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Aggregate Implications of Firm Heterogeneity: A Nonparametric Analysis of Monopolistic Competition Trade Model

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Abstract

We measure the role of firm heterogeneity in counterfactual predictions of monopolistic competition trade models without parametric restrictions on the distribution of firm fundamentals. We show that two bilateral elasticity functions are sufficient to nonparametrically compute the counterfactual aggregate impact of trade shocks, and recover changes in economic fundamentals from observed data. These functions are identified from two semiparametric gravity equations governing the impact of bilateral trade costs on the extensive and intensive margins of firm-level exports. Applying our methodology, we estimate elasticity functions that imply an impact of trade costs on trade flows that falls when more firms serve a market because of smaller extensive margin responses. Compared to a baseline where elasticities are constant, firm heterogeneity amplifies both the gains from trade in countries with more exporter firms, and the welfare gains of European market integration in 2003-2012.

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1 Introduction

The international trade field has been transformed by the study of firm heterogeneity. The observed correlation between firm characteristics and export participation is the cornerstone of the workhorse monopolistic competition model, linking decisions of heterogeneous firms to international trade and welfare – see [Melitz \(2003\)](#) and, for a review, [Melitz and Redding \(2014\)](#). Typically, to specify firm heterogeneity, the literature relies on restrictive parametric assumptions about the distribution of firm fundamentals. Such restrictions are useful in practice: approximations help with estimation and functional forms facilitate extrapolating aggregate counterfactual predictions from observed firm-level outcomes. However, an important question is whether these parametric distributional assumptions are essential for incorporating firm heterogeneity into empirical and counterfactual analyses in international trade. Furthermore, to what extent do these assumptions determine the implications of firm heterogeneity for the economy’s aggregate response to international trade shocks?¹

This paper aims to measure the aggregate importance of firm heterogeneity by developing a foundation for a widely used class of monopolistic competition trade models of firm heterogeneity, without any parametric restrictions on the distribution of firm fundamentals. We show that two elasticity functions are sufficient to nonparametrically compute the counterfactual aggregate impact of trade shocks, and recover changes in economic fundamentals from observed data. These functions are identified from two semiparametric gravity equations governing the impact of bilateral trade costs on the extensive and intensive margins of firm-level exports. Estimating these equations only requires bilateral data on a trade cost shifter, the share of a country’s firms exporting to a destination, and their average exports. Conditional on these elasticity functions, more information on cross-section variation in firm-level outcomes does not affect the model’s aggregate counterfactual predictions. We exploit this characterization to provide estimates of these two sufficient elasticity functions and quantify the importance of firm heterogeneity in measuring the aggregate implications of international market integration.

Our starting point is an augmented version of the multi-country workhorse monopolistic competition model with constant elasticity of substitution (CES) preferences. Firms are heterogeneous with respect to productivity, demand, and variable and fixed trade costs across destinations. An extensive literature has imposed different parametric distributional

¹A series of recent papers have shown that parametric distributional assumptions are important to determine both the adjustment margins driving the model’s counterfactual predictions and the set of moments used for identification – e.g., [Chaney \(2008\)](#); [Arkolakis, Costinot and Rodríguez-Clare \(2012\)](#); [Melitz and Redding \(2014, 2015\)](#); [Bas, Mayer and Thoenig \(2017\)](#); [Fernandes, Klenow, Denisse Periola, Meleshchuk and Rodríguez-Clare \(2019\)](#).

assumptions on some of these dimensions of firm heterogeneity to generate the observed patterns of cross-firm variation in productivity, sales, and entry in different markets – e.g., [Chaney \(2008\)](#), [Helpman, Melitz and Rubinstein \(2008\)](#), [Eaton, Kortum and Kramarz \(2011\)](#), [Redding \(2011\)](#), [Head, Mayer and Thoenig \(2014\)](#). In contrast, we do not restrict the distribution of these firm fundamentals. We thus only acknowledge that such heterogeneity exists and evaluate how it affects the response of aggregate outcomes to trade shocks.

We show that the distribution of all these sources of firm heterogeneity can be folded into two functions that yield semiparametric gravity equations for the extensive margin (i.e., exporter firm share) and intensive margin (i.e., average firm exports) of firm sales in each market. The extensive margin gravity equation links a flexible function of the exporter firm share to exogenous bilateral trade costs, and endogenous origin and destination fixed-effects (i.e., wages and price indices). The intensive margin gravity equation connects average firm exports to a flexible function of the exporter firm share, as well as bilateral trade costs and origin and destination fixed-effects.² The functions in the two gravity equations control the elasticities of the extensive and intensive margins of firm exports to changes in bilateral trade costs. The elasticity of each margin is an arbitrary function of the initial exporter firm share. These functions also determine the aggregate trade elasticity, which combines the elasticities of the two margins and, thus, may vary arbitrarily with the initial exporter firm share.

With this characterization, we establish that the two gravity elasticity functions of firm exports summarize the role that firm heterogeneity plays in the model’s counterfactual predictions following shocks in trade costs and productivity. Formally, three objects are sufficient for conducting counterfactual exercises: (i) bilateral data on trade flows and export firm shares, (ii) the elasticity of substitution across varieties, and (iii) the gravity elasticity functions of the extensive and intensive margins of firm exports. Thus, the distribution of firm fundamentals only affects the model’s aggregate counterfactual predictions through the shape and level of the two gravity elasticity functions.

We provide theoretical results that synthesize the debate on the aggregate implications of firm heterogeneity. For small shocks, the aggregate trade elasticity is sufficient to characterize the response of aggregate outcomes, in line with [Arkolakis et al. \(2012\)](#). For large shocks, the trade elasticity changes along the adjustment path to the new equilibrium due to changes in exporter firm shares, implying that firm heterogeneity plays an important role for the model’s predictions, in line with [Melitz and Redding \(2015\)](#) and [Feenstra \(2018\)](#). We show, however, that this role is summarized by the gravity elasticities of the firm export margins and their

²We derive these semiparametric gravity equations using the same type of inversion argument in [Berry and Haile \(2014\)](#) and [Adão \(2015\)](#). The function of the exporter firm share in the intensive margin equation captures composition effects in exports generated by endogenous firm entry: average firm exports depend on the average sales cost of the endogenous pool of exporter firms.

dependence on the firm exporter share. When both elasticity functions are constant, the aggregate trade elasticity is also constant and the model’s predictions are isomorphic to those of the constant-elasticity gravity trade models in [Arkolakis et al. \(2012\)](#).

We then derive expressions to nonparametrically recover changes in economic fundamentals from observed data. Given the two gravity elasticity functions, the inversion of changes in bilateral (variable and fixed) trade costs only requires data on changes in trade flows, firm exporter shares, and wages. This result does not hinge on the type of symmetry assumption imposed by [Head and Ries \(2001\)](#). The inversion of domestic productivity shifters also requires observing changes in the country’s price index, as in [Eaton, Kortum, Neiman and Romalis \(2016\)](#).

We conclude our theoretical analysis by deriving nonparametric expressions for the impact of trade shocks on welfare that extend sufficient statistics in the literature – e.g., [Melitz \(2003\)](#), [Arkolakis et al. \(2012\)](#) and [Melitz and Redding \(2015\)](#). In particular, we derive an extension of the formula in [Arkolakis et al. \(2012\)](#) for small shocks in which welfare gains combine the initial trade elasticity, and changes in the domestic spending share and number of entrants. By integrating this formula for large shocks, we show that welfare gains must account for the correlation between changes in the trade elasticity and changes in domestic spending share and firm entry, which depends on the two elasticity functions of firm exports.

Our theoretical results indicate that different parametric assumptions in the literature only affect the model’s aggregate predictions through what they imply for the shape and level of the gravity elasticity functions of the firm export margins. Moreover, such parametric assumptions play the key role of allowing the identification of the export margin elasticities from micro cross-sectional variation in observed firm outcomes.³ Without parametric distributional assumptions, cross-firm variation in sales and exports does not identify the two functions necessary for counterfactual analysis. Instead, we argue that, in a more general environment without parametric restrictions in the distribution of firm fundamental, one should directly estimate the gravity elasticity functions of the firm export margins.

We do so by extending standard tools for gravity estimation to recover the two sufficient elasticity functions using the model-implied semiparametric gravity equations of firm exports. We exploit cross-country variation in exporter firm share and average firm exports induced by observed shifters of bilateral trade costs (conditional on origin and destination fixed-effects). This requires two main assumptions. First, different country groups must have common trade

³For instance, the elasticities of all margins do not vary with the exporter firm share if firm productivity has the Pareto distribution in [Chaney \(2008\)](#). The elasticities are decreasing in the exporter firm share if productivity has the Truncated Pareto distribution in [Melitz and Redding \(2015\)](#) or the Log-normal distribution in [Head et al. \(2014\)](#). These parametric assumptions allow the identification of the elasticity functions and the extrapolation of counterfactual predictions from cross-firm variation in sales and exports.

elasticity functions. Second, observed cost shifters must satisfy the same set of assumptions necessary for the consistent estimation of constant-elasticity gravity models – for a review, see [Head and Mayer \(2014\)](#). Intuitively, our methodology extends that of standard constant-elasticity gravity estimation by specifying the elasticity of each export margin to be a flexible function of the observed exporter firm share.

Our estimates yield three main insights about the impact of trade costs on the different export margins.⁴ First, the extensive margin of firm entry is more responsive when few firms serve a market (in line with [Kehoe and Ruhl \(2013\)](#) and [Kehoe, Rossbach and Ruhl \(2015\)](#)). We estimate an extensive margin elasticity of six for low levels of the exporter firm share, but this elasticity is only two if more than 10% of the country’s firms export to a destination. Second, we estimate an intensive margin elasticity that is positive (as in [Fernandes et al. \(2019\)](#)) but not sensitive to the number of exporter firms. As in [Melitz \(2003\)](#), our intensive margin elasticity entails strong composition effects: an increase of 1% in the number of exporters is associated with a reduction of 0.2% in average firm exports due to the higher cost of marginal exporters. Lastly, the combination of these two margins yields an aggregate trade elasticity that declines with the firm exporter share, varying between seven and four.

We conclude by studying the welfare consequences of two types of trade shocks in 2012: (i) moving from the current equilibrium to autarky by setting trade costs to infinity, and (ii) reverting trade costs within the European single market to its level of 2003. We measure the importance of firm heterogeneity for the shock’s impact by comparing results implied by our baseline estimates of the gravity elasticity functions and those implied by the benchmark constant-elasticity gravity model of bilateral trade flows.⁵

In our first exercise, the welfare impact of moving to autarky obtained with our baseline estimates differs on absolute value by an average of 7% from those implied by the constant-elasticity gravity benchmark. Firm heterogeneity has a substantial impact on some countries in which our baseline gains differ by more than 25% from the gains in the benchmark model. We find that, because of the non-linear trade elasticities, our baseline gains are systematically larger than those in the benchmark for countries with a higher fraction of firms exporting.

Our second exercise quantifies the welfare impact of changes in trade costs within the European single market from 2003 to 2012. Using our theoretical inversion result, we use changes in observed outcomes to recover actual changes in fixed and variable costs of selling in different markets. We find that the impact of such shocks was highly heterogeneous across

⁴We use a sample of exporter-importer pairs in 2012 covering 88% of world trade. Following the gravity estimation literature, our trade cost shifter is bilateral distance.

⁵As discussed above, firm heterogeneity does not matter in this benchmark since it is isomorphic to the gravity trade models in [Arkolakis et al. \(2012\)](#). So, counterfactual predictions only require the aggregate constant trade elasticity and the initial trade flow matrix.

countries. While the average welfare gain was only 0.5% in Western Europe, it was 3.4% in East Europe (Estonia and Hungary both saw gains of over 5%). Isolating the impact of different shocks, we show that changes in fixed costs had only a small welfare impact. Most of the gains came from reductions in variable costs, with the larger gains in East Europe driven by stronger reductions in their export costs.⁶ Compared to the constant-elasticity gravity benchmark, the average welfare impact of European market integration is 8% higher with our baseline estimates. This is because trade shocks affect several country pairs whose exporter firm shares yield a trade elasticity above the benchmark constant trade elasticity.

Our paper is related to an extensive literature using variations of the framework in [Melitz \(2003\)](#) together with parametric distributional assumptions to conduct empirical and counterfactual analyses – for a review, see [Melitz and Redding \(2014\)](#). We complement this literature by showing how to introduce firm heterogeneity in such analyses while dispensing parametric restrictions on the distribution of firm fundamentals. Our analysis indicates that the role of firm heterogeneity is summarized by two trade elasticity functions that are identified from semiparametric gravity equations for the margins of firm exports.⁷ Parametric assumptions made in the literature are central to extrapolate counterfactual predictions from micro moments associated with cross-firm heterogeneity. Our empirical results show that such an approach may have a large quantitative impact on the contribution of firm heterogeneity for the model’s predictions as it effectively restricts the two elasticity functions.

Our empirical analysis relies on two semiparametric gravity equations of firm exports. It is thus related to the literature estimating extensions of the log-linear gravity equation of bilateral trade flows – e.g., [Novy \(2013\)](#), [Fajgelbaum and Khandelwal \(2016\)](#), and [Lind and Ramondo \(2018\)](#). We also show that semiparametric gravity equations arise when we extend our baseline environment to allow for (i) tariffs, multiple sectors, multiple factors, and input-output links as in [Costinot and Rodriguez-Clare \(2013\)](#) and [Caliendo and Parro \(2014\)](#), (ii) zero bilateral trade flows as in [Helpman et al. \(2008\)](#), (iii) multi-product firms as in [Bernard, Redding and Schott \(2011\)](#) and [Arkolakis, Ganapati and Muendler \(2019b\)](#), and (iv) non-CES demand functions with a single price aggregator as in [Matsuyama and Ushchev \(2017\)](#) and [Arkolakis, Costinot, Donaldson and Rodríguez-Clare \(2019a\)](#).

We also contribute to a recent literature focusing on nonparametric counterfactual analysis in international trade models ([Adao, Costinot and Donaldson, 2017](#); [Bartelme, Lan and Levchenko, 2019](#)). Counterfactual predictions in these settings require knowledge of

⁶The impact of our baseline inverted shocks is larger and more heterogeneous than the impact of either observed changes in tariffs or symmetric trade shocks inverted with a nonparametric extension of the procedure in [Head and Ries \(2001\)](#).

⁷[Fernandes et al. \(2019\)](#) show that counterfactual analysis can be conducted for any given parametric distribution of firm productivity, but its implementation still requires the distribution to be specified.

multivariate functions whose nonparametric estimation is challenging in finite samples – for example, [Adao et al. \(2017\)](#) must estimate a country’s demand function for all factors in the world economy. Compared to these papers, we show how to construct nonparametric counterfactuals in a different class of models featuring monopolistic competition and increasing returns to scale. Moreover, it is simple to implement our methodology in finite samples since it only requires estimating two univariate functions of the exporter firm share using semiparametric gravity equations. Further work that offers sufficient statistics for welfare and trade in general environments requires parametric assumptions for the actual implementation of empirical and counterfactual analyses (e.g. [Baqee and Farhi \(2019\)](#); [Kleinman, Liu and Redding \(2020\)](#); [Adao, Arkolakis and Esposito \(2020\)](#)).

Finally, we offer formal results on how to recover changes in economic fundamentals that can be used in counterfactual analysis. These results constitute a nonparametric extension of the identification procedure in [Eaton et al. \(2016\)](#) in which we recover (potentially asymmetric) changes in fixed and variable trade costs, as well as changes in productivity. This is different from the focus on symmetric bilateral variable trade shifters, as in the approach of [Adao et al. \(2017\)](#), or productivity and amenity shocks (given bilateral trade shifters), as for example in the spatial models in [Allen and Arkolakis \(2014\)](#) and [Redding \(2016\)](#).

