of matrices of idiosyncratic shocks—to both productivity and menu costs—to all firms in all sectors, and consider some simulated path of money growth $\{\varepsilon_t^g\}_{t=0}^T$. I simulate two economies, both under $\{E_t\}_{t=0}^T$ and with the same initial distribution of markups, but one under $\{\varepsilon_t^g\}_{t=0}^T$ and the other under $g_t = \overline{g}$ for all t. From the second simulation, I then compute the sequence of aggregate markups and call this $\overline{\mu}_t$, with corresponding μ_t from the first simulation. I then run the following regression on simulated data from $\underline{T}$ to

[62] Note that when solving the problem for a firm, the problem is only solved *on* the collocation nodes, which means that a competitor's policy is never evaluated *off* the collocation nodes. The only computations that involve the splines are evaluating the expected value function for proposed μ_i^* values in the maximization step, and the simulation of sectors.

[63] "Quadrature" methods, by contrast, use only a small handful of points in the approximation of the integral. Working with continuous splines and iterating on the expected value function allow a much more precise computation of the integral.

T:

$$(\log \mu_t - \log \overline{\mu}_t) = \alpha_1 (\log g_t - \log \overline{g}) + \alpha_2 \left(\log \mu_{t-1} - \log \overline{\mu}_{t-1}\right) + \eta_t.$$

I also compute the average aggregate markup $\overline{\mu} = 1/(T - \underline{T}) \sum_{t=\underline{T}}^{T} \mu_t$. When solving the model on the next iteration, I renormalize the aggregate state space to $S = (\log g - \log \overline{g}, \log \mu_{-1} - \log \overline{\mu})$ and provide firms with the forecasting rule

$$\log \mu(S) = \log \overline{\mu} + \widehat{\alpha}_1 S_1 + \widehat{\alpha}_2 S_2.$$

In practice, I simulate 10,000 sectors, set $T = 2,000$, and $\underline{T} = 500$, and iterate to convergence on $\{\overline{\mu}, \alpha_1, \alpha_2\}$. In the monopolistically competitive model, I simulate a single sector with 20,000 firms (recall that all monopolistically competitive sectors are the same, so simulating one sector is sufficient). This approach controls for simulation error, and allows me to keep the nodes of the state space for S_2 the same across solutions of the model, while incorporating changes in the forecast of the average markup.

The algorithm converges quickly and the rule provides a high R^2 in simulation. This works especially well in the context of this model for a number of reasons, which all relate to the role of μ_t in the firm's problem. First, μ_t simply shifts the level of the firm's profit function, which implies that in a static model, it only affects the value of a price change, not the firm's optimal markup. Second, if θ is close to one, then this movement in the profit function is small for any given fluctuations in μ_t. Third, these fluctuations in μ_t are in fact small, given the empirical magnitude of money growth shocks. From a robustness perspective, this is reassuring: if the rule used by firms was incorrect, then this misspecification would have little impact on the policies of the firm. In practice, this means that the coefficients for $\{\overline{\mu}, \alpha_1, \alpha_2\}$ from the first solution of the model under the rule $\mu_t = \mu^*$, are very close to the final coefficients.

Computing aggregate fluctuations. I carefully correct the computation of other moments for simulation error, which might otherwise bias one toward finding larger time-series fluctuations. For example, the key statistic of $\sigma(\log C_t)$ is computed using $std\left[\log C_t - \log \overline{C}_t\right]$, where $\overline{C}_t$ is aggregate consumption computed under the simulation with aggregate money growth equal to $\overline{g}$ in all periods. In this "steady-state" economy, there are still fluctuations in aggregate consumption, but these are due only to large shocks to firms not washing out in a simulation of finitely many firms. The same approach is taken when computing impulse response functions for moments such as the frequency of price adjustment of low-markup firms in Figure 6.

D Discussion of model assumptions

1. CES demand structure An alternative formulation of the demand system could have been chosen. A pertinent example is a nested logit system commonly used in structural estimation of demand systems. However, as shown by Anderson, De Palma, and Thisse (1992), the representative agent nested CES structure delivers a demand system that is isomorphic to that which stems from a population of consumers $k \in [0,1]$ with nested logit preferences, income $y^k \sim F(y^k)$, that purchase $c_{ij}^k = y^k / p_{ij}$ units of their chosen good-ij. That is, consumers may have identical preferences for Kraft and Hellmann's mayonnaise, up to an *iid* taste shock that shifts each consumer's tastes toward one or the other each period.[64]

2. Random menu costs Random menu costs serve two purposes in the model. First, they generate some small price changes. Some firms, having recently changed their price and accumulating little change in sectoral productivity, draw a small menu cost and again adjust their price. Figure 8 shows that a monopolistically competitive model with random menu costs gives a distribution of price changes that appear as smoothed versions of the bimodal spikes of GL. Midrigan (2011b) explicitly models multiproduct firms and shows that the implications for aggregate price and quantity dynamics are—when calibrated to the same price-change data—the same as in a model with random menu costs. What is important for these dynamics is that the model generates small price changes—which dampen the extensive margin effect—leading to the statement that the conclusions drawn are not sensitive to the exact mechanism used to generate small price changes. In this sense, one can think of the random menu costs in my model as standing in for an unmodeled multiproduct pricing problem.

