

Online Appendix

A Theory Appendix: Proofs and Additional Results

A.1 Proof of Lemma 1

Part 1. The extensive and intensive margins of firm-level sales, n_{ij} and $\bar{x}_{ij}$, satisfy (11) and (14) for all i and j . Together with N_i , they determine bilateral trade flows, $X_{ij} = N_i n_{ij} \bar{x}_{ij}$.

Part 2. For all i , total spending, E_i , satisfies (8).

Part 3. To derive the labor market clearing condition notice that there are three sources of demand for labor: production of goods, fixed-cost of entering a market and fixed-cost of creating a variety. Thus,

$$w_i \bar{L}_i = \sum_j N_i Pr[\omega \in \Omega_{ij}] \left(1 - \frac{1}{\sigma} \right) E[R_{ij}(\omega) | \omega \in \Omega_{ij}] + \sum_j N_i Pr[\omega \in \Omega_{ij}] w_i \bar{f}_{ij} E[f_{ij}(\omega) | \omega \in \Omega_{ij}] + N_i w_i \bar{F}_i$$

From the free entry condition, we know that

$$w_i \bar{F}_i = \sum_j \mathbb{E}[\max\{\pi_{ij}(\omega); 0\}] = \sum_j Pr[\omega \in \Omega_{ij}] \left(\frac{1}{\sigma} E[R_{ij}(\omega) | \omega \in \Omega_{ij}] - w_i \bar{f}_{ij} E[f_{ij}(\omega) | \omega \in \Omega_{ij}] \right),$$

which implies that

$$w_i \bar{L}_i = \sum_j N_i Pr[\omega \in \Omega_{ij}] E[R_{ij}(\omega) | \omega \in \Omega_{ij}].$$

Thus, since $\bar{x}_{ij} \equiv E[R_{ij}(\omega) | \omega \in \Omega_{ij}]$ and $n_{ij} = Pr[\omega \in \Omega_{ij}]$, this immediately implies

$$w_i \bar{L}_i = \sum_j N_i n_{ij} \bar{x}_{ij}. \quad (\text{OA.1})$$

Thus, the only exogenous element in this expression is $\bar{L}_i$.

Part 4. Since $p_{ij}(\omega) = \frac{\sigma}{\sigma-1} \frac{\bar{\tau}_{ij} w_i}{\bar{a}_i} \frac{\tau_{ij}(\omega)}{a_i(\omega)}$, the expression for $P_j^{1-\sigma}$ in (2) implies that

$$P_j^{1-\sigma} = \sum_i \left[\bar{b}_{ij} \left(\frac{\sigma}{\sigma-1} \frac{\bar{\tau}_{ij}}{\bar{a}_i} \right)^{1-\sigma} \right] (w_i^{1-\sigma}) \int_{\Omega_{ij}} (b_{ij}(\omega)) \left(\frac{\tau_{ij}(\omega)}{a_i(\omega)} \right)^{1-\sigma} d\omega$$

Using the definitions in (4), we can write this expression as

$$P_j^{1-\sigma} = \sum_i \bar{r}_{ij} (w_i^{1-\sigma}) \int_{\Omega_{ij}} r_{ij}(\omega) d\omega$$

Notice that $\int_{\Omega_{ij}} r_{ij}(\omega) d\omega = N_i Pr[\omega \in \Omega_{ij}] E[r | \omega \in \Omega_{ij}] = N_i n_{ij} \rho_{ij}(n_{ij})$. This immediately yields

$$P_j^{1-\sigma} = \sum_i \bar{r}_{ij} w_i^{1-\sigma} \rho_{ij}(n_{ij}) n_{ij} N_i. \quad (\text{OA.2})$$

Thus, the only exogenous elements in this expression are σ , $\bar{r}_{ij}$, and $\rho_{ij}(n)$.

Part 5. We start by writing

$$\begin{aligned}\mathbb{E}[\max\{\pi_{ij}(\omega); 0\}] &= Pr[\omega \in \Omega_{ij}]E[\pi_{ij}(\omega) | \omega \in \Omega_{ij}] + Pr[\omega \notin \Omega_{ij}]0 \\ &= Pr[\omega \in \Omega_{ij}] \left(\frac{1}{\sigma} E[R_{ij}(\omega) | \omega \in \Omega_{ij}] - w_i \bar{f}_{ij} E[f_{ij}(\omega) | \omega \in \Omega_{ij}] \right) \\ &= n_{ij} \left(\frac{1}{\sigma} \bar{x}_{ij} - w_i \bar{f}_{ij} E[r_{ij}(\omega)/e_{ij}(\omega) | \omega \in \Omega_{ij}] \right)\end{aligned}$$

where the second equality follows from the expression for $\pi_{ij}(\omega) = (1/\sigma)R_{ij}(\omega) - w_i \bar{f}_{ij} f_{ij}(\omega)$, and the third equality follows from the definitions of $\bar{x}_{ij} \equiv E[R_{ij}(\omega) | \omega \in \Omega_{ij}]$ and $e_{ij}(\omega) \equiv r_{ij}(\omega)/f_{ij}(\omega)$.