Our paper is organized as follows. Section 2 derives the semiparametric gravity equations for the extensive and intensive margins of firm exports. In Section 3, we present our nonparametric counterfactual analysis and inversion of economic fundamentals. Section 4 outlines the methodology to estimate the two main elasticity functions in the model. In Section 5, we report our baseline estimation results. Section 6 conducts counterfactual exercises. Section 7 concludes.

2 Model

We consider an economy in which firms are heterogeneous in terms of productivity, demand, and trade (variable and fixed) costs. The equilibrium of this economy entails two semiparametric gravity equations for the extensive and intensive margins of firm exports. In general equilibrium, the functions in these two gravity equations along with country-level fundamentals determine trade flows, firm entry, price indices, and wages.

2.1 Environment

Demand. Each country j has a representative household that inelastically supplies $\bar{L}_j$ units of labor. In each country j , the representative household has Constant Elasticity of

Substitution (CES) preferences over the continuum of available varieties from different origins i , $\omega \in \Omega_{ij}$, with elasticity of substitution $\sigma > 1$. This yields j 's demand for variety ω from country i :

$$q_{ij}(\omega) = (\bar{b}_{ij} b_{ij}(\omega)) \left(\frac{p_{ij}(\omega)}{P_j} \right)^{-\sigma} \frac{E_j}{P_j}, \quad (1)$$

where, in market j , E_j is the total spending, $p_{ij}(\omega)$ is the price of variety ω of country i , and P_j is the CES price index,

$$P_j^{1-\sigma} = \sum_i \int_{\Omega_{ij}} (\bar{b}_{ij} b_{ij}(\omega)) (p_{ij}(\omega))^{1-\sigma} d\omega. \quad (2)$$

The demand shifters, $b_{ij}(\omega)$, generate dispersion in sales across varieties conditional on prices. The term $\bar{b}_{ij}$ is the component of bilateral taste shifters that is common to all varieties.

Production. Each variety is produced by a single firm, so we refer to a variety as a firm-specific good. The production function implies that, in order to sell q units in country j , firm ω from country i incurs in a labor cost of

$$C_{ij}(\omega, q) = w_i \frac{\tau_{ij}(\omega)}{a_i(\omega)} \frac{\bar{\tau}_{ij}}{\bar{a}_i} q + w_i \bar{f}_{ij} f_{ij}(\omega).$$

The first term is the variable cost of selling q units in country j , including both firm-specific iceberg shipping costs, $\bar{\tau}_{ij} \tau_{ij}(\omega)$, and productivity, $\bar{a}_i a_i(\omega)$. The second term is the fixed cost of labor necessary to enter j . As in [Melitz \(2003\)](#), we specify the fixed entry cost in terms of labor in the origin country. However, we depart from [Melitz \(2003\)](#) by allowing firms to be different not only in their productivity, but also in their variable and fixed costs of exporting. [Eaton et al. \(2011\)](#) show that these additional sources of firm heterogeneity are important to generate observed patterns of firm-level exports to different countries.

We consider a monopolistic competitive environment in which firms maximize profits given the demand in (1). For firm ω of country i , the optimal price in market j is $p_{ij}(\omega) = \frac{\sigma}{\sigma-1} \frac{\bar{\tau}_{ij} w_i}{\bar{a}_i} \frac{\tau_{ij}(\omega)}{a_i(\omega)}$ with an associated revenue of

$$R_{ij}(\omega) = \bar{r}_{ij} r_{ij}(\omega) \left[\left(\frac{w_i}{P_j} \right)^{1-\sigma} E_j \right] \quad \text{where} \quad (3)$$

$$r_{ij}(\omega) \equiv b_{ij}(\omega) \left(\frac{\tau_{ij}(\omega)}{a_i(\omega)} \right)^{1-\sigma} \quad \text{and} \quad \bar{r}_{ij} \equiv \bar{b}_{ij} \left(\frac{\sigma}{\sigma-1} \frac{\bar{\tau}_{ij}}{\bar{a}_i} \right)^{1-\sigma}. \quad (4)$$

We refer to $r_{ij}(\omega)$ as the *revenue potential* in j of firm ω from i and $\bar{r}_{ij}$ as the bilateral revenue shifter that is common to all firms. Conditional on entering market j , $r_{ij}(\omega)$ is the ω -specific sales shifter in j that combines different sources of firm heterogeneity.

The firm's entry decision depends on the profit generated by the revenue in (3), $(1/\sigma)R_{ij}(\omega)$, and the fixed-cost of entry, $w_i \bar{f}_{ij} f_{ij}(\omega)$. Specifically, firm ω of i enters j if, and only if, $\pi_{ij}(\omega) = \frac{1}{\sigma}R_{ij}(\omega) - w_i \bar{f}_{ij} f_{ij}(\omega) \geq 0$. This yields the set of firms from i selling in j , Ω_{ij} :

$$\omega \in \Omega_{ij} \Leftrightarrow e_{ij}(\omega) \equiv \frac{r_{ij}(\omega)}{f_{ij}(\omega)} \geq \sigma \frac{\bar{f}_{ij}}{\bar{r}_{ij}} \left[\left(\frac{w_i}{P_j} \right)^\sigma \frac{P_j}{E_j} \right]. \quad (5)$$

We refer to $e_{ij}(\omega)$ as the *entry potential* of firm ω of i in j . Among firms with identical revenue potential, heterogeneity in the fixed-cost of entry generates heterogeneity in entry potentials and, therefore, in decisions to enter different destination markets. The difference between revenue and entry potentials of firms allows for imperfect cross-firm correlation between entry and sales across markets.

General Equilibrium. Firms in country i hire $\bar{F}_i$ units of domestic labor to create a new variety whose characteristics are a random draw from an arbitrary distribution:

$$v_i(\omega) \equiv \{a_i(\omega), b_{ij}(\omega), \tau_{ij}(\omega), f_{ij}(\omega)\}_j \sim G_i(v). \quad (6)$$

In the free entry equilibrium, N_i enter and receive an expected profit of zero,

$$\sum_j E[\max\{\pi_{ij}(\omega); 0\}] = w_i \bar{F}_i. \quad (7)$$

As in [Dekle et al. \(2008\)](#), we allow for exogenous international transfers $\{\bar{T}_i\}_i$, so that

$$E_i = w_i \bar{L}_i + \bar{T}_i, \quad \sum_i \bar{T}_i = 0. \quad (8)$$

Since labor is the only factor of production, labor income in i equals the total revenue of firms from i : $w_i \bar{L}_i = \int_{\omega \in \Omega_{ij}} R_{ij}(\omega) d\omega$. Given the expression in (3),

$$w_i \bar{L}_i = \bar{r}_{ij} \left(\frac{w_i}{P_j} \right)^{1-\sigma} E_j \left[\int_{\omega \in \Omega_{ij}} r_{ij}(\omega) d\omega \right]. \quad (9)$$

Summarizing, given the arbitrary distribution in (6), the equilibrium is defined as the vector $\{P_i, \{\Omega_{ij}\}_j, N_i, E_i, w_i\}_i$ satisfying equations (2), (5), (7), (8), (9) for all i .

2.2 The Extensive and Intensive Margins of Firm Exports

We now use two aggregate bilateral outcomes, the share of firms from i selling in j , $n_{ij} = \Pr[\omega \in \Omega_{ij}]$, and their average sales, $\bar{x}_{ij} \equiv E[R_{ij}(\omega) | \omega \in \Omega_{ij}]$, to define two corresponding aggregate elasticity functions that control how the intensive and extensive margins of firm-level exports respond to changes in bilateral trade costs. In the next section, we use these elasticity functions in our sufficient statistics characterization of the counterfactual impact of trade shocks on aggregate outcomes. In the rest of the paper, we refer to n_{ij} and $\bar{x}_{ij}$ as exporter firm share and average firm exports.

We consider the CDF of $(r_{ij}(\omega), e_{ij}(\omega))$ implied by $G_i(v)$. Without loss of generality, this CDF can be decomposed as

$$r_{ij}(\omega) \sim H_{ij}^r(r|e), \quad \text{and} \quad e_{ij}(\omega) \sim H_{ij}^e(e). \quad (10)$$

Intuitively, firms draw their entry potential e from $H_{ij}^e(e)$. Conditional on having an entry potential of e , firms draw their revenue potential r from $H_{ij}^r(r|e)$. We impose the following regularity condition on the distribution of entry potentials.

Assumption 1. *Assume that $H_{ij}^e(e)$ is continuous and strictly increasing in $\mathbb{R}_+$ with $\lim_{e \rightarrow \infty} H_{ij}^e(e) = 1$.*

This assumption implies that that H_{ij}^e has no mass points, which guarantees that any trade cost change induces a positive mass of firms to switch entry decisions. This is central for the change of variables necessary for our characterization of the equilibrium below.⁸

Our specification allows for any pattern of heterogeneity and correlation in $(r_{ij}(\omega), e_{ij}(\omega))$. It thus encompasses several distributional assumptions in the literature. For instance, in [Melitz \(2003\)](#), the only source of firm heterogeneity is productivity such that $r_{ij}(\omega) = e_{ij}(\omega) = (a_i(\omega))^{\sigma-1}$. The single source of heterogeneity implies a strict hierarchy of entry across destinations and a perfect cross-firm correlation between intensive and extensive margins of export. The distribution of e_{ij} can be specified to be Pareto, as in [Chaney \(2008\)](#) and [Arkolakis \(2010\)](#), truncated Pareto, as in [Helpman et al. \(2008\)](#) and [Melitz and Redding \(2015\)](#), or log-normal, as in [Head et al. \(2014\)](#) and [Bas et al. \(2017\)](#). Multiple papers incorporate additional sources of heterogeneity across firms that yield dispersion in both $r_{ij}(\omega)$ and $e_{ij}(\omega)$. For example, the demand and entry cost heterogeneity in [Eaton et al. \(2011\)](#) are modeled so that $a_i(\omega)$ is Pareto distributed while $b_{ij}(\omega)$ and $f_{ij}(\omega)$ are joint log-normally distributed while [Fernandes et al. \(2019\)](#) assume a multivariate log-normal

⁸The assumption of no upper bound on the support of e also implies positive trade flows between all origin-destination pairs. However, this is not essential. In Section 3.4, we allow for the possibility of zero bilateral trade by imposing that there exists $\bar{e}_{ij} < \infty$ such that $H_{ij}^e(\bar{e}_{ij}) = 1$.

distribution of bilateral sale shifters across destinations. [Arkolakis et al. \(2019b\)](#) extend these setups by considering a further layer of product-firm heterogeneity.

We now use Assumption 1 to characterize the extensive and intensive margins of firm exports. Focusing on the extensive margin first, we define the inverse distribution of entry potential: $\epsilon_{ij}(n) \equiv (H_{ij}^e)^{-1}(1 - n)$ where $\bar{\epsilon}_{ij}(n)$ is strictly decreasing, $\epsilon_{ij}(1) = 0$, and $\lim_{n \rightarrow 0} \epsilon_{ij}(n) = \infty$. Applying this definition to (5),

$$\ln \epsilon_{ij}(n_{ij}) = \ln (\sigma \bar{f}_{ij} / \bar{r}_{ij}) + \ln (w_i^\sigma) - \ln (E_j P_j^{\sigma-1}). \quad (11)$$

This expression is a semiparametric gravity equation since it relates the function $\epsilon_{ij}(n_{ij})$ to a log-linear combination of exogenous bilateral trade shifters and endogenous outcomes in the origin and destination markets. Notice that its elasticity, $\varepsilon_{ij}(n_{ij}) \equiv \frac{\partial \ln \epsilon_{ij}(n)}{\partial \ln n} \Big|_{n=n_{ij}}$, controls the sensitivity of the exporter firm share to changes in bilateral trade costs (holding constant other endogenous variables),

$$\frac{\partial \ln n_{ij}}{\partial \ln \bar{r}_{ij}} = \frac{\sigma - 1}{\varepsilon_{ij}(n_{ij})} < 0. \quad (12)$$

Turning to the intensive margin, we define the average revenue potential when a share n_{ij} of i 's firms sell in j as

$$\rho_{ij}(n_{ij}) \equiv \frac{1}{n_{ij}} \int_0^{n_{ij}} E[r|e = \epsilon_{ij}(n)] dn \quad (13)$$

where $E[r|e = \bar{\epsilon}_{ij}(n)]$ is the mean revenue potential of firms in quantile n of the entry potential distribution. Average bilateral firm sales can be written as

$$\bar{x}_{ij} = \bar{r}_{ij} \left[\left(\frac{w_i}{P_j} \right)^{1-\sigma} E_j \int_{e_{ij}^*}^{\infty} E[r|e] \frac{dH^e(e)}{1 - H^e(e_{ij}^*)}, \quad \text{where} \quad e_{ij}^* \equiv \sigma \frac{\bar{f}_{ij}}{\bar{r}_{ij}} \left[\left(\frac{w_i}{P_j} \right)^\sigma \frac{P_j}{E_j} \right]. \right.$$

We now implement a change of variables in this expression by defining $n = 1 - H_{ij}^e(e)$ such that $n_{ij} = 1 - H_{ij}^e(e_{ij}^*)$. This yields our second semiparametric gravity equation:

$$\ln \bar{x}_{ij} - \ln \rho_{ij}(n_{ij}) = \ln (\bar{r}_{ij}) + \ln (w_i^{1-\sigma}) + \ln (E_j P_j^{\sigma-1}), \quad (14)$$

The expression relates the composition-adjusted average firm exports to a linear combination of exogenous bilateral trade shifters and endogenous outcomes in the origin and destination markets. The elasticity of $\rho_{ij}(n)$, $\varrho_{ij}(n_{ij}) \equiv \frac{\partial \ln \rho_{ij}(n)}{\partial \ln n_{ij}} \Big|_{n=n_{ij}}$, controls the sensitivity of average firm exports to changes in bilateral trade costs (holding constant other endogenous variables):

$$\frac{\partial \ln \bar{x}_{ij}}{\partial \ln \bar{r}_{ij}} = -(\sigma - 1) + \varrho_{ij}(n_{ij}) \frac{\partial \ln n_{ij}}{\partial \ln \bar{r}_{ij}}. \quad (15)$$

This elasticity combines two well-known forces. The first term is the reduction in the sales of the initial set of exporters in j arising from the constant elasticity of substitution across varieties. The second term measures how the change in the number of exporters affects the average revenue potential of firms selling in j . The sign of this term depends on how different marginal and infra-marginal exporters are in terms of revenue potential. Specifically,

$$\varrho_{ij}(n_{ij}) = \frac{E[r|e = \epsilon_{ij}(n_{ij})]}{\frac{1}{n_{ij}} \int_0^{n_{ij}} E[r|e = \epsilon_{ij}(n)] dn} - 1. \quad (16)$$

Notice that $\varrho_{ij}(n_{ij}) < 0$ if, and only if, the mean revenue potential of marginal exporters, $E[r|e = \epsilon_{ij}(n_{ij})]$, is lower than that of the average exporter, $\rho_{ij}(n_{ij}) = \frac{1}{n_{ij}} \int_0^{n_{ij}} E[r|e = \epsilon_{ij}(n)] dn$.⁹

The bilateral trade flow between i and j , X_{ij} , combines the extensive and intensive margins of firm exports, $X_{ij} = N_i n_{ij} \bar{x}_{ij}$. Thus, by definition, $\epsilon_{ij}(n_{ij})$ and $\rho_{ij}(n_{ij})$ determine the elasticity of bilateral trade flows to changes in bilateral trade costs (holding constant endogenous variables in the origin and destination countries):