Second, and most important, random menu costs that are private information allow me to avoid solving for mixed-strategy equilibria. This technique I borrow from Doraszelski and Satterthwaite (2010), who deploy it to address the computational infeasibility of solving the model of Ericson and Pakes (1995), which has potential equilibria in mixed strategies as well as issues with existence of equilibrium.[65] To see how these arise, consider solving the model under mixed strategies with fixed menu costs. Given the values of adjustment and non-adjustment and a fixed menu cost ξ, the firm may choose its probability of adjustment

$$\gamma_i(s, \mathbf{S}) = \arg \max_{\gamma_i \in [0,1]} \gamma_i \left[v_i^{adj}(s, \mathbf{S}) - \xi \right] + (1 - \gamma_i)\, v_i^{stay}(s, \mathbf{S}).$$

If firm $-i$ follows a mixed strategy such that $v_i^{adj}(s, \mathbf{S}) - \xi = v_i^{stay}(s, \mathbf{S})$, then a mixed strategy $\gamma_i \in (0,1)$ is a best response of firm i. If one believes that menu costs are fixed, then this provides an alternative rationale for small price changes. Some firms may not wish to adjust prices this period, yet their mixed strategy over adjustment leads them to change prices nonetheless. However, the solution of this model would be vastly more complicated and at this stage infeasible. The Online Appendix proves that even in a simple static game of price adjustment with menu costs, such multiple equilibria may arise.

3. Information I assume that the evolution of product demand within the sector (z_{1j}, z_{2j}) is known by both firms at the beginning of the period and only menu costs are private information. An alternative case

[64] For estimation of alternative static demand systems using scanner data similar to that used in this paper, see Beck and Lein (2020) (nested logit), Dossche, Heylen, and den Poel (2010) (AIDS), and Hottman, Redding, and Weinstein (2014) (nested CES). Only the latter studies an equilibrium, imperfectly competitive model.

[65] This technique is also used by Nakamura and Zerom (2010) and Neiman (2011) in menu cost models.

is that menu costs are fixed, but firms know only their own productivity and the past prices of both firms. This would add significant complexity to the problem. First, if productivity is persistent, then firms would face a filtering problem and a state vector that includes a prior over their competitor's productivity. Second, computation is still complicated even if productivity is *iid*. From firm 1's perspective, z_{2j} would be given by a known distribution, which firm 1 must integrate over when computing expected payoffs. Integrating over firm 2's policy functions—which depend on z_{2j}—would be computationally costly. Since the menu cost is sunk, such issues are avoided.

4. Idiosyncratic shocks Three key assumptions are made regarding idiosyncratic shocks: they (i) follow a random walk, (ii) move both marginal revenue and marginal productivity schedules of the firm, and (iii) are idiosyncratic rather than sectoral. These are made for tractability but are not unrealistic.

The first is plausible given that the model is solved monthly. It achieves tractability in that future states depend on growth rates of z_{ij}, which are *iid*. An alternative assumption deployed in similar studies is a random walk in money growth and AR(1) in firm-level shocks, which reduces the *aggregate* state variables of a monopolistically competitive model in the same way, removing g_t from the state.[66] In the duopoly model, this would leave the overall state vector with five elements

$$(s_{ijt}, S_t) = (p_{ijt-1}, p_{-ijt-1}, z_{ijt}, z_{-ijt}, \mu_t) .$$

This leaves four state variables in the sectoral problem which is infeasible. The random walk assumptions on z_{ijt} and AR(1) in money growth implies $(s_{ijt}, S_t) = (\mu_{ijt}, \mu_{-ijt}, g_t, \mu_t)$, with two state variables in the sectoral problem which is feasible. Additionally, since at a monthly frequency the estimated persistence of money growth is significantly less than one ($\rho_g = 0.61$, see Section 4), this is preferred.

The second seems acceptable if one does not hold a strong view on whether demand or productivity shocks drive firm price changes, a reasonable stance given that only revenue productivity is observed in the data for all but a small number of sectors. Midrigan (2011b) interprets ε_{ij}'s as shocks to "quality": the good has higher demand but is more costly to produce. This assumption is necessary—along with random walk shocks—to express the sectoral state vector in two rather than four states.

The third assumption is not for tractability of the duopoly model but the monopolistically competitive model. The latter with sectoral shocks would introduce two additional state variables to the firm's problem: the sectoral markup and sectoral shock. Firms would require forecasting rules for *both* of these on top of forecasting rules for the aggregate markup. This would render the problem infeasible. In addition, the existing literature does not take this approach, and assumes symmetric sectors.

[66] Specifically, such an assumption would allow the aggregate state—following the Krusell-Smith approximation—to be captured by only the aggregate markup.