By defining $e_{ij}^* \equiv \sigma \frac{\bar{f}_{ij}}{\bar{r}_{ij}} \left[\left(\frac{w_i}{\bar{P}_j} \right)^\sigma \frac{\bar{P}_j}{\bar{E}_j} \right]$, we can write

$$E[r_{ij}(\omega)/e_{ij}(\omega) | \omega \in \Omega_{ij}] = \int_{e_{ij}^*}^{\infty} \frac{1}{e} \left[\int_0^{\infty} r dH_{ij}^r(r|e) \right] \frac{dH^e(e)}{1 - H^e(e_{ij}^*)}$$

Consider the transformation $n = 1 - H_{ij}(e)$ such that $e = \bar{e}_{ij}(n)$. In this case, $dH_{ij}(e) = -dn$ and $n_{ij} = 1 - H_{ij}(e_{ij}^*)$, which implies that

$$E[r_{ij}(\omega)/e_{ij}(\omega) | \omega \in \Omega_{ij}] = \frac{1}{n_{ij}} \int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn,$$

where, to simplify exposition, we define the mean revenue potential of firms in quantile n of the entry potential distribution as

$$\rho_{ij}^m(n) \equiv E[r|e = \epsilon_{ij}(n)]. \quad (\text{OA.3})$$

Thus,

$$\mathbb{E}[\max\{\pi_{ij}(\omega); 0\}] = \frac{1}{\sigma} n_{ij} \bar{x}_{ij} - w_i \bar{f}_{ij} \int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn.$$

Thus, the free entry condition is

$$\sigma w_i \bar{F}_i = \sum_j n_{ij} \bar{x}_{ij} - \sum_j (\sigma w_i \bar{f}_{ij}) \int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn. \quad (\text{OA.4})$$

Notice that the summation of (11) and (14) implies that

$$\ln(\sigma w_i \bar{f}_{ij}) = \ln \bar{x}_{ij} - \ln \rho_{ij}(n_{ij}) + \ln \epsilon_{ij}(n_{ij})$$

which yields

$$\sigma w_i \bar{F}_i = \sum_j n_{ij} \bar{x}_{ij} - \sum_j \bar{x}_{ij} \frac{\epsilon_{ij}(n_{ij})}{\rho_{ij}(n_{ij})} \int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn.$$

By substituting the definition of $\rho_{ij}(n)$, we can write the free entry condition as

$$\sigma w_i \bar{F}_i = \sum_j n_{ij} \bar{x}_{ij} - \sum_j n_{ij} \bar{x}_{ij} \frac{\epsilon_{ij}(n_{ij})}{\int_0^{n_{ij}} \rho_{ij}^m(n) dn} \int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn. \quad (\text{OA.5})$$

Using the market clearing condition in (OA.1), we have that

$$\frac{1}{N_i} = \sigma \frac{\bar{F}_i}{\bar{L}_i} + \sum_j \frac{n_{ij} \bar{x}_{ij}}{w_i \bar{L}_i} \frac{\epsilon_{ij}(n_{ij})}{\int_0^{n_{ij}} \rho_{ij}^m(n) dn} \int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn, \quad (\text{OA.6})$$

which immediately yields

$$N_i = \left[\sigma \frac{\bar{F}_i}{\bar{L}_i} + \sum_j \frac{n_{ij} \bar{x}_{ij}}{w_i \bar{L}_i} \frac{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn}{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ij})} dn} \right]^{-1}. \quad (\text{OA.7})$$

Notice that, by (16), $\rho_{ij}^m(n)$ is uniquely determined by $\rho_{ij}(n)$. Thus, the only exogenous elements in this expression are $\sigma \bar{F}_i$, $\bar{L}_i$, $\epsilon_{ij}(n)$, and $\rho_{ij}(n)$.