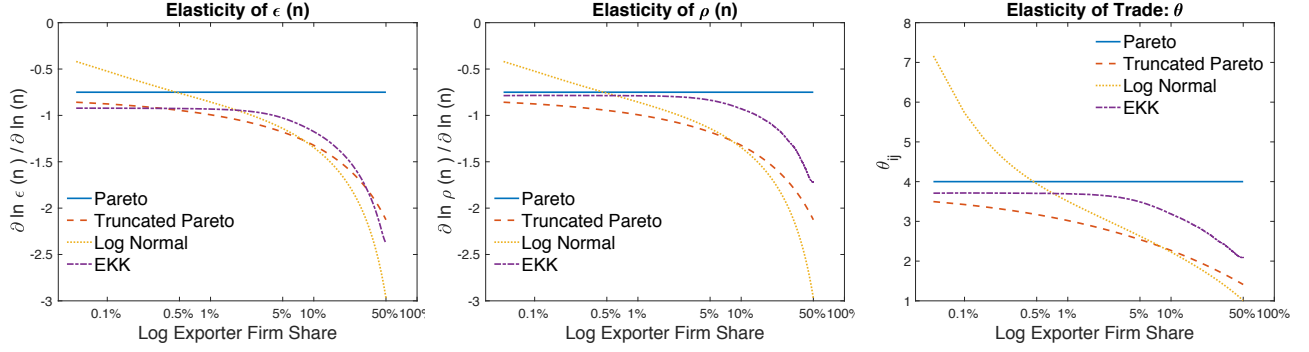
$$\theta_{ij}(n_{ij}) \equiv -\frac{\partial \ln X_{ij}}{\partial \ln \bar{\tau}_{ij}} = -\left(\frac{\partial \ln n_{ij}}{\partial \ln \bar{\tau}_{ij}} + \frac{\partial \ln \bar{x}_{ij}}{\partial \ln \bar{\tau}_{ij}} \right) = (\sigma - 1) \left(1 - \frac{1 + \varrho_{ij}(n_{ij})}{\varepsilon_{ij}(n_{ij})} \right). \quad (17)$$

This expression indicates that n_{ij} acts like a state variable that determines the elasticity of bilateral trade flows to changes in trade costs – the so-called trade elasticity. This occurs because n_{ij} controls the elasticity of the extensive and intensive margins of firm exports. Notice that the trade elasticity is positive for all n_{ij} since $\varepsilon_{ij}(n_{ij}) < 0$ and $\varrho_{ij}(n_{ij}) > -1$.

Distributional assumptions and elasticity of trade flows. Overall, one of our key findings is that the extensive and intensive margin elasticities, and thus the bilateral trade elasticity, are univariate functions of the exporter firm share, n_{ij} . We do not impose restrictions on these functions other than the regularity conditions in Assumption 1. Before characterizing the non-parametric sufficient statistics in our model, we show how different parametric assumptions on the distribution of firm fundamentals yield specific patterns of dependence between the extensive and intensive margin elasticities of firm exports and the exporter firm share. In particular, Figure 1 illustrates the elasticity functions implied by productivity distributions from the Pareto family (Chaney, 2008), the truncated Pareto family (Melitz and Redding, 2015), and the log-normal family (Head et al., 2014). We also

⁹In Melitz (2003), the single source of firm heterogeneity ($r_{ij}(\omega) = e_{ij}(\omega)$) implies that $E[r|e = \epsilon_{ij}(n_{ij})] = \epsilon_{ij}(n_{ij})$. In this case, marginal exporters are worse than existing infra-marginal exporters since $\frac{\partial \rho_{ij}(n)}{\partial n} = \frac{1}{n^2} \int_0^n (\epsilon_{ij}(n) - \epsilon_{ij}(x)) dx < 0$ and $\epsilon_{ij}(n) < \epsilon_{ij}(x)$ for all $x < n$.

Figure 1: Distributional assumptions and Elasticity of different margins of trade flows



Note. Left panel reports the elasticity of $\epsilon_{ij}(n)$. Center panel reports the elasticity of $\rho_{ij}(n)$. Right panel reports the trade elasticity $\theta_{ij}(n)$ in (17). Each line corresponds to the elasticity as a function of n implied by different parametric restrictions on the distribution of firm fundamentals. See main text for a description of each parametrization.

consider the specification in Eaton et al. (2011) where productivity has a Pareto distribution and shifters of demand and entry costs have a joint log-normal distribution. In all cases, we use the baseline parameters reported in each paper.

The first plot indicates that the Pareto assumption yields constant elasticities of all margins. The other parameterizations yield a declining elasticity of $\epsilon(n)$, which, by equation (12), implies that the extensive margin elasticity is more sensitive when the exporter firm share is low. Similarly, all other parameterizations yield a declining elasticity of $\rho(n)$, indicating that new entrants and incumbents are more similar to each other when n_{ij} is small. This implies that composition effects are weaker when few firms export to a particular destination. The third panel combines these two margins to show that the trade elasticity is higher when n_{ij} is low. In all parametrizations, the trade elasticity falls below two when n_{ij} is above 50%. We show below that this implies a low elasticity in the domestic market where n_{ii} is high, which has important implications for the measurement of the gains from trade.

2.3 Sufficient Statistics of Firm Heterogeneity

Having characterized individual firm decisions and aggregate trade flows as a function of $\epsilon_{ij}(n)$ and $\rho_{ij}(n)$, the following lemma establishes that all aggregate variables in equilibrium only depend on firm heterogeneity through $\epsilon_{ij}(n)$ and $\rho_{ij}(n)$. Specifically, we show in Appendix A.1 that the equilibrium equations determining price indices, number of entrants, expenditures and wages can be expressed in terms of model fundamentals $\{\bar{T}_i, \bar{L}_i, \bar{F}_i, \bar{f}_{ij}, \bar{r}_{ij}\}$, the elasticity of substitution σ , and the elasticity functions $\{\epsilon_{ij}(n), \rho_{ij}(n)\}_{i,j}$.

Lemma 1. *Suppose Assumption 1 holds. The equilibrium vector $\{n_{ij}, \bar{x}_{ij}, X_{ij}, P_i, N_i, E_i, w_i\}_{i,j}$ can be fully characterized as a function of (i) country fundamentals $\{\bar{T}_i, \bar{L}_i, \bar{F}_i, \bar{f}_{ij}, \bar{r}_{ij}\}_{i,j}$, (ii) the elasticity of substitution σ , and (iii) the bilateral elasticity functions, $\{\epsilon_{ij}(n), \rho_{ij}(n)\}_{i,j}$.*

Proof. See Appendix A.1.

The intuition for this lemma follows from the equilibrium characterization in [Melitz \(2003\)](#). We first show that all aggregate variables depend on the set of firms operating in each country pair and their average sales, which can be written as functions of the equilibrium entry cutoff. We then use the same inversion argument used in Section 2.2 to establish a one-to-one mapping between these functions of the entry cutoff and $(\epsilon_{ij}(n), \rho_{ij}(n))$.

The results of this section indicate that the functions $\epsilon_{ij}(n)$ and $\rho_{ij}(n)$ summarize the aggregate implications of the different dimensions of firm heterogeneity in the model. Thus, any parametric restriction on the distribution of firm fundamentals affects the economy's equilibrium insofar it determines the shape of these functions. We summarize this discussion in the following remark.

Remark 1. All dimensions of heterogeneity can be folded into the functions $\epsilon_{ij}(n)$ and $\rho_{ij}(n)$ that govern the semiparametric gravity equations for the extensive and intensive margins of firm exports, (11) and (14), and all aggregate variables in general equilibrium.

The rest of the paper exploits this insight in two ways. In the next section, we build directly on Lemma 1 by using $\{\epsilon_{ij}(n), \rho_{ij}(n)\}_{i,j}$ to provide ex-ante and ex-post sufficient statistics for the counterfactual impact of trade shocks on aggregate outcomes and welfare given bilateral trade and exporter entry data. We also show how to invert changes in the economy's fundamentals given knowledge of $\{\epsilon_{ij}(n), \rho_{ij}(n)\}_{i,j}$ and observed changes in bilateral trade and exporter entry. In Section 4, we will exploit the semiparametric gravity equations in (11) and (14) to estimate $\epsilon_{ij}(n)$ and $\rho_{ij}(n)$ without imposing any parametric restrictions on the distribution of firm fundamentals.

3 Nonparametric Counterfactual Analysis and Identification of Economic Fundamentals

This section uses the equilibrium characterization above for counterfactual analysis and identification of changes in fundamentals without imposing parametric restrictions on the distribution of firm fundamentals. We first show how to construct nonparametric counterfactual changes in aggregate outcomes in response to changes in fundamentals using observed trade data in the initial equilibrium and $(\sigma, \epsilon_{ij}(n), \rho_{ij}(n))$. We then discuss how to nonparametrically identify shocks to economic fundamentals from changes in observed outcomes given knowledge of $(\sigma, \epsilon_{ij}(n), \rho_{ij}(n))$. Finally, we use $(\sigma, \epsilon_{ij}(n), \rho_{ij}(n))$ and trade data to construct sufficient statistics for welfare gains implied by shocks in trade cost.

3.1 Counterfactual Responses to Changes in Fundamentals

We start by characterizing how to compute counterfactual changes in aggregate outcomes following exogenous shocks in the economy’s fundamentals. We consider shocks in international transfers $\bar{T}_i$, population $\bar{L}_i$, entry costs $\bar{F}_i$, fixed costs of exporting $\bar{f}_{ij}$, and bilateral revenue shifters $\bar{r}_{ij}$. We use $\hat{y}_j \equiv y'_j/y_j^0$ to express changes in any variable between its level in the initial observed equilibrium, y_i^0 , and the counterfactual equilibrium, y'_i . We also use bold letters to denote vectors, $\mathbf{y} = [y_i]_i$, and bold bar variables to denote matrices, $\bar{\mathbf{y}} = [y_{ij}]_{i,j}$.

Proposition 1. *Consider any change in fundamentals, $\{\hat{\mathbf{T}}, \hat{\mathbf{L}}, \hat{\mathbf{F}}, \hat{\mathbf{f}}, \hat{\mathbf{r}}\}$. Given the matrices of exporter firm shares and bilateral trade flows in the initial equilibrium $(\bar{\mathbf{n}}^0, \bar{\mathbf{X}}^0)$, the substitution elasticity σ and the functions $(\bar{\epsilon}(\bar{\mathbf{n}}), \bar{\rho}(\bar{\mathbf{n}}))$ (up to a scalar) are “sufficient statistics” that characterize the counterfactual changes in equilibrium outcomes, $\{\hat{\mathbf{n}}, \hat{\mathbf{x}}, \hat{\mathbf{X}}, \hat{\mathbf{P}}, \hat{\mathbf{N}}, \hat{\mathbf{w}}\}$.*

Proof. See Appendix A.2.

The proposition generalizes the sufficient statistics result of [Arkolakis et al. \(2012\)](#) beyond the class of constant-elasticity gravity models by exploiting the characterization of the equilibrium in Lemma 1. It outlines two sufficient requirements to compute counterfactual changes in aggregate outcomes. First, it is not necessary to know the entire distribution of firm fundamentals in the initial equilibrium. Instead, one only needs to obtain the exporter firm share n_{ij} , and the two functions controlling the extensive and intensive margins of firm exports, $\epsilon_{ij}(n)$ and $\rho_{ij}(n)$. Second, it is also necessary to obtain bilateral trade flows between countries, as in the original “hat algebra” methodology in [Dekle et al. \(2008\)](#), and the elasticity of substitution σ , as in the “hat algebra” for heterogeneous firm models (see [Costinot and Rodriguez-Clare \(2013\)](#)).¹⁰

According to Proposition 1, the trade elasticity, $\theta_{ij}(n_{ij})$, does not play a direct role in the model’s counterfactual predictions without parametric restriction on the distribution of firm fundamentals. However, as the next proposition shows, the trade elasticity is the channel through which the extensive and intensive margin elasticities of firm exports affect counterfactual changes in aggregate outcomes in response to trade cost shocks.

Proposition 2. *Consider a small shock to the bilateral revenue shifters between origin o and destination d , $d \ln \bar{r}_{od}$. Denote the vector of aggregate outcomes as $Y_i \equiv \{\{X_{ij}\}_j, P_i, N_i, w_i\}$.*

1. *The elasticity of any element of Y_i to $\bar{r}_{od}$ is a function of $(\sigma, \bar{\boldsymbol{\theta}}(\bar{\mathbf{n}}^0), \bar{\mathbf{X}}^0)$:*

$$\frac{d \ln Y_i}{d \ln \bar{r}_{od}} = \Psi_{i,od}^z \left(\sigma, \bar{\boldsymbol{\theta}}(\bar{\mathbf{n}}^0), \bar{\mathbf{X}}^0 \right), \quad (18)$$

¹⁰The elasticity of substitution is necessary when the entry cost is set in terms of the origin country wage. In this case, gravity origin fixed-effects contain wages with an elasticity determined by σ .

where $\theta_{ij}(n)$ is the trade elasticity function defined in (17).

2. The elasticity of n_{ij} to $\bar{r}_{od}$ is a function of $(\sigma, \bar{\theta}(\bar{n}^0), \bar{X}^0)$ and $\varepsilon_{ij}(n_{ij}^0)$:

$$\frac{d \ln n_{ij}}{d \ln \bar{r}_{od}} = \tilde{\Psi}_{ij,od}^z \left(\sigma, \bar{\theta}(\bar{n}^0), \bar{X}^0, \varepsilon_{ij}(n_{ij}^0) \right). \quad (19)$$

Proof. See Appendix A.3.

The first part of the proposition establishes that the elasticity of aggregate outcomes, $\{\{X_{ij}\}_j, P_i, N_i, E_i, w_i\}$, to bilateral trade costs is a function of the initial aggregate trade matrix, $\bar{X}^0$, the elasticity of substitution σ , and the bilateral trade elasticity matrix, $\bar{\theta}(\bar{n}^0)$. Thus, in contrast to Proposition 1, the first part of Proposition 2 indicates that, at least for local responses, separate knowledge of the extensive and intensive margin elasticities – and thus the distribution of firm-specific fundamentals – is not required conditional on knowing the trade elasticity matrix, $\bar{\theta}(\bar{n}^0)$.¹¹ The second part of Proposition 2 explains exactly why computing counterfactual responses for large changes in parameters requires separate knowledge of $\varepsilon_{ij}(n)$ and $\rho_{ij}(n)$. The extensive margin elasticity, $\varepsilon_{ij}(n_{ij}^0)$, pins down the impact of the shock on the exporter firm share, n_{ij} . Along the path to the new equilibrium, changes in n_{ij} then imply changes in the trade elasticity $\theta_{ij}(n_{ij})$ through both the intensive and extensive margins of firm exports – see equation (17). This determines the local responses of aggregate outcomes for any sequence of small shocks (as shown in the first part of the proposition).

Propositions 1 and 2 provide a synthesis of the results in Melitz and Redding (2015), who stress the importance of varying trade elasticities and firm entry, with the results of Arkolakis et al. (2012), who stress the sufficient role of the constant trade elasticity for responses in aggregate outcomes to trade shocks. According to Proposition 2, firm heterogeneity matters to the extent that it controls how much trade shocks affect the trade elasticity through (large) changes in the exporter firm share, ultimately regulated by the functions $\varepsilon_{ij}(n)$ and $\rho_{ij}(n)$.¹²

This point is illustrated by the special case in which the intensive and extensive margin elasticities are constant,

¹¹In Appendix A.3, we show that the same data and elasticity requirements are sufficient to compute the elasticity of aggregate outcomes to changes in population $d \ln \bar{L}_o$ and trade imbalances $d \ln \bar{T}_o$. However, to compute elasticities to changes in fixed cost of exporting $d \ln \bar{f}_{od}$ and entry cost $d \ln \bar{F}_o$, we need also the initial share of the country's labor force employed to cover fixed costs of exporting, which depends on the functions $(\varepsilon_{ij}(n), \rho_{ij}(n))$ and initial firm export shares $\{n_{ij}^0\}_{i,j}$ in equilibrium.