Appendix II - For Online Publication

Market Structure and Monetary Non-neutrality

Simon Mongey

OA.1 Introduction

In this appendix, I study a two-player price-setting game in which the profit function of the firm features complementarities in prices, firms face a fixed cost of changing prices, and initial prices are above the frictionless Nash equilibrium price. I establish that (i) the frictionless best response function of the firm has a positive gradient bounded between zero and one, (ii) menu costs can sustain higher prices than obtain in a frictionless setting, (iii) the only pure strategy equilibria that exist are ones in which both firms change their price or both keep them fixed, and (iv) for any given menu cost, there is always a range of initial prices for which both equilibria exist, (v) show how a monetary shock can reduce low priced firms incentive to increase prices when initial prices are dispersed. (vi) the profit functions—derived from nested CES preferences—in the body of the paper satisfy the sufficient assumptions for these results.

OA.2 Static model

Environment. Consider two firms with symmetric profit functions $\pi^1(p_1, p_2) = \pi^2(p_2, p_1)$. In what follows, I drop the superscripts on the profit function and prices, with the second argument always referring to the competitor's price. Assume that π is twice continuously differentiable and that the derivatives of π have the following properties for all positive prices: $\pi_{11} < 0$, $\pi_{12} > 0$, and $|\pi_{11}| > |\pi_{12}|$. The second assumption is the definition of complementarity in prices. Below I comment on the third assumption: that own price effects dominate.

There is one period. I start by assuming both firms begin the period with initial price $\overline{p}$, which is greater than the frictionless Nash equilibrium price p^* that solves $\pi_1(p^*, p^*) = 0$. To deviate from this price, a firm must pay a cost ξ. The objective function of firm i is therefore $v(p_i, p_j) = \pi(p_i, p_j) - \mathbf{1}[p_i \neq \overline{p}]\,\xi$.

Static best-response. The *frictionless best response function* $p^*(p)$ is the best response of a firm to its competitor's price p when $\xi = 0$. A key property discussed in the text is that this function has a positive gradient between zero and one: a firm follows and undercuts its competitor. To prove this, take the firm's first-order condition: $\pi_1(p^*(p), p) = 0$. By the implicit function theorem, the derivative of $p^*(p)$ can be obtained by rearranging the total derivative of the first-order condition:

$$\frac{\partial p^*(p)}{\partial p} = -\frac{\pi_{12}(p^*(p), p)}{\pi_{11}(p^*(p), p)}.$$

The frictionless Nash equilibrium price $p^* = p^*(p^*)$ solves both firms' first-order conditions simultaneously. The second-order conditions must hold at (p^*, p^*), requiring:

$$\pi_{11}(p^*, p^*) < 0, \quad \text{and} \quad \pi_{12}(p^*, p^*)^2 < \pi_{11}(p^*, p^*)^2.$$

The first condition holds by assumption. A sufficient condition for the second order conditions to hold is that $|\pi_{12}| < |\pi_{11}|$. The third assumption is therefore equivalent to assuming that the conditions for a local maximum at (p^*, p^*) also hold globally.

The second order condition jointly with the assumption that $\pi_{11} < 0$ and $\pi_{12} > 0$, gives the result that at any Nash equilibrium

$$\frac{\partial p^*(p)}{\partial p}\Big|_{p=p^*} = -\frac{\pi_{12}(p^*, p^*)}{\pi_{11}(p^*, p^*)} \in (0, 1).$$

Multiple equilibria would therefore require $p^{*\prime}(p^*)$ to have a slope greater than one at some other equilibria, so clearly the equilibrium is also unique. The additional assumption that $|\pi_{12}| < |\pi_{11}|$ *globally*, implies that $p^*(p) \in (p^*, p)$ for $p > p^*$, that is, the frictionless best response function exhibits "undercutting."

Equilibria of the menu cost game. I categorize possible pure strategy equilibria into three types : (I) neither firm changes its price, (II) both firms change their price, (III) one firm changes its price.

A necessary and sufficient condition for a Type-I equilibrium is

$$\pi(\overline{p}, \overline{p}) \geq \max_p \pi(p, \overline{p}) - \xi, \tag{OA.1}$$

or equivalently

$$\xi \geq \Delta_I(\overline{p}) := \pi\left(p^*(\overline{p}), \overline{p}\right) - \pi\left(\overline{p}, \overline{p}\right). \tag{OA.2}$$

This condition for a Type-I equilibrium holds when (i) ξ is very large or (ii) $\overline{p}$ is sufficiently close to p^*. To show that $\Delta_I(\overline{p})$ is increasing in $\overline{p}$, it is useful to represent $\Delta_I(\overline{p})$ as an integral. Some calculus then delivers the following convenient expression for the derivative of the value of the optimal downward deviation:

$$\frac{\partial \Delta_I(\overline{p})}{\partial \overline{p}} = \frac{\partial}{\partial \overline{p}}\left[\int_{\overline{p}}^{p^*(\overline{p})} \pi_1(u, \overline{p})\, du\right] = \int_{p^*(\overline{p})}^{\overline{p}} \underbrace{\pi_{11}(u, \overline{p})}_{(-)} \underbrace{\left[\frac{\partial p^*(p)}{\partial p}\Big|_{p=u} - 1\right]}_{(-)} du > 0 \tag{OA.3}$$

The final expression is a positive integrand integrated over an increasing support since $p^*(\overline{p}) < \overline{p}$, and so positive. The change in value that accompanies the optimal deviation from $p^*(\overline{p})$ increases in $\overline{p}$. Sustaining initial deviations from the frictionless Nash equilibrium requires the initial deviation to be not too large or menu costs to be not too small.