Part 6. The equilibrium vector $\{n_{ij}, \bar{x}_{ij}, E_i, w_i, P_i, N_i\}_{i,j}$ is determined by equations (11), (14), (8), (OA.1), (OA.2), and (OA.7). The system is thus a function of the vector of country fundamentals $\{\bar{T}_i, \bar{L}_i, \bar{F}_i, \bar{f}_{ij}, \bar{r}_{ij}\}$, the elasticity of substitution σ , and the bilateral functions, $\{\epsilon_{ij}(n), \rho_{ij}(n)\}_{i,j}$. ■

A.2 Proof of Proposition 1

Part 1. We start by pointing out that equation (16) implies that knowledge of $\rho_{ij}(n)$ implies knowledge of $\rho_{ij}^m(n)$ (as defined in (OA.3)). We then use the equilibrium conditions in Proposition 1 to obtain a system of equations for the changes in $\{n_{ij}, \bar{x}_{ij}\}_j, P_i, N_i, E_i, w_i\}$ given changes in $\{\bar{T}_i, \bar{L}_i, \bar{F}_i, \{\bar{r}_{ij}, \bar{f}_{ij}\}_j\}_i$.

1. The extensive and intensive margins of firm-level sales, n_{ij} and $\bar{x}_{ij}$, in (11) and (14) imply

$$\frac{\epsilon_{ij}(n_{ij} \hat{n}_{ij})}{\epsilon_{ij}(n_{ij})} = \frac{\hat{f}_{ij}}{\hat{r}_{ij}} \left[\left(\frac{\hat{w}_i}{\hat{P}_j} \right)^\sigma \frac{\hat{P}_j}{\hat{E}_j} \right], \quad (\text{OA.8})$$

$$\hat{x}_{ij} = \hat{r}_{ij} \frac{\rho_{ij}(n_{ij} \hat{n}_{ij})}{\rho_{ij}(n_{ij})} \left[\left(\frac{\hat{w}_i}{\hat{P}_j} \right)^{1-\sigma} \hat{E}_j \right]. \quad (\text{OA.9})$$

2. Let $\iota_i \equiv w_i L_i / E_i = (\sum_d X_{id}) / (\sum_o X_{oi})$ be the output-spending ratio in country i in the initial equilibrium. The spending equation in (8) implies

$$\hat{E}_i = \iota_i \left(\hat{w}_i \hat{L}_i \right) + (1 - \iota_i) \hat{T}_i, \quad (\text{OA.10})$$

3. Let $y_{ij} \equiv (N_i n_{ij} \bar{x}_{ij}) / (w_i L_i) = X_{ij} / (\sum_{j'} X_{ij'})$ be the share of i 's revenue from sales to j . The labor market clearing condition in (OA.1) implies

$$\hat{w}_i \hat{L}_i = \sum_j y_{ij} \left(\hat{N}_i \hat{n}_{ij} \hat{x}_{ij} \right). \quad (\text{OA.11})$$

4. The price index (OA.2) implies

$$\begin{aligned} \hat{P}_j^{1-\sigma} &= \sum_i \frac{\bar{r}_{ij} w_i^{1-\sigma} \rho_{ij}(n_{ij}) n_{ij} N_i}{P_j^{1-\sigma}} \left(\hat{r}_{ij} \frac{\rho_{ij}(n_{ij} \hat{n}_{ij})}{\rho_{ij}(n_{ij})} \hat{w}_i^{1-\sigma} \hat{n}_{ij} \hat{N}_i \right) \\ &= \sum_i \frac{\bar{r}_{ij} w_i^{1-\sigma} \rho_{ij}(n_{ij}) n_{ij} N_i E_j P_j^{\sigma-1}}{\sum_o \bar{r}_{oj} w_o^{1-\sigma} \rho_{oj}(n_{oj}) n_{oj} N_o E_j P_j^{\sigma-1}} \left(\hat{r}_{ij} \frac{\rho_{ij}(n_{ij} \hat{n}_{ij})}{\rho_{ij}(n_{ij})} \hat{w}_i^{1-\sigma} \hat{n}_{ij} \hat{N}_i \right) \\ &= \sum_i \frac{\bar{x}_{ij} n_{ij} N_i}{\sum_o \bar{x}_{oj} n_{oj} N_o} \left(\hat{r}_{ij} \frac{\rho_{ij}(n_{ij} \hat{n}_{ij})}{\rho_{ij}(n_{ij})} \hat{w}_i^{1-\sigma} \hat{n}_{ij} \hat{N}_i \right) \end{aligned}$$