¹²It is also useful to compare Proposition 2 to the results of Arkolakis et al. (2019a) showing that knowledge of the trade elasticity is sufficient, locally, for nonparametric counterfactuals in the case of two symmetric countries. We show that their results hold locally, for many asymmetric countries, but they are not true when we consider large shocks.

$$\rho_{ij}(n) = n^{\varrho_{ij}} \quad \text{and} \quad \epsilon_{ij}(n) = n^{\varepsilon_{ij}}, \quad (20)$$

for $\varrho_{ij} > -1$ and $\varepsilon_{ij} < 0$. This specification is a flexible extension of the constant elasticity Pareto variant of Melitz (2003) studied by Chaney (2008).¹³ In this case, Proposition 2 implies that all aggregate outcomes can be computed by integrating the local responses in (18) without tracking the changes in the exporter firm share n_{ij} (even for large shocks). Thus, firm heterogeneity does not matter for aggregate outcomes conditional on knowing the trade elasticity matrix. We summarize the conclusions of this section under the following remark.

Remark 2. Firm heterogeneity only matters for counterfactual responses in aggregate outcomes to trade shocks through σ and the shape of $(\bar{\epsilon}(\bar{\mathbf{n}}), \bar{\rho}(\bar{\mathbf{n}}))$. For small trade cost shocks, $(\bar{\epsilon}(\bar{\mathbf{n}}), \bar{\rho}(\bar{\mathbf{n}}))$ matter only through their combined effect on the trade elasticity matrix $\bar{\boldsymbol{\theta}}(\bar{\mathbf{n}})$. If $(\bar{\epsilon}(\bar{\mathbf{n}}), \bar{\rho}(\bar{\mathbf{n}}))$ have constant elasticities, then trade elasticities θ_{ij} are constant and sufficient to compute counterfactual responses of aggregate outcomes to trade cost shocks.

3.2 Identification of Changes in Fundamentals

In the preceding discussion, we established how to conduct counterfactual experiments for any given change in economic fundamentals, like changes in productivity and trade costs. An important challenge in conducting these counterfactuals is to measure actual changes in economic fundamentals hitting the world economy. We thus show how to identify such changes in fundamentals from observed aggregate data without imposing parametric restrictions on the distribution of firm fundamentals or symmetry restrictions on trade cost changes.

Given data on n_{ij}^0 and $\{\hat{n}_{ij}^t, \hat{x}_{ij}^t, \hat{w}_i^t\}$ for all i and j , we can invert changes in fixed costs of exporting and relative revenue shifters using $\{\sigma, \epsilon_{ij}(n), \rho_{ij}(n)\}$, along with data on bilateral sales, firm entry, and wages. In particular, equations (11) and (14) immediately yield

$$\hat{f}_{ij}^t = \frac{\hat{x}_{ij}^t}{\rho_{ij}(n_{ij}^0 \hat{n}_{ij}^t) / \rho_{ij}(n_{ij}^0)} \frac{\epsilon_{ij}(n_{ij}^0 \hat{n}_{ij}^t) / \epsilon_{ij}(n_{ij}^0)}{\hat{w}_i^t}, \quad (21)$$

and equation (14) implies that

$$\frac{\hat{r}_{ij}^t}{\hat{r}_{jj}^t} = \frac{\hat{x}_{ij}^t}{\rho_{ij}(n_{ij}^0 \hat{n}_{ij}^t) / \rho_{ij}(n_{ij}^0)} \frac{\rho_{jj}(n_{jj}^0 \hat{n}_{jj}^t) / \rho_{jj}(n_{jj}^0)}{\hat{x}_{jj}^t} \left(\frac{\hat{w}_i^t}{\hat{w}_j^t} \right)^{\sigma-1}. \quad (22)$$

¹³The model in Chaney (2008) entails $r_{ij}(\omega) = \epsilon_{ij}(\omega) = (a_i(\omega))^{\sigma-1}$ and $a_i(\omega) \sim 1 - a^{-\theta}$, leading to $\varrho_{ij} = \varepsilon_{ij} = (\sigma - 1)/\theta$. This implies that average firm exports do not respond to changes in variable trade costs because $\ln \bar{x}_{ij}$ does not depend on $\ln \bar{r}_{ij}$.

Notice that we obtain these expressions without imposing the symmetry of trade cost changes used by [Head and Ries \(2001\)](#). We achieve this more general result at the cost of also requiring data on wage changes.¹⁴ In Appendix A.4, we show that one can use the free entry condition to invert changes in fixed entry costs, $\hat{F}_i$, using the same information.

Intuitively, equation (21) states that fixed entry costs must have increased whenever we observe that the composition-adjusted firm sales, $\frac{\hat{x}_{ij}^t}{\rho_{ij}(n_{ij}^0 \hat{n}_{ij}^t) / \rho_{ij}(n_{ij}^0)}$, has increased relative to the change in the entry cost of marginal firms, $\epsilon_{ij}(n_{ij}^0 \hat{n}_{ij}^t) / \epsilon_{ij}(n_{ij}^0) \hat{w}_i^t$. Similarly, in market j , the revenue shifter of i must have increased more than that of j whenever the composition-adjusted sales of firms from i in j increase more than those of domestic firms.

We use the expressions in (21)–(22) to formally establish the identification of changes in economic fundamentals from observed changes in aggregate outcomes.

Proposition 3. *Consider matrices of exporter firm shares and bilateral trade flows in the initial equilibrium $(\bar{\mathbf{n}}^0, \bar{\mathbf{X}}^0)$, the substitution elasticity σ , and the elasticity functions $(\bar{\epsilon}(\bar{\mathbf{n}}), \bar{\rho}(\bar{\mathbf{n}}))$. Observed changes in $\{\hat{\mathbf{n}}, \hat{\mathbf{x}}, \hat{\mathbf{X}}, \hat{\mathbf{w}}\}$ between two equilibria uniquely identify the set of shocks in fundamentals $\{\hat{\mathbf{T}}, \hat{\mathbf{L}}, \hat{\mathbf{F}}, \hat{\mathbf{f}}, \hat{\mathbf{r}}\}$ with $\hat{r}_{ij} = \hat{r}_{ij} / \hat{r}_{jj}$. Observing also the change in the price index $\hat{P}_j$ in country j between the two equilibria uniquely identifies the domestic revenue shock $\hat{r}_{jj}$ in country j .*

Proof. See Appendix A.4.

It is important to note that the full separation of location specific shocks (e.g. productivity gains) from bilateral revenue shocks (e.g. bilateral trade shocks) requires knowledge of price index changes (as the second part of the proposition shows). Without price index information, the two could not be separated because changes in relative output of countries can be generated by either a uniform change in exporting advantage or different combinations of changes in bilateral revenue shifters – a formal result that follows from the insights of [Eaton et al. \(2016\)](#).¹⁵

3.3 Sufficient Statistics for Welfare Gains

We finally derive expressions for welfare gains caused by shocks to fundamentals, $\{\hat{\mathbf{T}}, \hat{\mathbf{L}}, \hat{\mathbf{F}}, \hat{\mathbf{f}}, \hat{\mathbf{r}}\}$, under trade balance (i.e., $T_i = 0$). When $\hat{f}_{ii} = \hat{r}_{ii} = 1$, our expressions link real wage changes to observable variables and measurable functions, leading to sufficient statistics for the

¹⁴Our result differs from that in [Adao et al. \(2017\)](#) who use an invertible aggregate demand system to nonparametrically recover the effective bilateral prices in each destination – i.e., in our notation, $\hat{r}_{ij} / \hat{r}_{jj}$. We instead combine demand and supply equations to invert fixed costs and relative revenue shifters.

¹⁵See also [Allen et al. \(2014\)](#) and [Costinot et al. \(2010\)](#). Our identification result provides a formal treatment of that insight, extends it to cases where elasticities are not constant, and adapts it to monopolistic competition environments.

welfare consequences of trade shocks. We discuss here the main formulas and present their derivations in Appendix A.5.

We first express changes in the real wage in terms of the change in the share of domestic active firms, $\hat{n}_{ii}$, and the domestic elasticity function, $\epsilon_{ii}(n)$. From (11),

$$\ln \left(\frac{\hat{w}_i}{\hat{P}_i} \right) = \frac{1}{\sigma - 1} \ln \left(\frac{\hat{r}_{ii}}{\hat{f}_{ii}} \right) + \frac{1}{\sigma - 1} \ln \left(\frac{\epsilon_{ii}(n_{ii}\hat{n}_{ii})}{\epsilon_{ii}(n_{ii})} \right). \quad (23)$$

Since $\epsilon_{ii}(n)$ is decreasing and $\sigma > 1$, the real wage increases if, and only if, n_{ii} falls (i.e., $\hat{n}_{ii} < 1$). The second term in this expression illustrates the main new source of gains from trade in Melitz (2003): the consumption-equivalent gain of reallocating resources from domestic firms with a lower entry potential to firms with a higher entry potential. Given the change in the share of domestic active firms $\hat{n}_{ii}$, welfare gains are higher if $\bar{\epsilon}_{ii}(n)$ is more elastic. Intuitively, for a steeper $\epsilon_{ii}(n)$, the difference in entry potential between incumbent and marginal firms is larger, leading to larger reallocation gains.¹⁶

We obtain an alternative welfare formula by combining (11) and (14) to solve for the change in n_{ii} as a function of the changes in domestic spending share x_{ii} and number of entrants N_i . Locally,

$$d \ln \left(\frac{w_i}{P_i} \right) = \frac{1}{\sigma - 1} d \ln \frac{\bar{r}_{ii}}{\bar{f}_{ii}} - \frac{1}{\theta_{ii}(n_{ii})} \left(d \ln \frac{x_{ii}}{N_i} - d \ln \bar{f}_{ii} \right), \quad (24)$$

which can be written in terms of $(\sigma, \rho_{ii}(n), \epsilon_{ii}(n))$ due to the definition of $\theta_{ii}(n)$ in (17).

This expression shows that, for any given $d \ln (x_{ii}/N_i)$, the real wage change is stronger whenever the domestic trade elasticity $\theta_{ii}(n_{ii})$ is lower. Intuitively, the lower trade elasticity implies that it is harder for the economy to substitute consumption from foreign varieties to domestic varieties (through both the extensive and the intensive margins). This amplifies the cost of reducing the spending share on foreign varieties.

This expression is closely related to the welfare formulas in Arkolakis et al. (2009) (footnote 17) and Melitz and Redding (2013) (equation 33). Our characterization shows that the trade elasticity is a function of the observable share of active firms n_{ii} . Thus, for large shocks, the computation of welfare gains must account for the correlation between changes in the domestic trade elasticity and the domestic spending share (normalized by the number of domestic entrants). Such a correlation arises from endogenous changes in the share of domestic active

¹⁶Expression (23) is related to the characterization of the gains from trade in terms of the domestic productivity cutoff in Melitz (2003). Notice however that such characterization lacks empirical analogs due to the lack of measures of the productivity cutoff. Instead, our formula expresses the gains from trade in terms of the change in the share of active domestic firms $\hat{n}_{ii}$ and the extensive margin elasticity function $\epsilon_{ii}(n)$. The next section shows that it is possible to obtain measures of both of these elements.

firms and its implied effect on the domestic trade elasticity.¹⁷ Most importantly, perhaps, the domestic trade elasticity function in equation (24) has a precise empirical content, as we discuss in the next section.

Remark 3. The elasticity of substitution, σ , and the domestic elasticity functions, $\epsilon_{ii}(n)$ and $\rho_{ii}(n)$, can be used to compute nonparametric sufficient statistics for the welfare gains implied by trade shocks.

Gains from Trade. It is possible to use the welfare expressions above to compute the welfare gains of moving from autarky to the trade equilibrium. This requires solving for the counterfactual changes in n_{ii} and N_i . In Appendix A.6, we show that such changes are the solution of a nonlinear system of two equations and two unknowns. This system is a special case of the general system used to compute the nonparametric counterfactual changes in Proposition 1. It depends on three ingredients: (i) data on exporter firm shares and export flows of country i in the initial equilibrium $\{n_{ij}^0, X_{ij}^0\}_j$, (ii) the elasticity of substitution σ , and (iii) the two elasticity functions of firm exports for country i , $\{\epsilon_{ij}(n), \rho_{ij}(n)\}_j$.

3.4 Extensions

Online Supplemental Material A presents five extensions of our baseline framework.

Multiple sectors, multiple factors, input-output links. We extend our baseline model to include multiple factors of production and input-output links between multiple sectors. Specifically, we extend the multi-sector multi-factor gravity model of [Costinot and Rodriguez-Clare \(2013\)](#) in which, as in our baseline, firms in each sector are heterogeneous with respect to productivity, preferences, and variable and fixed trade costs. We restrict all firms in a sector to have the same nested constant elasticity of substitution (CES) production technology that uses multiple factors and multiple sectoral composite goods. In this setting, we derive sector-specific analogs of (11) and (14) that can be used to perform nonparametric counterfactual analysis with respect to trade cost shocks.¹⁸

Allowing for zero bilateral trade flows. We extend our baseline framework to allow for zero trade flows between two countries. As in [Helpman et al. \(2008\)](#), we allow the support

¹⁷As shown in Appendix A.5, expression (24) reduces to the sufficient statistic for the gains from trade in [Arkolakis et al. \(2012\)](#) when the trade elasticity is restricted to be constant –as in our CES benchmark– and also identical across all destinations –as in the Melitz-Pareto baseline. This equivalence arises because N_i does not depend on trade costs in this case.

¹⁸This framework also accommodates nested CES preferences for the goods produced by firms with different observable characteristics – for instance, sector affiliation or country of origin.

of the entry potential distribution to be bounded: $H_{ij}^e(e)$ has full support over $[0, \bar{e}_{ij}]$. The bounded support does not affect the intensive margin gravity equation (14), but it introduces a censoring structure in the extensive margin equation (11). Under the assumption that zero trade flows remain equal to zero, we use these extended gravity equations to compute the model’s counterfactual predictions.

Allowing for import tariffs. Third, we follow [Costinot and Rodriguez-Clare \(2013\)](#) and introduce bilateral import tariffs in the model. In this setting, market clearing and spending must account for the fact that tariff revenue remains in the destination country. We show that the semiparametric gravity equations above still hold, but now bilateral trade costs also include ad-valorem import tariffs. We then characterize the system of equations that determines the model’s counterfactual predictions without parametric distributional assumptions. It depends on the same elements as before, with one addition, the tariff levels in the initial equilibrium.

Multi-product firms. We extend our model to allow firms to produce multiple products as in [Bernard et al. \(2011\)](#). As in the baseline, we allow for an arbitrary distribution of firm-specific fundamentals. In this extension however, firms face a convex labor cost of increasing the number of product varieties that they will sell in each destination (see e.g. [Arkolakis et al. \(2019b\)](#)). This yields three semiparametric gravity equations: two extended versions of the baseline expressions for the extensive and intensive margins of firm exports, and one additional equation for the average number of products sold per firm across destinations. We establish that the elasticity functions in these equations are sufficient to construct counterfactual predictions given the same information required by our baseline setting (i.e., initial trade flows and firm exporter shares, and the elasticity of substitution across varieties).