In a Type-II equilibrium, in which both firms change their price, it must be that the prices chosen are

(p^*, p^*). Given that both firms are changing their prices, then the price chosen by each firm must be a best response to its competitor. We then need to check that it is not optimal for a firm to leave its price at $\overline{p}$, which requires

$$\xi \leq \Delta_{II}(\overline{p}) := \pi\left(p^*, p^*\right) - \pi\left(\overline{p}, p^*\right). \tag{OA.4}$$

This condition for a Type-II equilibrium holds when (i) ξ is small or (ii) $\overline{p}$ is large. To see that $\Delta_{II}(\overline{p})$ is increasing in $\overline{p}$, note that $\pi(\overline{p}, p^*)$ is decreasing in $\overline{p}$ for all $\overline{p} > p^*$. The frictionless equilibrium will still obtain when $\overline{p}$ is large relative to the menu cost. The menu cost limits the size of permissible deviations from $\overline{p}$.

Type-III equilibria do not exist. Observe that in a Type-III equilibrium the firm that changes its price chooses $p^*(\overline{p})$. There are therefore two conditions for a Type-III equilibrium. First, firm 2 must find it profitable to change its price given that firm 1's price remains at $\overline{p}$:

$$\pi\left(p^*(\overline{p}), \overline{p}\right) - \xi \geq \pi\left(\overline{p}, \overline{p}\right). \tag{OA.5}$$

This holds when (i) ξ is small or (ii) $\overline{p}$ is large. Second, the frictionless best response of firm 1 to firm 2's price must not be a best response under a positive menu cost. Letting $p^{**}(\overline{p})$ denote the frictionless best response to $p^*(\overline{p})$, we then require

$$\pi\left(p^{**}(\overline{p}), p^*(\overline{p})\right) - \xi \leq \pi\left(\overline{p}, p^*(\overline{p})\right). \tag{OA.6}$$

This holds when (i) ξ is large or (ii) $\overline{p}$ is small. Intuitively, it seems that these conditions should not simultaneously hold. If one firm finds it valuable to undercut its competitor, then its competitor should find it valuable to respond. This can be proven, with the proof found at the end of this appendix.

Multiple equilibria. Having asserted that the only pure strategy equilbria are of Type-I and Type-II, we can also show that for any value of ξ, there exist an interval of $\overline{p}$ for which both Type-I equilibria and Type-II equilibria may exist. First note that $\Delta_I(p^*) = \Delta_{II}(p^*) = 0$. That is, both equilibria trivially exist for zero menu costs at $\overline{p} = p^*$. Both equilibria exist if $\xi \in [\Delta_I(\overline{p}), \Delta_{II}(\overline{p})]$ We can show that for all $\overline{p} > p^*$, $\Delta_{II}(\overline{p}) > \Delta_I(\overline{p})$:

$$\pi(p^*, p^*) - \pi(\overline{p}, p^*) > \pi(p^*(\overline{p}), \overline{p}) - \pi(\overline{p}, \overline{p}). \tag{OA.7}$$

Since p^* is the best response to p^* then $\pi(p^*, p^*) > \pi(p^*(\overline{p}), p^*)$, so showing the following is sufficient:

$$\pi(p^*(\overline{p}), p^*) - \pi(\overline{p}, p^*) > \pi(p^*(\overline{p}), \overline{p}) - \pi(\overline{p}, \overline{p}). \tag{OA.8}$$

If π displays complementarity, then this holds.[67]

[67] To see this, express both sides as integrals:

$$\begin{aligned} \int_{\overline{p}}^{p^*(\overline{p})} \pi_1(u, p^*)du &> \int_{\overline{p}}^{p^*(\overline{p})} \pi_1(u, \overline{p})du \\ \int_{p^*(\overline{p})}^{\overline{p}} \pi_1(u, p^*)du &< \int_{p^*(\overline{p})}^{\overline{p}} \pi_1(u, \overline{p})du \end{aligned}$$

Due to complementarity, $p^* < \overline{p}$ implies $\pi_1(u, p^*) < \pi_1(u, \overline{p})$. Since both integrals are of positive integrands over the same increasing support, then the inequality must always hold.

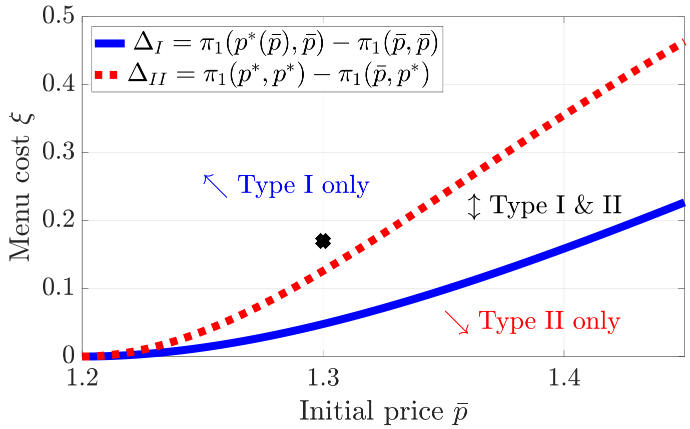

Figure OA.1: Regions of equilibria in a static price-setting game

Notes: Figure summarizes regions of equilibria in the static price setting game with initial prices $p_1 = p_2 = \overline{p}$ and menu cost ξ solved in this appendix. Type I equilibria involve both prices remaining fixed at $\overline{p}$. Type II equilibria involve both firms paying ξ and changing their price to the frictionless Nash equilibrium price $p^* = 1.20$. In the intermediate region either equilibrium may be obtained.

Characterization. These results characterize equilibria in $(\overline{p}, \xi)$-space. Consider fixing $\overline{p}$ and start at a high value of ξ. In this region, only the Type-I equilibrium exists. Menu costs are sufficiently high that the best response of each firm to the initially high price of its competitor is to keep a high price. As ξ decreases, we reach a point at which Type-II equilibria are also feasible. In this region, if firm 2 changes its price, then the best response of firm 1 is to also change its price (Type-II), but if firm 2 leaves its price fixed, then the best response of firm 1 is to also leave its price fixed (Type-I). As ξ decreases further, the Type-I equilibrium can no longer be sustained as the menu cost is insufficient to commit firms not to respond to a price decrease at their competitor. Alternatively, fixing ξ and increasing $\overline{p}$, first only the Type-I equilibrium exists, then both, then as the value of a price decrease becomes large, only the Type-II equilibrium exists. Figure OA.1 plots these regions for a profit function discussed below.

Ranking. In the case of the existence of multiple equilibria, the equilibria are ranked as we would expect: firms prefer the fixed price Type-I equilibrium. This requires that $\pi(\overline{p}, \overline{p}) > \pi(p^*, p^*) - \xi$. Since the Type-I equilibrium exists, then $\xi \geq \pi(p^*(\overline{p}), \overline{p}) - \pi(\overline{p}, \overline{p})$, and therefore this ranking holds if $\pi(p^*(\overline{p}), \overline{p}) > \pi(p^*, p^*)$. Since prices are complements, this is true: the best response to a high price yields a larger profit than the best response to a low price.

From this static game we learn that for a given menu cost ξ, high prices $\overline{p}$ can be sustained so long as they are not too far from the frictionless Nash equilibrium. If the initial price is too high, one firm has a profitable deviation even it pays the menu cost. If the value of one firm's deviation exceeds the menu cost, then the value of an iterative undercutting strategy from its competitor must also exceed the menu cost. Both firms change their prices, and only the frictionless Nash equilibrium price is attainable. If initial prices are not too high, then the menu cost is enough to negate the small value of the optimal frictionless downward deviation in price, making the high-priced strategy credible. We also learn that the equilibrium is not unique for certain combinations of ξ and $\overline{p}$, while these equilibria are clearly Pareto ranked: if firms could coordinate on an equilibrium, they would choose not to change their prices.

Two periods with simultaneous moves from p^*. Consider the game when the firms' prices are initially at p^*. Regardless of the size of ξ, the only equilibrium is (p^*, p^*). One firm increasing its price is not an equilibrium, since p^* is already the best response to p^*. Both firms raising their prices to the same price $\overline{p}$ is not an equilibrium since conditional on changing price the best response is $p^*(\overline{p}) \in (p^*, \overline{p})$.

Two periods with sequential moves from p^*. Now assume instead a Stackleberg game starting from (p^*, p^*). Firm 1 moves first, firm 2 follows and then the simultaneous one period game is played. The following is an equilibrium for an appropriate ξ. The first firm chooses $\overline{p}$ to maximize $\pi(\overline{p}, p^*(\overline{p}))$ and the second firm chooses $p^*(\overline{p})$. If $\overline{p}$ is chosen such that prices $(\overline{p}, p^*(\overline{p})$ yields a Type-I equilibrium and $\pi(\overline{p}, p^*(\overline{p})) - \pi(p^*, p^*) > \xi$, then the equilibrium consists of the first firm paying the menu cost to raise its price, and the second firm paying the menu cost to then undercut the first firm.[68] Firm 2 earns greater profits than firm 1, but profits—including the menu cost—are higher that an (p^*, p^*) for both firms.

The dynamic duopoly model with random menu costs is similar. From initially low prices if firm 2 draws a low menu cost it can "take the high road" by posting a high price today, which is credible under its competitor's policy to increase prices in the future. Once both prices are increased, menu costs wipe out the value of price cuts.

CES demand. In the main text, the profit function of the firm is

$$\begin{aligned} \pi_1(p_1, p_2) &= \left(\frac{p_1}{p(p_1, p_2)}\right)^{-\eta} \left(\frac{p(p_1, p_2)}{P}\right)^{-\theta} (p_1 - 1)C, \\ p(p_1, p_2) &= \left[p_1^{1-\eta} + p_2^{1-\eta}\right]^{1/1-\eta}. \end{aligned}$$

To be consistent with notation in this appendix, I have replaced markups with prices and a unit marginal cost. From this profit function we can solve in closed form for the Nash equilibrium price as follows.