Non-CES Preferences. We adapt our framework to general Marshallian demand functions that can be written as a function of the destination’s price aggregator and income level. Our demand system subsumes the settings in [Arkolakis et al. \(2019a\)](#) and [Matsuyama and Ushchev \(2017\)](#). In this case, we abstract from fixed entry costs and incorporate endogenous firm entry through a choke price in demand. We show how to extend our inversion argument to derive an extensive margin gravity equation analogous to the one in (11). Because revenue and entry potentials are identical, the same function in the extensive margin gravity equation determines the intensive margin of average firm exports.

4 Estimation Strategy: Semiparametric Gravity

In the previous section, we derived two main insights. First, the functions $\rho_{ij}(n)$ and $\epsilon_{ij}(n)$ are sufficient to summarize how firm heterogeneity affects counterfactual responses of aggregate outcomes to trade shocks. Second, these two functions control semiparametric gravity equations for the extensive and intensive margins of firm exports. This section outlines a strategy to estimate $\rho_{ij}(n)$ and $\epsilon_{ij}(n)$ using these gravity equations. We then use these estimates for counterfactual analysis.

4.1 Semiparametric Gravity Equations of Firm Exports

The gravity equations in (11) and (14) imply the following semiparametric specifications:

$$\ln \epsilon_{ij}(n_{ij}) = \ln (\bar{f}_{ij} \bar{\tau}_{ij}^{\sigma-1}) + \tilde{\delta}_i^\epsilon + \tilde{\zeta}_j^\epsilon, \quad (25)$$

$$\ln \bar{x}_{ij} - \ln \rho_{ij}(n_{ij}) = \ln (\bar{\tau}_{ij}^{1-\sigma}) + \tilde{\delta}_i^\rho + \tilde{\zeta}_j^\rho. \quad (26)$$

where $\tilde{\delta}_i^\epsilon \equiv \ln(\sigma^\sigma(\sigma-1)^{1-\sigma} \bar{a}_i^{1-\sigma} w_i^\sigma)$, $\tilde{\zeta}_j^\epsilon \equiv -\ln(E_j P_j^{\sigma-1})$, $\tilde{\delta}_i^\rho \equiv \ln(\sigma w_i) - \tilde{\delta}_i^\epsilon$, and $\tilde{\zeta}_j^\rho \equiv -\tilde{\zeta}_j^\epsilon$. Without loss of generality, we normalize $\bar{b}_{ij} \equiv 1$ since bilateral shifters of demand and trade costs are isomorphic in the model – i.e., the equilibrium only depend on $\bar{\tau}_{ij}^{1-\sigma} \bar{b}_{ij}$.

These two equations form the basis of our empirical strategy. They link average firm exports and the two functions of the exporter firm share to bilateral shifters of variable and fixed costs of exporting, as well as exporter and importer fixed-effects. We can then estimate $\rho_{ij}(n)$ and $\epsilon_{ij}(n)$ using these equations along with bilateral data on average firm exports, exporter firm shares, and trade cost shifters.

Remark 4. $\rho_{ij}(n)$ and $\epsilon_{ij}(n)$ can be estimated with the semiparametric specifications (25)–(26).

In the rest of this section, we first describe sufficient assumptions on the data generating process that allow us to estimate $\rho_{ij}(n)$ and $\epsilon_{ij}(n)$ using the semiparametric equations in (25) and (26). We then outline an estimator of $\rho_{ij}(n)$ and $\epsilon_{ij}(n)$ based on cross-country variation in bilateral trade cost shifters.

4.2 Data Generating Process

Our goal is to estimate the functions $\rho_{ij}(n)$ and $\epsilon_{ij}(n)$. Throughout our analysis, we use estimates in the literature to calibrate the elasticity of substitution, σ . In particular, we set $\sigma = 3.9$ to match the median estimate in [Hottman et al. \(2016\)](#).

We start by describing the observed and unobserved variables in the economy. For a set of origin-destination pairs (i, j) , we observe the share of firms of i selling in j , n_{ij} , and their average sales, $\bar{x}_{ij}$. We make assumptions on the relationship between the variable trade costs in the model and their observed counterparts (denoted by τ_{ij}). We also assume that there is an exogenous shifter of the variable and fixed trade costs (denoted by z_{ij}).

Assumption 2. *Assume that we observe a component of variable trade costs, τ_{ij} , such that*

$$\ln \bar{\tau}_{ij} = \ln \tau_{ij} + \eta_{ij}^u. \quad (27)$$

Assume also that there exists an observed bilateral trade shifter, z_{ij} , such that

$$\begin{aligned} \ln \bar{\tau}_{ij} &= z_{ij} \kappa^\tau + \delta_i^\tau + \zeta_j^\tau + \eta_{ij}^\tau, \\ \ln \bar{f}_{ij} &= z_{ij} \kappa^f + \delta_i^f + \zeta_j^f + \eta_{ij}^f. \end{aligned} \quad (28)$$

These equations are the first-stage of the estimation of the semiparametric gravity equations. They link variable and fixed trade costs to an observed shifter while accounting for the fact that we may not observe all components of trade costs.¹⁹ Previewing our empirical application, we use data on bilateral freight costs to measure τ_{ij} , and data on bilateral distance to measure z_{ij} .

We further restrict the data generating process of trade shocks.

Assumption 3. *Assume that $E[\eta_{ij}^\tau | z_{ij}, D_{ij}] = E[\eta_{ij}^f | z_{ij}, D_{ij}] = E[\eta_{ij}^u | z_{ij}, D_{ij}] = 0$, where D_{ij} is a vector of origin and destination fixed-effects.*

This orthogonality assumption is the basis of the estimation of constant elasticity gravity equations of international trade flows – for a review, see [Head and Mayer \(2014\)](#). Conditional on origin and destination fixed-effects, the observed shifter must be mean independent from unobserved shifters of trade costs.

Finally, we impose the following restrictions on the functions $\rho_{ij}(n)$ and $\epsilon_{ij}(n)$ that reduce their dimensionality but allow for their flexible estimation.

Assumption 4. *Assume that origin-destination pairs are divided into groups ($g = 1, \dots, G$) such that, for all $(i, j) \in g$,*

$$\begin{bmatrix} \ln \rho_{ij}(n) \\ \ln \epsilon_{ij}(n) \end{bmatrix} = \begin{bmatrix} \ln \rho_g(n) \\ \ln \epsilon_g(n) \end{bmatrix} = \sum_{k=1}^K \begin{bmatrix} \gamma_{g,k}^\rho f_k(\ln n) \\ \gamma_{g,k}^\epsilon f_k(\ln n) \end{bmatrix} \quad (29)$$

¹⁹This specification allows z_{ij} to affect the fixed cost of entering foreign markets and, therefore, it is weaker than the requirement in [Helpman et al. \(2008\)](#) that the instrument cannot affect entry costs.

where $f_k(\ln n)$ denotes restricted cubic splines over knots $k = 1, \dots, K$.

This assumption imposes two types of restrictions on $\rho_{ij}(n)$ and $\epsilon_{ij}(n)$. First, these functions are identical among origin-destination pairs in the same group g . This allows us to estimate $\rho_g(n)$ and $\epsilon_g(n)$ using variation in the observed shifters of trade costs across origin-destination pairs at a point in time.²⁰ In our empirical application, we specify that all countries belong to a single group, so that $\epsilon_{ij}(n) = \epsilon(n)$ and $\rho_{ij}(n) = \rho(n)$ for all i and j . In Supplementary Material B, we provide estimates with multiple country groups defined in terms of characteristics of origin and destination countries.

Second, Assumption 4 specifies a flexible function basis for $\rho_g(n)$ and $\epsilon_g(n)$. We approximate the shape of these functions with a series of restricted cubic polynomials. We specify K knots that form intervals, $\mathcal{U}_k \equiv [u_k, u_{k+1}]$, over which a cubic spline function governs the behavior of the elasticity function – see Appendix B.2. We discuss below how our functional form choice affects the interpretation and generality of our estimates.

4.3 Estimating Moment Conditions

We now construct moment conditions for the estimation of $\rho_g(n)$ and $\epsilon_g(n)$.

Pass-through from observed shifter to variable trade cost. We first specify an equation for the estimation of the pass-through from the observed cost shifter z_{ij} to the observed component of variable trade costs τ_{ij} . Assumption 2, by combining equations (27) and (28), implies that

$$v_{ij}^\tau = \ln \tau_{ij} - z_{ij} \kappa^\tau - \delta_i^\tau - \zeta_j^\tau. \quad (30)$$

Here, $v_{ij}^\tau = \eta_{ij}^\tau - \eta_{ij}^u$, implying that $E[v_{ij}^\tau | z_{ij}, D_{ij}] = 0$ by Assumption 3. We exploit this condition to estimate κ^τ using the linear equation in (30).

Semiparametric gravity equations of firm exports. To estimate $\bar{\rho}_g(n)$ and $\bar{\epsilon}_g(n)$, we show in Appendix A.7 that, under Assumptions 2 and 4, equations (25)–(26) yield

$$\begin{bmatrix} v_{ij}^\epsilon \\ v_{ij}^\rho \end{bmatrix} = \begin{bmatrix} z_{ij} \\ \ln \bar{x}_{ij} + \tilde{\sigma} \kappa^\tau z_{ij} \end{bmatrix} - \sum_{k=1}^K \begin{bmatrix} \kappa^\epsilon \gamma_{g,k}^\epsilon f_k(\ln n) \\ \gamma_{g,k}^\rho f_k(\ln n) \end{bmatrix} - \begin{bmatrix} \delta_i^\epsilon + \zeta_j^\epsilon \\ \delta_i^\rho + \zeta_j^\rho \end{bmatrix}, \quad (31)$$

where $\tilde{\sigma} \equiv \sigma - 1$ and $\kappa^\epsilon \equiv 1 / (\tilde{\sigma} \kappa^\tau + \kappa^f)$.

²⁰Our notation allows groups to be defined as destination-origin country pairs over different years. In this case, one can easily extend our strategy to exploit variation over time in the observed trade cost shifter to obtain bilateral-specific estimates of the elasticity functions.

In terms of the structural unobserved shifters introduced above, $v_{ij}^\epsilon \equiv \kappa^\epsilon(v_{ij}^\rho - \eta_{ij}^f)$ and $v_{ij}^\rho \equiv -\tilde{\sigma}\eta_{ij}^\tau$. Thus, Assumption 3 implies that $E[v_{ij}^\epsilon|z_{ij}, D_{ij}] = E[v_{ij}^\rho|z_{ij}, D_{ij}] = 0$. Combined with (31), these moment conditions can be used to estimate the parameters $\gamma_{g,k}^\rho$ and $\gamma_{g,k}^\epsilon$.²¹

Pass-through from observed shifter to fixed entry cost. To estimate the scale parameter κ^ϵ , we exploit the restriction imposed by the specification of entry costs in terms of labor in the origin country. Under this assumption, we show in Appendix A.7 that

$$v_j^f = \kappa^\epsilon \zeta_j^\rho - \zeta_j^\epsilon. \quad (32)$$

Here, $v_j^f \equiv \kappa^\epsilon \zeta_j^f$ is the destination fixed-effect in the first-stage specification for the entry cost in (28) and, because (28) contains a constant, $E[v_j^f] = 0$. We use this moment condition to estimate κ^ϵ .

Estimator. Expressions (30)–(32) can be used to compute $(v_{ij}^\tau, v_{ij}^\epsilon, v_{ij}^\rho, v_j^f)$ conditional on our main parameters of interest, $\Theta \equiv (\kappa^\epsilon, \kappa^\tau, \{\gamma_{g,k}^\rho, \gamma_{g,k}^\epsilon\}_{g,k=1}^{G,K})$, as well as the set of origin fixed-effects, $\delta \equiv \{\delta_i^\tau, \delta_i^\epsilon, \delta_i^\rho\}_{i=1}^N$ and destination fixed-effects, $\zeta \equiv \{\zeta_i^\tau, \zeta_j^\epsilon, \zeta_j^\rho\}_{j=1}^N$. We use the recovered structural residuals to construct the following GMM estimator for (Θ, δ, ζ) :

$$\min_{(\Theta, \delta, \zeta)} h(\Theta, \delta, \zeta)' \hat{\Omega} h(\Theta, \delta, \zeta), \quad \text{where} \quad h(\Theta, \delta, \zeta) \equiv \begin{bmatrix} \sum_{ij} (v_{ij}^\tau z_{ij}, v_{ij}^\tau D_{ij})' \\ \sum_{ij} (v_{ij}^\epsilon F(z_{ij}), v_{ij}^\epsilon D_{ij})' \\ \sum_{ij} (v_{ij}^\rho F(z_{ij}), v_{ij}^\rho D_{ij})' \\ \sum_j v_j^f \end{bmatrix}, \quad (33)$$

and $\hat{\Omega}$ is the two-step optimal matrix of moment weights and $F(z_{ij})$ is the following function,

$$F(z_{ij}) \equiv \left\{ \mathbb{I}_{(ij \in g)} \mathbb{I}_{(n \in \mathcal{U}_k)} (z_{ij})^d \right\}_{g=1, k=1, d=1}^{G, K, 3}.$$

There are two ways of perceiving our estimation procedure to recover $\rho_g(n)$ and $\epsilon_g(n)$. First, imposing that $\rho_g(n)$ and $\epsilon_g(n)$ are given by the flexible functional form in Assumption 4 implies that identification, consistency, and inference follow from usual results for GMM. As such, identification requires the typical GMM rank condition. Our functional form choice fits in the general class of sieve functions, which can be represented as $h(x) = \sum_{d=1}^D \alpha_d f_d(x)$ where $\{f_d(x)\}_d$ is a set of known functional basis. We choose a series of cubic polynomials

²¹Conditional on observing κ^τ , the assumption that $E[\eta_{ij}^u|z_{ij}, D_{ij}] = 0$ is not necessary for the estimation of $\rho_g(n)$ and $\epsilon_g(n)$ using (31). Accordingly, as in [Adao et al. \(2017\)](#), it is possible to estimate $\rho_g(n)$ and $\epsilon_g(n)$ with the alternative assumption of perfect pass-through from z_{ij} to $\bar{\tau}_{ij}$ (i.e., $\kappa^\tau \equiv 1$).

over intervals, as in [Ryan \(2012\)](#), but restrict the bottom and upper intervals to have a linear slope to improve on the precision of our estimates at extreme values of the support.

Alternatively, the functional form in Assumption 4 can be seen as a functional basis for the nonparametric estimation of $\rho_g(n)$ and $\epsilon_g(n)$. Under this interpretation, our estimator is the sieve nonparametric instrumental variable (NPIV) estimator in [Chen and Qiu \(2016\)](#), [Chen and Christensen \(2018\)](#), and [Compiani \(2019\)](#). In this case, identification requires the assumption of completeness in [Newey and Powell \(2003\)](#) or, in the case of our model with a linear component, the weaker version of this assumption in [Florens et al. \(2012\)](#).²² [Chen and Christensen \(2018\)](#) derive converge rates and confidence intervals for this type of sieve NPIV estimator. In Appendix B.4, we implement their procedure to compute confidence bands accounting for the fact that the function basis approximates $\rho_g(n)$ and $\epsilon_g(n)$.