The first-order condition of the firm's problem is

$$\left[p_1^{-\eta} - \eta p_1^{-\eta-1}(p_1 - 1)\right] p^{\eta-\theta} + (\eta - \theta) p_1^{-\eta} p^{\eta-\theta-1}(p_1 - 1)\frac{\partial p}{\partial p_1} = 0,$$

where the term in square brackets gives the first order condition of a monopolistically competitive firm facing elasticity of demand η. The second term gives the marginal profit due to the firm increasing the sectoral price. Since $\eta > \theta$, this second term is positive, implying that the term in brackets is negative, and so the equilibrium price must be larger than the monopolistically competitive price under η.

[68] Since $\pi(\overline{p}, p^*(\overline{p})) - \pi(p^*, p^*) > \xi$, then firm 2's net pay off is also positive since $\pi(p^*(\overline{p}), \overline{p}) > \pi(\overline{p}, p^*(\overline{p}))$. Therefore firm 2's policy is a best response and the game is subgame perfect.

Two additional results for a CES demand system allow us to solve the first-order condition in closed form. First,

$$\frac{\partial p}{\partial p_1} = \left[p_1^{1-\eta} + p_2^{1-\eta}\right]^{\frac{1}{1-\eta}-1} p_1^{-\eta} = \left(\frac{p_1}{p}\right)^{-\eta}.$$

Second, the revenue of the firm is $r_1 = p_1(p_1/p)^{-\eta}(p/P)^{-\theta}C$, which gives the following revenue share:

$$s_1 = \frac{r_1}{r_1 + r_2} = \frac{p_1^{1-\eta}}{p_1^{1-\eta} + p_2^{1-\eta}} = \left(\frac{p_1}{p}\right)^{-\eta}\frac{p_1}{p} = \frac{\partial p}{\partial p_1}\frac{p_1}{p}.$$

Using these results in the first order condition, we obtain

$$p_1 - \eta(p_1 - 1) + (\eta - \theta)(p_1 - 1)s_1 = 0.$$

Since firms are symmetric, the equilibrium will yield equal revenue shares $s_1 = 0.5$, and $p^* = \varepsilon/(\varepsilon - 1)$, where ε is an average of the within- and across-sector demand elasticities $\varepsilon = 0.5 \times (\eta + \theta)$. The form of the solution implies that markups are consistent with those chosen by a monopolistically competitive firm facing an elasticity of demand equal to ε. Note that since P and C are first order terms in the firm's profit function, they do not affect the Nash equilibrium markup: there is *no complementarity between firm and aggregate prices and quantities*.

Numerical example. The calibration of the dynamic duopoly model yielded $\theta = 1.5$ and $\eta = 10.5$ (see Table 1). For these values, $\varepsilon = 6$ and $p^* = 1.2$.[69] I apply these values to the equilibrium profit function from the text (7), in which $P^{\theta-1}$ would multiply the profit function instead of $PC^{-\theta}$. Setting P to the average markup 1.30, Figure OA.1 shows how $(\xi, \overline{p})$-space separates across different equilibria for this profit function. It is entirely consistent with the theoretical results. Recall that the model was calibrated to the average size and frequency of price change, so the menu cost was not chosen with a particular equilibrium in mind. The average markup in the model is $\overline{p} = 1.3$, and the upper bound on the menu cost is $\overline{\xi} = 0.17$ (marked with an x in the figure). Zbaracki, Ritson, Levy, Dutta, and Bergen (2004) find that total price adjustment costs make up 1.2 percent of firm revenue. As a benchmark, $\Delta_{II}(\overline{p})/rev(\overline{p}, \overline{p}) = 0.012$ at $\overline{p} = 1.27$, so a menu cost around empirical estimates as a share of revenue would, in this static game under the calibrated parameters of the model, guarantee a Type-I equilibrium. Figure A5 plots various features of this profit function for firm 1, varying p_2.

Summary. From only this exercise, the following is a heuristic understanding of the dynamic model. Nominal rigidity allows firm markups to fluctuate around an average markup that is larger than the frictionless Nash equilibrium. However, this is constrained by the size of the menu cost, which is pinned down

[69] Recall that the MC_{III} calibration of the monopolistically competitive model set $\eta = 6$ to deliver this as a frictionless markup.

by the average frequency of price change. Given a menu cost ξ, firms choose reset prices around a real price $\overline{p}$ that supports a Type-I equilibrium, but not so high as to risk a Type-II equilibrium. Idiosyncratic shocks force firms' real prices apart, but firms keep on adjusting their prices so as to not let them get too far away from $\overline{p}$. Prices that are too high invite undercutting, and prices that are too low reduce profitability. Menu costs in the range of empirical estimates can sustain markups in the range of empirical estimates. Finally, getting to these high prices requires firms to reduce profit in the short run in order to lay the incentives for their competitor to choose a price that maintains higher profits in the long run.