4.3.1 The Constant Elasticity Benchmark

To gain intuition for the estimation strategy, we return to the constant elasticity benchmark where $\epsilon_{ij}(n) = \epsilon(n)$ and $\rho_{ij}(n) = \rho(n)$ are log-linear (as in (20)). In this case, the equations in (31) yield two constant-elasticity gravity equations, one for the exporter firm share n_{ij} , and another for the average firm exports $\bar{x}_{ij}$:

$$\begin{aligned}\ln n_{ij} &= \beta^\epsilon z_{ij} + \tilde{\delta}_i^\epsilon + \tilde{\zeta}_j^\epsilon + \tilde{\eta}_{ij}^\epsilon \\ \ln \bar{x}_{ij} &= \beta^\rho z_{ij} + \tilde{\delta}_i^\rho + \tilde{\zeta}_j^\rho + \tilde{\eta}_{ij}^\rho,\end{aligned}\tag{34}$$

where

$$\beta^\epsilon \equiv (\kappa^\epsilon \varepsilon)^{-1} \quad \text{and} \quad \beta^\rho \equiv -(\sigma - 1)\kappa^\tau + \varrho\beta^\epsilon.\tag{35}$$

These expressions illustrate how our estimation strategy identifies the parameters controlling the functions $\epsilon_{ij}(n) = \epsilon(n)$ and $\rho_{ij}(n) = \rho(n)$. The parameter ε of $\epsilon(n)$ is proportional to β^ϵ : the constant gravity elasticity of the bilateral exporter firm share to the bilateral cost shifter (conditional on origin and destination fixed-effects). Given the fact that $\tilde{\eta}_{ij}^\epsilon \equiv \beta^\epsilon \kappa^\epsilon (\eta_{ij}^f + \tilde{\sigma} \eta_{ij}^\tau)$, the consistent estimation of β^ϵ requires the orthogonality between the observed shifter z_{ij} and the unobserved shifters of bilateral trade costs $(\eta_{ij}^f, \eta_{ij}^\tau)$ in (28). This is the same type of assumption necessary for the causal interpretation of the estimates of any gravity equation. Notice that, in order to independently recover ε from the definition of β^ϵ in (35), we need the pass-through of the cost shifter to the fixed entry cost κ^ϵ , which we estimate using the relationship described above between the destination fixed-effects in the

²²The completeness assumption is not testable ([Canay et al., 2013](#)), but it is generically satisfied ([Andrews, 2011](#); [Chen and Christensen, 2018](#)). By imposing that $\rho_g(n)$ and $\epsilon_g(n)$ are bounded, identification can be achieved by the weaker condition of bounded completeness ([Blundell et al., 2007](#)).

two gravity equations.²³

In addition, the parameter ϱ in $\rho(n)$ regulates β^ρ : the constant gravity elasticity of the bilateral average firm exports to the bilateral cost shifter (conditional on origin and destination fixed-effects). Again, because $\tilde{\eta}_{ij}^\rho$ is a linear combination of the unobserved shifters of variable and fixed trade costs in (28), the consistent estimation of β^ρ requires the observed shifter z_{ij} to be orthogonal to $(\eta_{ij}^f, \eta_{ij}^\tau)$. Finally, to separately recover ϱ from β^ρ in (35), we need the pass-through of z_{ij} to the variable entry cost κ^τ , which we estimate from the log-linear specification in (30) with an observable component of variable trade costs, τ_{ij} .

Our GMM estimator in (33) is an extension of the estimator for this log-linear specification. The only difference is that, instead of imposing a unique interval with a log-linear function, we specify that the elasticities are determined by cubic splines that may differ over K intervals: the parameters $\{\gamma_{g,k}^\rho, \gamma_{g,k}^\epsilon\}_{g,k=1}^{G,K}$ governing the functions $\epsilon_{ij}(n) = \epsilon_g(n)$ and $\rho_{ij}(n) = \rho_g(n)$ in Assumption 4. Thus, the parameters controlling $\epsilon_g(n)$ and $\rho_g(n)$ are identified from the fact that the extensive and intensive margin elasticities may differ across country pairs (in the same group) that have different levels of the exporter firm share.

5 Empirical Estimation

We use the strategy above to estimate $\rho_{ij}(n)$ and $\epsilon_{ij}(n)$ using the semiparametric gravity equations for the extensive and intensive margins of firm exports. Our results show how the two elasticity functions of firm exports vary with the exporter firm share. In the next section, we use our estimates to evaluate how much firm heterogeneity matters for computing counterfactual changes in aggregate outcomes following trade shocks.

5.1 Data

Our baseline data source for bilateral trade flows is the 2016 release of the World Input-Output Database (WIOD). It contains domestic sales, X_{ii} , as well as bilateral trade flows, X_{ij} , for 43 countries. The first columns in Table OA.1 in Appendix B.1 presents the list of countries with trade flows in the WIOD. Our sample of countries accounts for 90% of world trade and entails positive bilateral flows for almost all exporter-importer pairs.²⁴

The estimator in equation (33) requires four bilateral variables: (i) the exporter firm share, n_{ij} ; (ii) the average firm revenue, $\bar{x}_{ij}$; (iii) the trade cost shifter, z_{ij} ; and (iv) the

²³In this case, this relationship can be written as $v_j^f = \kappa^\epsilon (\tilde{\zeta}_j^\rho - \varrho \tilde{\zeta}_j^\epsilon) + \beta^\epsilon \tilde{\zeta}_j^\epsilon$ with $E[v_j^f] = 0$.

²⁴This attenuates concerns related to the estimation of gravity equations with zero trade flows, as in Helpman et al. (2008) and Silva and Tenreyro (2006).

observed component of trade costs, τ_{ij} . We now describe how we construct these variables for a subset of the countries in the WIOD in 2012.

We use various sources to construct n_{ij} and $\bar{x}_{ij}$ for a subset of 37 origin countries in the WIOD – for the full list of countries, see columns (2) and (3) of Table OA.1 in Appendix B.1. We construct the data in two steps. We first use the OECD Trade by Enterprise Characteristics (TEC) database to obtain the number of manufacturing firms from i selling in j , N_{ij} for $i \neq j$. For origin-destination pairs not in the OECD TEC database, we obtain N_{ij} from the World Bank Exporter Dynamics Database (EDD). These datasets also contain the total exports of the same set of firms from i exporting to j . We use this information to compute the average revenue of firms from i selling to j , $\bar{x}_{ij}$.²⁵

The second step is the construction of the number of entrants N_i , which is not readily available in national statistics. Together with the number of exporters N_{ij} , we use N_i to construct the exporter firm shares, $n_{ij} = N_{ij}/N_i$.²⁶ We compute the number of entrants as $N_i = N_{ii}/n_{ii}$ where, in country i , N_{ii} is the number of active manufacturing firms and n_{ii} is the survival probability of new manufacturing firms. Our approach assumes that a low survival rate represents a large pool of entrants that pay the sunk entry cost but fail to be productive enough to survive. A high survival rate reflects instead that most firms paying the entry cost are successful in production. To maximize country coverage, we obtain N_{ii} from several datasets: the OECD Demographic Business Statistics (SDBS), the OECD Structural Statistics for Industry and Services (SSIS), and the World Bank Enterprise Surveys. In addition, we obtain n_{ii} from the one-year survival rate of manufacturing firms in the OECD SDBS.²⁷

Our measure of the bilateral trade cost shifter, z_{ij} , is the log of bilateral distance (population-weighted) in the Centre d’Etudes Prospectives et d’Informations Internationales (CEPII). This dataset includes not only distance between countries, but also distances within a country z_{ii} due to the nature of population weighting. We use this information to include observations associated with domestic trade in our baseline sample.

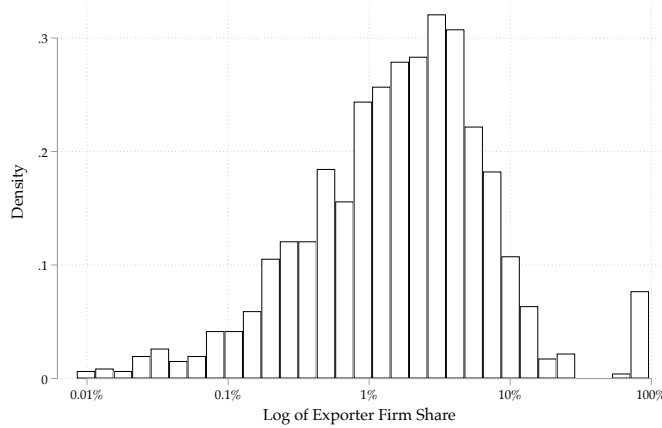
Finally, we use the bilateral freight cost to measure the observed component of variable trade costs τ_{ij} . This is only necessary for the estimation of κ^τ using the linear specification in (30). We consider a subset of countries for which we observe CIF/FOB import margins in

²⁵For Australia and China, we directly construct N_{ij} and $\bar{x}_{ij}$ from national statistic agencies.

²⁶Prior research circumvents this data requirement by assuming that $N_i = N_{ii}$ and $n_{ii} = 1$ – e.g., see [Fernandes et al. \(2019\)](#). However, this limits the potential sources of gains from trade in our model by shutting down welfare gains implied by changes in domestic firm composition – see Section 3.3.

²⁷This data is only available for 80% of the origin countries in our sample. We impute the survival rate for the remaining countries using the simple average of the survival rate for countries with available data. We show that our results are robust to excluding from the sample countries without survival rate data. We also show that our results are similar when we use two-year or three-year survival rates.

Figure 2: Empirical distribution of $\ln n_{ij}$, 2012



Note. Empirical distribution of $\ln(n_{ij})$ in the cross-section of origin-destination pairs in 2012. For the list of countries, see Table OA.1 in Appendix B.1.

the OECD freight cost database. For each country pair, we compute τ_{ij} as the ratio of the total CIF and FOB imports for manufacturing products. Columns (4) and (5) of Table OA.1 in Appendix B.1 report the list of countries with available data on bilateral freight costs.

The availability of data on $\bar{x}_{ij}$, n_{ij} , and z_{ij} determines our sample for the estimation of the last three moment conditions in (33). Table OA.1 in Appendix B.1 reports the list of countries in our baseline sample. Figure 2 summarizes the distribution of $\ln(n_{ij})$ for all bilateral pairs in 2012.²⁸ The empirical distribution of n_{ij} is central for our analysis: because n_{ij} is the only input of the elasticity functions $\rho_g(n)$ and $\epsilon_g(n)$, we are only able to precisely estimate these functions in the part of the support in which we observe values of n_{ij} .

5.2 Pass-Through of Distance to Freight Costs

We start by estimating the pass-through parameter κ^τ from the linear specification in (30). We consider the pooled sample of exporter-importer-year triples in 2008-2014. Table 1 reports our pass-through estimates along with standard errors clustered at the destination-origin level. We estimate an elasticity of trade costs to distance of roughly 0.35. We obtain similar pass-through estimates in the presence of different sets of fixed-effects. This is reassuring given that the fixed-effects absorb a great deal of variation in freight costs in our sample – the R^2 increases from 0.48 in columns (1) to 0.81 in column (3).

²⁸We obtain a similar country coverage for $(\bar{x}_{ij}, n_{ij}, z_{ij})$ in every year between 2010 and 2014. In addition, 2012 is the year with the most observations of the freight cost τ_{ij} used in estimation. In the appendix, we show that our results are similar when we use data for 2010 and 2014.

Table 1: Estimation of κ^τ

	<i>Dep. Var.: Log of Freight Cost</i>		
	(1)	(2)	(3)
Log of Distance	0.349*** (0.057)	0.360*** (0.083)	0.369*** (0.097)
R^2	0.483	0.726	0.813
<u>Fixed-Effects:</u>			
Year	Yes	Yes	No
Origin, Destination	No	Yes	No
Origin-Year, Destination-Year	No	No	Yes

Note. Sample of 547 origin-destination-year triples described in Table OA.1 of Appendix B.1. Standard errors clustered by origin-destination pair. *** $p < 0.01$

5.3 Results: Constant Elasticity Gravity

As a benchmark, Table 2 presents the estimates of ε and ϱ obtained with the GMM estimator in (33) under the constant elasticity assumption in (20). Using (11), our estimate of ε indicates that a 1% increase in bilateral trade costs triggers a change of $(\sigma - 1)/\varepsilon = -3.5\%$ in the exporter firm share (conditional on origin and destination fixed-effects). This is consistent with a skewed distribution of firm entry potentials such that the exporter firm share falls sharply with the increase in cost caused by higher bilateral distance. Our estimate of ρ indicates that a 1% increase in the exporter firm share is associated with a 0.21% decline in the mean revenue potential of exporters, implying that marginal exporters are worse than infra-marginal exporters. The combination of the definition in (17) and our estimates of ε and ϱ implies a trade elasticity of 5, which is within the range of estimates in the literature – see [Costinot and Rodriguez-Clare \(2013\)](#).

As shown in Section 4.3.1, our structural estimates are closely related to the gravity elasticity of the extensive and intensive margin of firm exports to distance, which we report in Table OA.2 of Appendix B.3.1. Columns (1) and (2) indicate that 1% higher distance is associated with declines of $\beta^\varepsilon = 1.2\%$ in exporter firm shares and $\beta^\rho = 0.8\%$ in average firm exports. The extensive margin gravity coefficient is $\beta^\varepsilon = 1/\kappa^\varepsilon\varepsilon$, which yields $\varepsilon = 1.1$ because the ratio of the average destination fixed-effects is $\kappa^\varepsilon \approx 0.75$. The intensive margin gravity coefficient is $\beta^\rho \equiv \varrho\beta^\varepsilon - (\sigma - 1)\kappa^\tau$, which yields $\rho = -0.21$ because we set $\sigma = 3.9$ and estimate $\kappa^\tau = 0.36$ in Table 1. Thus, due to the non-zero intensive margin elasticity, we reject the restriction of $\varrho = \varepsilon$ imposed in the Melitz-Pareto model of [Chaney \(2008\)](#).

Table 2: Constant Elasticity Gravity of Firm Exports with $\epsilon_{ij}(n) = n^\varepsilon$ and $\rho_{ij}(n) = n^\varrho$

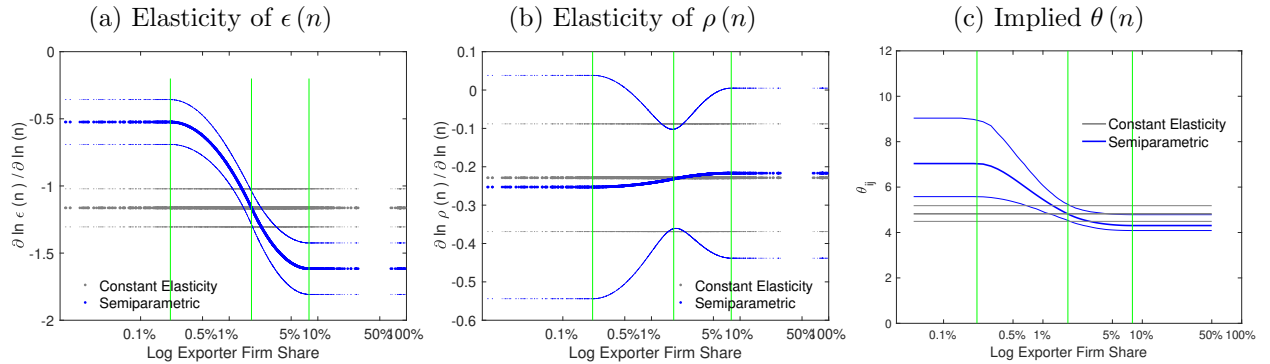
ε	ϱ	θ	κ^ε	κ^τ
-1.16	-0.23	4.82	0.75	0.36
(0.030)	(0.029)	(0.180)	(0.038)	(0.083)

Note. Estimates obtained with GMM estimator in (33) in the 2012 sample of 1,522 origin-destination pairs described in Table OA.1 of Appendix B.1. Calibration of $\tilde{\sigma} = \sigma - 1 = 2.9$ from [Hottman, Redding and Weinstein \(2016\)](#). Standard errors in parenthesis and 95% confidence intervals in brackets. For ϱ , ε and κ^ε , we report robust standard errors implied estimation of (33). For θ , we report standard error obtained from 1,000 bootstrapped draws. For κ_τ , we report standard errors clustered by origin-destination, as in Table 1.

5.4 Results: Semiparametric Gravity

We now turn to our semiparametric estimates of $\epsilon(n)$ and $\rho(n)$ for a single group pooling all countries. Figure 3 presents estimates of the elasticities of $\epsilon(n)$ and $\rho(n)$ with respect to the exporter firm share. We use green bars to denote the estimation knots, and overlay our baseline estimates with the estimates of the constant elasticity specification presented in Table 2. We report the elasticity of $\epsilon(n)$ in Panel (a), the elasticity of $\rho(n)$ in Panel (b), and the implied trade elasticity $\theta(n)$ in Panel (c) (obtained from (17)).

Figure 3: Semiparametric Gravity of Firm Exports with $\epsilon_{ij}(n) = \epsilon(n)$ and $\rho_{ij}(n) = \rho(n)$



Note. Estimates obtained with GMM estimator in (33) in the 2012 sample of 1,522 origin-destination pairs described in Table OA.1 of Appendix B.1. Estimates obtained with a cubic spline over three intervals ($K = 3$) for a single group ($G = 1$). Calibration of $\tilde{\sigma} = \sigma - 1 = 2.9$ from [Hottman, Redding and Weinstein \(2016\)](#). Thick lines are the point estimates and thin lines are the 95% confidence intervals computed with robust standard errors for $\epsilon(n)$ and $\rho(n)$ and 1,000 bootstrap draws for $\theta(n)$.

Our estimates show that the extensive margin elasticity varies with the share of firms exporting to a market. Since $\frac{\partial \ln n_{ij}}{\partial \ln \tau_{ij}}$ is inversely proportional to $\varepsilon(n)$ (see equation (12)), Panel (a) shows that the extensive margin becomes less responsive as more firms serve a market. For low levels of entry, the expression in (12) implies that a 1 log-point increase in trade costs reduces the share of exporting firms by 5.8 log-points. This elasticity is lower for

higher levels of firm entry. In the top knot, a 1 log-point increase in trade costs reduces the exporter firm share by 2.1 log-point. This implies that, at first, exporters are very sensitive to changes in trade frictions. However, high levels of entry potential are rare in the economy: when many firms export, small changes in trade frictions lead to smaller responses in the share of firms that decide to export.

Panel (b) indicates that selection patterns do not change substantially with the exporter firm share. For all levels of n_{ij} , the entry of 1% more exporters into a market induces a reduction in the mean revenue potential of around 0.2%. Thus, for different levels of entry potential, marginal exporters are worse than incumbent exporters in terms of average sales by a similar amount.

Panel (c) shows what the elasticities of the two margins of firm exports imply for the response of bilateral trade flows to changes in bilateral trade costs. Due to the declining extensive margin, the trade elasticity is lower when the exporter firm share is high, falling from 7 in pairs with n_{ij} in the lower knot to 4 in pairs with n_{ij} in the top knot.²⁹ Thus, in line with the product-level evidence in [Kehoe and Ruhl \(2013\)](#) and [Kehoe et al. \(2015\)](#), the trade elasticity tends to be lower when trade volumes are high.

As discussed in Section 4.3.1, our estimation strategy extends the constant elasticity gravity specification widely used in the literature. As such, we obtain variation in the estimated coefficients around the values obtained when we restrict the elasticities to be constant for all values of n_{ij} . The patterns shown in Figure 3 arise because the impact of distance on firm level exports varies across different parts of the support of n_{ij} . In Table OA.3 of Appendix OA.2, we show that the extensive margin elasticity is less sensitive to distance among country pairs with high n_{ij} , while the intensive margin elasticity varies little with n_{ij} . The decreasing the extensive margin elasticity then implies that bilateral trade flows become less sensitive to distance among countries with high values of n_{ij} .

In Figure OA.10 of Appendix B.5.1, we compare our estimated trade elasticity function to that implied by parametric assumptions and their associated estimates about the distribution of firm fundamentals in the existing literature. The log-normal assumption in [Bas et al. \(2017\)](#) and [Head et al. \(2014\)](#) implies a much steeper trade elasticity function. The trade elasticity for high levels of n_{ij} is below two, while it is above twelve for low levels of n_{ij} . The truncated Pareto assumption in [Melitz and Redding \(2015\)](#) yields a trade elasticity function that is uniformly low. It is always below four and falls below two for high levels of n_{ij} . In both cases, the sharp decline in the trade elasticity is driven by strong reductions in $\varepsilon(n_{ij})$

²⁹It is possible that existing trade elasticity estimates are average treatment effects obtained from variation in particular parts of the support of exporter firm shares. Our approach then just captures how the trade elasticity varies across the support of values of n_{ij} .

and $\rho(n_{ij})$ when n_{ij} increases.

These comparisons highlight the difference between our approach based on semiparametric gravity equations and approaches based on parametrizations of cross-section variation in firm-level outcomes. While we directly estimate the elasticity functions driving the model’s aggregate predictions, the parametric micro approach extrapolates from heterogeneity in firm-level outcomes to obtain these elasticity functions. Our results indicate that this extrapolation may lead to elasticity functions that are substantially different from those implied by estimates of the semiparametric gravity equations of firm exports. In the next section, we investigate the quantitative implications of such differences for the model’s counterfactual predictions.

Robustness of baseline estimates. In Appendix B.4, we investigate the robustness of the baseline estimates in Figure 3. We show that confidence intervals are similar when we implement the inference method in [Chen and Christensen \(2018\)](#) that accounts for the fact that the function basis in Assumption 4 approximates the nonparametric functions $\epsilon(n)$ and $\rho(n)$. We show that estimates are similar when (i) we use data for alternative years with similar country coverage, (ii) we exclude observations associated with domestic sales, (iii) we measure n_{ii} using two-year or three-year survival rates, (iv) we exclude from the sample origin countries with imputed survival rates, (v) we assume that all entrants sell in the domestic market (i.e., $n_{ii} = 1$ and $\bar{f}_{ii} = 0$), and (vi) we use a higher elasticity of substitution given by $\sigma = 5$.

Additional estimates for multiple country groups. Our baseline estimates impose identical elasticity functions across all exporter-destination pairs ($G = 1$). In the Online Supplementary Material B, we allow the elasticity functions to vary across country groups. Specifically, we investigate whether the trade elasticity functions vary with the country’s per capita income, as in [Adao et al. \(2017\)](#). For developed origin countries, all elasticity functions are similar to those reported in Figure 3. However, for developing origins, the extensive margin elasticity almost does not vary with n_{ij} , leading to a trade elasticity that is roughly constant around six. We further show that the elasticity estimates do not vary with the destination’s development level. Finally, we also find that the elasticity functions do not differ across country pairs that belong to free trade areas, and share a common language or currency.

6 Quantifying The Importance of Firm Heterogeneity for The Impact of Trade Shocks on Welfare

We conclude by applying our methodology to study the welfare consequences of two types of trade shocks. In Section 6.1, we use the results in Sections 3.1 and 3.3 to quantify the impact of reducing trade costs from infinity to the levels observed in the current trade equilibrium – i.e., the so-called gains from trade. In Section 6.2, we use the results in Section 3.2 to identify the changes in economic fundamentals in the world economy between 2003 and 2012. We then use the inverted shocks to quantify the welfare consequences of good market integration among countries in the European single market. In both cases, we measure the importance of firm heterogeneity for the shock’s impact by comparing results implied by our semiparametric estimates of the gravity equations of firm exports and those implied by the benchmark constant elasticity gravity model of bilateral trade flows.

6.1 Welfare Impact of Moving to Autarky

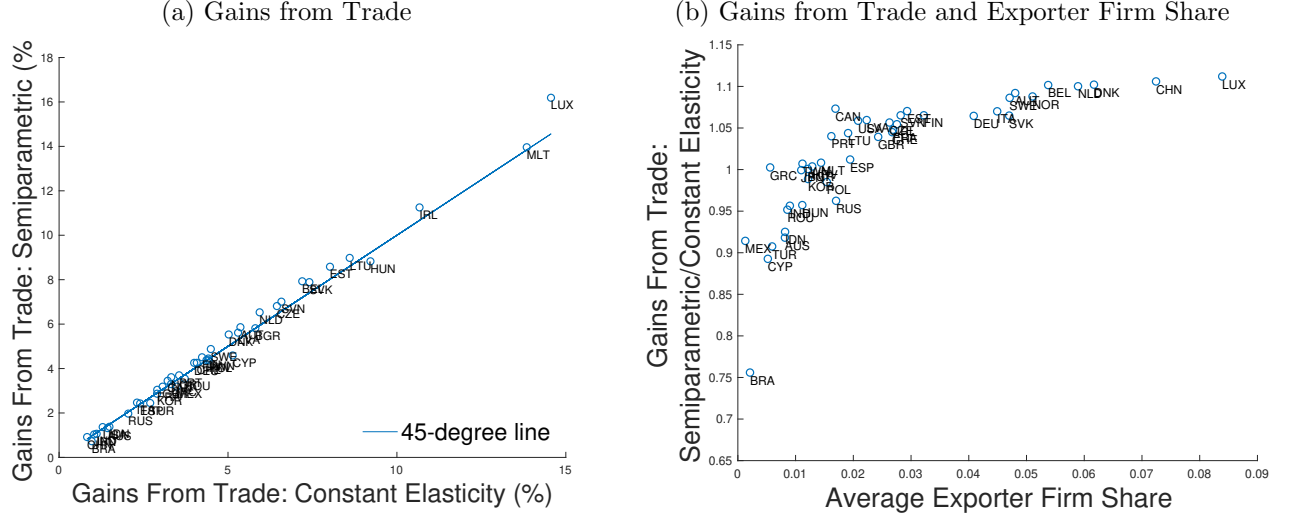
To investigate how firm heterogeneity affects the gains from trade, we consider results under two scenarios. First, the benchmark constant elasticity specification in Table 2, where the gains are given by the aggregate sufficient statistics in [Arkolakis et al. \(2012\)](#), and firm heterogeneity does not play any role for the gains from trade. Second, the semiparametric gravity estimates in Figure 3, where the gains are given by the nonparametric sufficient statistics in Section 3.3, and depend on the export decisions of firms.³⁰

Figure 4a compares the gains under the two scenarios implied by a move from the counterfactual autarky equilibrium to the trade equilibrium of 2012. They yield highly correlated gains from trade – the cross-country correlation is 0.99. As pointed out in Section 3.3, this is a consequence of the fact that the domestic trade share remains an important driver of the gains from trade in our general specification. Notice that firm heterogeneity may still have a substantial impact on the gains from trade of some countries. It yields gains from trade that are roughly 10% higher for Luxembourg, China, Denmark, Belgium and Netherlands. However, the gains are more than 10% lower for Brazil, Mexico, Turkey, India and Australia. Overall, when we account for firm heterogeneity, the absolute average change in the gains from trade is 6%.

In Figure 4b, we investigate how firm heterogeneity affects the ratio between the gains

³⁰We solve $\hat{n}_{ii}$ and $\hat{N}_i$ using the system in Appendix A.6. We have data on n_{ij} for 81% of country pairs in our baseline sample, accounting for 88% world trade flows in 2012. To compute gains from trade, we impute the exporter firm share for the subset of pairs with missing data using estimates of the constant elasticity gravity equations reported in column of (1) of Table OA.2.

Figure 4: Importance of Firm Heterogeneity for the Gains from Trade



Note. Gains from trade is the percentage change in the real wage implied by moving from autarky to the observed equilibrium in 2012. Gains from trade for semiparametric specification computed with the formula in Section 3.3 for $\hat{n}_{ii}$ and $\hat{N}_i$ solving the system in Appendix A.6 and the baseline semiparametric estimates in Figure 3. Gains from trade for constant elasticity specification computed with the formula in Section 3.3 with $\hat{N}_i = 1$ and $\theta_{ii}(n) = \theta$ reported in Table 2.

from trade implied by the semiparametric and the constant elasticity specifications. For each country, we compute this ratio and plot it against the average exporter firm share (i.e., average n_{ij} across $j \neq i$). The use of this statistic is motivated by our welfare gains derivation, equation (23), and a large tradition in international trade that stresses the link between firm heterogeneity and exporter success (see e.g. [Bernard and Jensen \(1999\)](#); [Bernard et al. \(2007\)](#); [Melitz and Redding \(2014\)](#)). The plot shows that firm heterogeneity amplifies the gains from trade in countries with a higher share of exporting firms. Economies of scale drive this relationship. When the fraction of firms exporting is high, more resources are allocated to covering the fixed cost of entering foreign markets. This strengthens competition in the domestic labor market and, therefore, amplifies the decline in domestic survival rate. This in turn creates higher welfare gains, as discussed in Section 3.3.

The Importance of Parametric Assumptions. In Appendix B.5.1, we investigate the quantitative importance of using our semiparametric approach to recover the two elasticity functions that are sufficient for counterfactual analysis. We compare our baseline estimates of the gains from trade to those implied by parametric assumptions about the firm productivity distribution and their associated estimates in the literature – specifically, the Truncated Pareto distribution in [Melitz and Redding \(2015\)](#) and the Log-normal distribution in [Bas et al. \(2017\)](#). Both parametric assumptions have quantitatively large impacts on the gains

from trade. They yield gains from trade that are too large because they imply a low trade elasticity for all levels of the exporter firm share – see Figure OA.10 of Appendix B.5.1. These results indicate that one should be cautious when extrapolating elasticity functions from cross-sectional dispersion in firm-level outcomes.

6.2 The Welfare Impact of Observed Changes in Trade Costs

Our second exercise investigates the welfare consequences of actual changes in trade costs that recently hit the world economy. We use Proposition 3 to recover the changes in fixed costs of exporting $\hat{f}_{ij}$ and revenue shifters $\hat{r}_{ij}$ between 2003 and 2012.³¹ We focus on measuring the welfare consequences of changes in trade costs between countries in the European single market, including the admission of 12 new members that took place in this period. Specifically, we ask “For any country j , how much higher (or lower) would welfare have been in 2012 if fixed costs of exporting and revenue shifters between members of the European single market were those of 2003 rather than those of 2012?”.³²

We first invert the changes in economic fundamentals between the EU-28, Norway, and Switzerland. In our model, they can be recovered from changes in bilateral trade outcomes, wages and prices. In fact, up to a first order approximation, (21)–(22) imply that

$$\begin{aligned}\log \hat{f}_{ij}^t &\approx \log \hat{x}_{ij}^t + (\varepsilon(n_{ij}^0) - \varrho(n_{ij}^0)) \log \hat{n}_{ij}^t - \log \hat{w}_i^t \\ \log \hat{r}_{ij}^t &\approx \log \hat{x}_{ij}^t - \varrho(n_{ij}^0) \log \hat{n}_{ij}^t + (\sigma - 1) \log \hat{w}_i^t + \zeta_j^t\end{aligned}\tag{36}$$

where ζ_j^t is a destination-specific component. Given our baseline estimates, these expressions clearly illustrate what features in the data are associated with changes in economic fundamentals through the lens of our model. The fixed cost of exporting must have increased whenever we observe firm-level exports increasing ($\hat{x}_{ij}^t$ increasing), but firms are not entering foreign markets ($\hat{n}_{ij}^t$ decreasing) or production costs are not increasing ($\hat{w}_i^t$ decreasing). Similarly, we recover an increase in the bilateral revenue shifter whenever we observe increases in either firm

³¹We construct changes in trade outcomes using the same data described in Section 5.1. The inversion of economic fundamentals also requires changes in wages and prices (relative to the economy’s numeraire). We construct these variables using data from the Penn Tables while normalizing variables so that the U.S. wage is the numeraire. Since our model entails a single factor, our wage measure is the output-side real GDP at current PPPs divided by the number of engaged persons. Our price index measure is the corresponding GDP price deflator PPP-adjusted to U.S. prices of 2011. We normalize both variables by the U.S. per-worker GDP in each year.