Calvo model. Finally, consider a Calvo version of the static model, where each firm changes its price with probability α. Let $\widetilde{p}$ be the optimal reset price of the firm. A Nash equilibrium requires that each firm's first order condition be satisfied at $\widetilde{p}$:

$$\alpha\pi_1(\widetilde{p},\widetilde{p}) + (1-\alpha)\pi_1(\widetilde{p},\overline{p}) = 0.$$

It is straightforward to show that $p^* < \widetilde{p} < p^*(\overline{p})$ for $\alpha < 1$. A sufficient condition is that $\pi_1(\widetilde{p},\overline{p}) < 0$, since $\pi_1(p^*(\overline{p}),\overline{p}) = 0$. The first order condition implies that this is true if $\pi_1(\widetilde{p},\overline{p}) > \pi_1(\widetilde{p},\widetilde{p})$, which is true due to complementarity and $\overline{p} > \widetilde{p}$. Note that as $\alpha \to 1$, then $\widetilde{p} \to p^*$.

Proof. For the Type-III equilibrium to exist, conditions ($OA.5$) and ($OA.6$) must hold simultaneously. The following condition is therefore necessary:

$$\pi(p^{**}(\overline{p}), p^*(\overline{p})) - \pi(\overline{p}, p^*(\overline{p})) \leq \pi(p^*(\overline{p}),\overline{p}) - \pi(\overline{p},\overline{p}).$$

I prove that this inequality always holds but with a positive inequality, concluding that Type-III equilibria do not exist.

Note that the expression on the left-hand side can be decomposed as follows:

$$\begin{aligned}\pi(p^{**}(\overline{p}), p^*(\overline{p})) - \pi(\overline{p}, p^*(\overline{p})) \quad = \quad & \left[\pi(p^{**}(\overline{p}), p^*(\overline{p})) - \pi(p^*(\overline{p}), p^*(\overline{p}))\right] \\ & + \left[\pi(p^*(\overline{p}), p^*(\overline{p})) - \pi(\overline{p}, p^*(\overline{p}))\right].\end{aligned}$$

The first term is positive by the definition of $p^{**}(\overline{p})$ being the best response to $p^*(\overline{p})$. The second term is positive since $\pi(p, p^*(\overline{p}))$ is decreasing in p for $p > p^{**}(\overline{p})$ and $p^*\overline{p} < \overline{p}$.

Since both terms are positive, then a sufficient condition for the non-existence of a Type-III equilibrium is

$$\pi(p^*(\overline{p}), p^*(\overline{p})) - \pi(\overline{p}, p^*(\overline{p})) \geq \pi(p^*(\overline{p}),\overline{p}) - \pi(\overline{p},\overline{p}).$$

Expressing both sides as integrals and then multiplying through by minus one:

$$\int_{p^*\overline{p}}^{\overline{p}} \pi_1(u, p^*(\overline{p}))\, du \leq \int_{p^*\overline{p}}^{\overline{p}} \pi_1(u, \overline{p})\, du.$$

Since $p^*\overline{p} < \overline{p}$ and $\pi_{12} > 0$, then the integrand on the left is always less than that on the right. Both integrals are evaluated on the same, increasing, supports, so the inequality always holds.

OA.3 A monetary shock in a static model

One can use the above framework to reason through how the value of price changes at low and high priced firms respond following a monetary shock. Consider real prices of firms. Let $P_1 < P_2$, and suppose that firms' profits depend on nominal prices relative to the nominal money supply M: $p_i = P_i/M$. The frictionless Nash equilibrium moves one for one with M such that $p^* = P^*/M$ is a constant. The initial states are (p_1, p_2) and following the monetary increase are (p'_1, p'_2) with $p'_i < p_i$. For convenience denote $p = p_2$ and express $p_1 < p_2$ as a ratio of p_2: $p_1 = \Delta p$, with $\Delta \in (0, 1)$. A *smaller* Δ is a *wider gap* between prices. With this notation the initial state can be expressed as (p, Δ), and after the shock as (p', Δ), with $p' < p$. A monetary shock reduces both firms' real prices (absent adjustment), but keeps their ratio constant.

Consider the following two functions:

$$\begin{aligned} f(p,\Delta) &= \pi(p^*(\Delta p), \Delta p) - \pi(p, \Delta p) = \int_{p}^{p^*(\Delta p)} \pi_1(u, \Delta p)\, du \\ g(p,\Delta) &= \pi(p^*(p), p) - \pi(\Delta p, p) \quad = \int_{\Delta p}^{p^*(p)} \pi_1(u, p)\, du \end{aligned}$$

The first function gives the value of the optimal, static, downward best response from the high priced firm: $p^*(\Delta p) \in (p^*, \Delta p)$. The optimal best response is to undercut its competitor. The second function gives the value of the optimal, static, best response from the low priced firm: $p^*(p) < p$. Note that depending on Δ, this best response may be a price increase or decrease depending on whether $\Delta p \gtrless p^*(p)$.