³²Our exercise differs from that in [Caliendo et al. \(2017\)](#) for two main reasons. First, we focus on the consequences of firm heterogeneity and ignore changes in labor mobility frictions across countries. Second, we consider all changes in economic fundamentals instead of changes in tariffs associated with the EU enlargement, so our exercise accounts for changes in (variable and fixed) trade costs and productivity experienced by both old and new members.

average exports (higher $\hat{x}_{ij}^t$) or wages (higher $\hat{w}_i^t$) given the change in firm composition (lower $\varrho(n_{ij}^0) \log \hat{n}_{ij}^t$). Note that such changes in revenue shifters are proportional to productivity and inversely proportional to variable trade costs.

Table 3 displays the results of regressing inverted changes in bilateral fundamentals on the changes in firm average exports, firm entry share, and origin's labor cost. Not surprisingly, all coefficient estimates are close to the values in the first-order approximation above. These variables explain most of the variation in fundamentals across country pairs: 93% for $\hat{f}_{ij}^t$ and 72% for $\hat{r}_{ij}^t$. More interesting, we use a Shapley decomposition of the R^2 to uncover which variables drive most of the cross-country variation in changes in fundamentals. For fixed exporting costs, the main driver is the growth in firm average exports, with a smaller relevant contribution of firm entry shares. In contrast, growth in firm average exports and labor costs have equal contributions for the variation in the growth of bilateral revenue shifters.

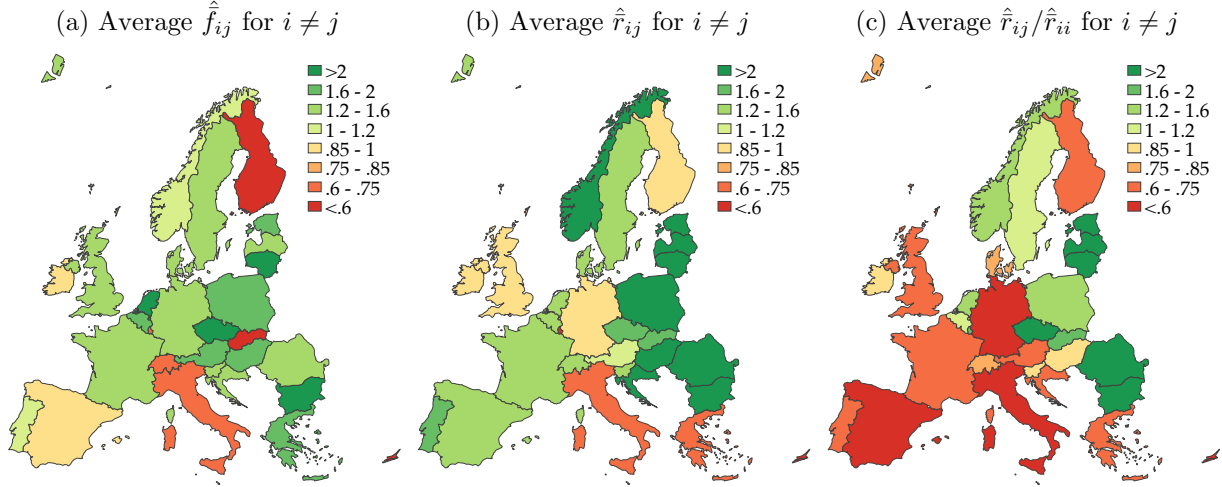
Table 3: Determinants of Changes in Bilateral Shifters

<i>Dep. Var.</i>	$\log \hat{f}_{ij}^t$ (1)	$\log \hat{r}_{ij}^t$ (2)
$\log \hat{x}_{ij}$	0.976*** (0.015) [79.1%]	0.723*** (0.027) [49.6%]
$\log \hat{n}_{ij}$	-0.498*** (0.052) [19.3%]	-0.092* (0.046) [2.1%]
$\log \hat{w}_i$	-0.853*** (0.005) [1.7%]	3.499*** (0.123) [48.3%]
Constant	-0.039** (0.013)	0.753*** (0.037)
R^2	0.934	0.719

Note. Sample of changes in bilateral shifters between 870 country pairs in the European single market (where origin and destination are different). We report the robust standard errors in parenthesis, and the Shapley contribution to the R^2 of each regressor in brackets. * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$

Figure 5 displays each origin's average inverted shock for the 30 countries of our sample in the European single market. Panel (a) shows that, through the lens of our model, firms in most countries had to hire more labor to cover the fixed cost necessary to enter foreign markets. This follows from the fact that, in almost every country, the increase in bilateral average firm exports was much stronger than average growth in firm export shares and wages – see Figure OA.12 in Appendix B.5.2. Panel (b) reports the average change in bilateral revenue shifters for each origin country. This panel shows that revenue shifters increased substantially in most countries mainly because of their strong growth in average firm export

Figure 5: Average Change in Bilateral Shifters of Exporting to Countries in The European Single Market, 2003-2012



Note: Changes in economic fundamentals computed with the expressions in Proposition 3 using changes in outcomes between 2003 and 2012. For each origin country i , we report the simple average of the changes in bilateral fundamentals across all other destination countries j in the European single market (with $i \neq j$).

and wages – see Figure OA.12 in Appendix B.5.2.³³ This growth was particularly strong in Eastern Europe, exceeding 100% in several countries. Notice that, by the definition in (4), $\hat{r}_{ij}$ incorporates changes in both variable trade costs and productivity. To measure only changes in trade costs, panel (c) presents the average change in the normalized bilateral revenue shifter, $\hat{r}_{ij}/\hat{r}_{ii} = \hat{b}_{ij}(\hat{\tau}_{ij})^{1-\sigma}$. Our results indicate that countries in Eastern Europe experienced strong average increases in normalized revenue shifters (and thus reductions in trade costs). In contrast, countries in Western Europe had reductions in their average normalized revenue shifter, indicating that the export growth was particularly weak compared to the domestic performance of firms.

We now turn to the welfare consequences of the changes in bilateral trade shifters between members of the European single market. The top panels of Figure 6 report the negative of the percentage change in the real wage of each country implied by a different set of shocks. A positive (negative) number corresponds to welfare gains (losses). Panel (a) shows that changes in both $\hat{f}_{ij}$ and $\hat{r}_{ij}$ caused welfare gains in all member countries (except Greece). The average gain was 1.7% among all countries. The map also shows that, because of stronger increases in revenue shifters, gains were higher for countries in East Europe that experienced an average gain of 3.4%. The largest gains occurred in Slovakia (4.1%), Lithuania (4.5%), Estonia (5.1%), and Hungary (5.2%).

³³Notable exceptions are Greece and Italy. For these countries, the revenue shifter reduction follows from their weak growth in firm average exports and, more importantly, in wages (relative to the U.S.).

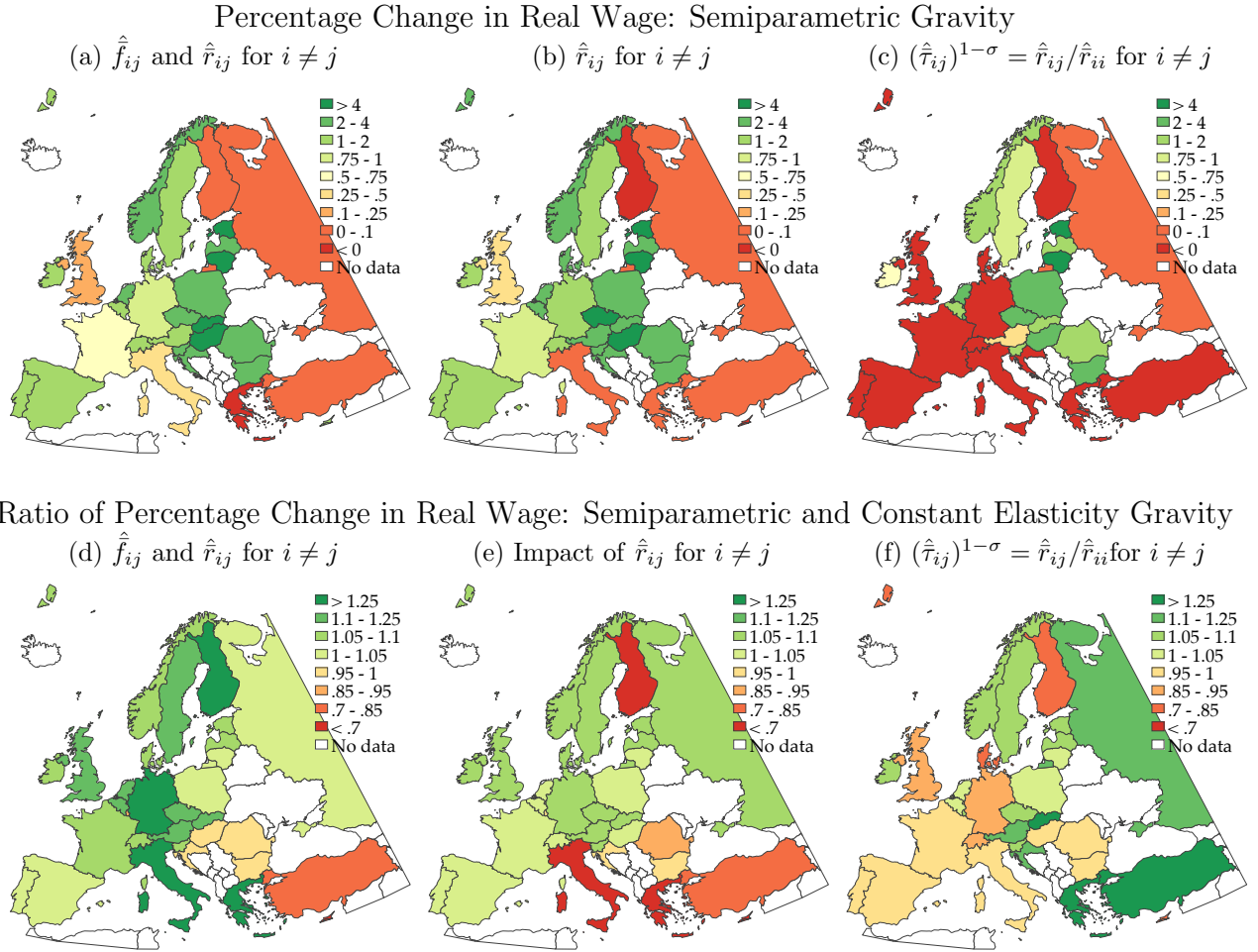
In panel (b), we display the real wage changes only considering the impact of changes in revenue shifters. Because we find that fixed exporting costs tended to increase in this period, welfare gains are typically higher when we do not feed shocks in fixed exporting costs. However, the similarity between panels (a) and (b) indicates that the change in fixed exporting costs had a quantitatively small impact on real wages throughout this period.

Finally, panel (c) reports the impact of changes in variable trade costs. The stark difference between panels (b) and (c) indicates that Western and Eastern Europe had different source of welfare gains. For countries in West Europe, gains are much lower in panel (c) than in panel (b) – the average gain is 0.5% in panel (b) and -0.1% in panel (c). This implies that these countries mostly benefited from productivity gains in their trade partners. In fact, as discussed above, they seem to have experienced average increases in their variable export costs (reduction in normalized revenue shifters). For countries in East Europe, the gains are similar in both panels. This indicates that their gains came mostly from gaining access to the European single market. Interestingly, the only Eastern country to suffer a welfare loss, Croatia, is also the only in country in our sample to join the EU after 2012.

Panels (e)–(f) of Figure 6 conclude our analysis by comparing the real wage change obtained by our baseline model based on the semiparametric estimates in Figure 3 and those obtained by the constant elasticity benchmark in Table 2. We consider the same set of changes in economic fundamentals. For the three sets of shocks, our baseline estimates yield larger gains than those implied by the constant elasticity benchmark: the median ratio is 1.08 in panel (a), 1.04 in panel (b) and 1.04 in panel (c). This difference combines two consequences of the non-linearity in the estimates of the elasticity function: different recovered shocks ($\hat{f}_{ij}$ and $\hat{r}_{ij}$), as well as different firm-level counterfactual responses. In particular, it is mainly driven by the fact that the changes in revenue shifters affect many country pairs for which the firm export share is in the range with a semiparametric trade elasticity above the benchmark constant trade elasticity.

The Importance of Recovering Asymmetric Trade Costs. Appendix B.5.2 investigates the quantitative importance of using our methodology to recover shocks in bilateral trade costs. We consider two alternative approaches in the existing literature. First, we measure trade cost shocks using tariff changes triggered by the EU enlargement, as in [Caliendo et al. \(2017\)](#). Since tariffs are only a fraction of trade costs, this approach yields shocks that are much smaller and more homogeneous (especially for pre-2003 EU members). This translates into much smaller welfare changes, with small gains for all countries. Finally, we consider a non-parametric extension of the approach in [Head and Ries \(2001\)](#) that imposes symmetry of trade cost shocks. This yields smaller welfare losses in West Europe and smaller welfare

Figure 6: The Welfare Impact of Changing Bilateral Shifters of Exporting to Countries in The European Single Market, 2003-2012



Note: Real wage changes caused by changes in bilateral trade shifters between 2003 and 2012 among 30 countries in the European single market. Bilateral trade shifters computed with the expressions in Proposition 3 for observed changes in outcomes between 2003 and 2012. Counterfactual exercise: starting from the equilibrium in 2012, we compute the impact of changing fundamentals back to their level of 2003 (while holding $\hat{r}_{ii} = \hat{f}_{ii} = 1$). Panels (a)–(c) report minus the real wage change obtained with the baseline semiparametric estimates in Figure 3. Panel (d)–(f) report the ratio of real wage changes obtained with the baseline semiparametric estimates in Figure 3 and the constant elasticity estimates in Table 2.

gains in East Europe because the symmetry assumption attenuates the increase (decline) in bilateral trade costs for West (East) Europe shown in Figure 5.

7 Conclusion

We propose a new way of modeling and estimating the aggregate implications of firm heterogeneity in the workhorse monopolistic competition framework of international trade that dispenses parametric restrictions on the distribution of firm fundamentals. We use this approach to revisit a number of open questions about the role of firm heterogeneity for the economy’s response to trade shocks through the lens of a new *not*-parametric point of view.

Instead of focusing on parametrically specifying the distribution of various firm-specific wedges, we show that they can be folded into two elasticity functions in the model’s semi-parametric gravity equations that intuitively shape how trade costs affect firm-level entry and sales across country pairs. Given the initial equilibrium, the different sources of firm heterogeneity, and any associated parametric assumption imposed, only matter for counterfactual predictions through the shape of these two gravity elasticity functions of firm exports. This characterization also allows us to (i) construct nonparametric counterfactual predictions to trade shocks, (ii) nonparametrically recover changes in economic fundamentals from observed trade and macroeconomic outcomes, and (iii) obtain nonparametric ex-post sufficient statistics for the impact of trade shocks on welfare.

Our results indicate that a key new statistic for aggregate gains from trade is the share of exporting firms. We evaluate its impact on the trade elasticity and the welfare impact of trade shocks. We find that firm heterogeneity amplifies both the gains from trade in countries with more exporter firms, and the welfare gains of European good market integration in 2003-2012.

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