Differentiating both expressions with respect to p and Δ we obtain:

$$\begin{aligned} f_p(p,\Delta) &= \int_{p^*(\Delta p)}^{p} \underbrace{\pi_{11}(u,\Delta p)}_{(-)} \left[\underbrace{\Delta}_{(-)} \underbrace{\left\{ -\frac{\pi_{12}(u,\Delta p)}{\pi_{11}(u,\Delta p)} \right\}}_{\in(0,1)} - 1 \right] du \quad , \quad f_\Delta(p,\Delta) = -\int_{p^*(\Delta p)}^{p} p\pi_{12}(u,\Delta p)\, du \\ g_p(p,\Delta) &= \int_{\Delta p}^{p^*(p)} \pi_{11}(u,\Delta p) \left[\Delta - \left\{ -\frac{\pi_{12}(u,\Delta p)}{\pi_{11}(u,\Delta p)} \right\} \right] du \quad , \quad g_\Delta(p,\Delta) = -p\pi_1(\Delta p, p) \end{aligned}$$

The derivatives of f are unambiguously signed: $f_p > 0$ and $f_\Delta < 0$. The value of a price decrease to the high priced firm is increasing in its initial price and increasing in the gap between prices. The derivatives of g depend on the initial gap between prices: Δ. In the relevant case where Δ is small—such that the prices are initially very dispersed—then $g_p > 0$. When Δ is such that $\Delta p < p^*(p)$, then $g_\Delta > 0$ too.

Idiosyncratic shock. An idiosyncratic shock that reduces Δ, widening the gap between prices has two effects on the static best-responses of the low priced firm. A direct effect through $g(p, \Delta)$, by which Δp moves further away from $p^*(p)$ and so the value of a price increase increases. This is similar to a monopolistically competitive firm facing a price lower than their optimal price and receiving a positive shock to their costs ($\overline{\mu}$ falls relative to μ^*). An indirect effect through $f(p, \Delta)$, by which the fall in the low priced firm's price relative to its competitor increases the value of a price cut to their competitor. In equilibrium this will increase the propensity of the low priced firm to increase their price to stave off a price-cut from their competitor.

Aggregate shock. An aggregate shock reduces p, and has two effects that can be similarly discussed. A direct effect through $g(p, \Delta)$ by which even when taking into account the decline in $p^*(p)$, the value of a price increase at the low priced firm falls. An indirect effect through $f(p, \Delta)$: the value of the optimal price cut at their competitor falls, which in equilibrium reduces the need for the low priced firm to increase their price. At the same time, the optimal price adjustment $p^*(p)$ falls, which implies that if p decreases by one, then the price adjustment $p^*(p) - \Delta p$ increases by less than one. These are consistent with behavior of the low-priced firm (in red) in Figure 4.

The direct effect is similar to what exists in a monopolistically competitive model. As M increases, p_1/M and p_2/M fall. At the low priced firm, the value—and so probability—of price increase increases. Here, despite the low priced and high priced firms' prices falling by the same amount, as they move toward p^*, the value of a downward adjustment at the high priced firm falls. If $f(p, \Delta)$ is initially larger than ξ, then the initial equilibrium consists of a price increase from the low priced firm, and no change from the high-priced firm. As p decreases to p' due to the monetary shock and this leads to $f(p', \Delta) < \xi$ then the price increase from the low priced firm is no longer required (indirect effect), which steeply reduces the steady-state adjustment hazard of the low priced firm. A monetary shock reduced the value of downward price adjustment at high priced, inframarginal firms, which reduces the value of upward adjustment from low priced, marginal firms.

Consider the following example. Let (p, Δ) be such that $f(p, \Delta) < \xi$ and $g(p, \Delta) < \xi$. In this case an equilibrium exists in which the initial prices persist. Now consider a negative shock to the price of the low priced firm such that $\Delta' < \Delta$ and $f(p, \Delta') > \xi$. Now the high priced firm has a credible threat of decreasing their price, yielding the equilibrium best response of a price increase from the low priced firm to increase Δ' back to Δ. Additionally $g(p, \Delta') > g(p, \Delta)$, so both the indirect and direct effects are toward a price increase. Low priced firms have a high probability of a price increase following a negative *idiosyncratic* shock, since they both move the firm away from its optimal price and increase the value of a price cut at their competitor.

Following an increase in the money supply, however, this price increase may no longer be necessary. Since f is increasing in p, there exists an increase in the money supply such that $f(p', \Delta') = f(p, \Delta)$. Similarly, as g is increasing in p, there exists an increase in the money supply such that $g(p', \Delta') = g(p, \Delta)$.

So although $\Delta' < \Delta$, the simultaneous decline in p reduces the value of a price cut at the high priced firm. In equilibrium the low priced firm absorbs the decrease in Δ but no longer increases their price.

In the presence of menu costs, firms with initially low prices before a monetary shock hits have a high value of increasing their price, precisely because their competitors have a high value of cutting theirs. A price increase staves off low sectoral prices. An increase in the money supply lowers both prices and, due to complementarity in prices, reduces the value of a price cut at the high priced firm. As this value falls below the menu cost, a low priced firm no longer needs to increase its price, and the aggregate price level increases by less.