Let $x_{ij} \equiv (N_i n_{ij} \bar{x}_{ij}) / (\sum_o \bar{x}_{oj} n_{oj} N_o) = X_{ij} / (\sum_o X_{oj})$ be the spending share of country j on country i . Thus,

$$\hat{P}_j^{1-\sigma} = \sum_i x_{ij} \left(\hat{r}_{ij} \frac{\rho_{ij}(n_{ij} \hat{n}_{ij})}{\rho_{ij}(n_{ij})} \hat{w}_i^{1-\sigma} \hat{n}_{ij} \hat{N}_i \right). \quad (\text{OA.12})$$

5. The free entry condition in (OA.7) implies

$$N_i \hat{N}_i = \left[\sigma \frac{\bar{F}_i}{\bar{L}_i} \frac{\hat{F}_i}{\hat{L}_i} + \sum_j \frac{n_{ij} \bar{x}_{ij}}{w_i \bar{L}_i} \frac{\hat{n}_{ij} \hat{x}_{ij}}{\hat{w}_i \hat{L}_i} \frac{\int_0^{n_{ij} \hat{n}_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn}{\int_0^{n_{ij} \hat{n}_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ij} \hat{n}_{ij})} dn} \right]^{-1}$$

Using (OA.7) to substitute for $\sigma \frac{\bar{F}_i}{\bar{L}_i}$,

$$\hat{N}_i = \left[\left(1 - \sum_j y_{ij} \frac{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn}{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ij})} dn} \right) \frac{\hat{F}_i}{\hat{L}_i} + \sum_j y_{ij} \frac{\hat{n}_{ij} \hat{x}_{ij}}{\hat{w}_i \hat{L}_i} \frac{\int_0^{n_{ij} \hat{n}_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn}{\int_0^{n_{ij} \hat{n}_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ij} \hat{n}_{ij})} dn} \right]^{-1}. \quad (\text{OA.13})$$

A.3 Proof of Proposition 2

Part 1. We start by totally differentiating the equilibrium equations in Lemma 1. Equation (11) implies

$$\varepsilon_{ij}(n_{ij}) d \ln n_{ij} = d \ln \bar{f}_{ij} - d \ln \bar{r}_{ij} + \sigma d \ln w_i - (\sigma - 1) d \ln P_j - d \ln E_j \quad (\text{OA.14})$$

Equation (14) yields

$$d \ln \bar{x}_{ij} = d \ln \bar{r}_{ij} + \varrho_{ij}(n_{ij}) d \ln n_{ij} - (\sigma - 1) d \ln w_i + (\sigma - 1) d \ln P_j + d \ln E_j.$$

The sum of the two equations above implies that

$$d \ln \bar{x}_{ij} = d \ln \bar{f}_{ij} + d \ln w_i + (\varrho_{ij}(n_{ij}) - \varepsilon_{ij}(n_{ij})) d \ln n_{ij} \quad (\text{OA.15})$$

Equation 8 implies that

$$d \ln E_j = \iota_j d \ln w_j + \iota_j d \ln \bar{L}_j + (1 - \iota_j) d \ln \bar{T}_j. \quad (\text{OA.16})$$

By combining the market clearing condition in (OA.1) with the version of the free entry condition in (OA.4), we have that

$$\frac{\sigma \bar{F}_i}{\bar{L}_i} = \frac{1}{N_i} - \sum_j \frac{\sigma \bar{f}_{ij}}{\bar{L}_i} \int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn,$$

which implies that

$$\begin{aligned} -\frac{1}{N_i} d \ln N_i - \frac{\sigma \bar{F}_i}{\bar{L}_i} (d \ln \bar{F}_i / \bar{L}_i) &= \left(\sum_j \frac{\sigma \bar{f}_{ij}}{\bar{L}_i} \int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn \right) (d \ln \bar{f}_{ij} - d \ln \bar{L}_i) + \sum_j \frac{\sigma \bar{f}_{ij}}{\bar{L}_i} \frac{\rho_{ij}^m(n_{ij})}{\epsilon_{ij}(n_{ij})} n_{ij} d \ln n_{ij} \\ &= \sum_j \left(\frac{\bar{x}_{ij}}{w_i \bar{L}_i} \frac{\epsilon_{ij}(n_{ij})}{\rho_{ij}(n_{ij})} \int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn \right) (d \ln \bar{f}_{ij} - d \ln \bar{L}_i) + \sum_j \frac{\bar{x}_{ij} n_{ij}}{w_i \bar{L}_i} \frac{\rho_{ij}^m(n_{ij})}{\rho_{ij}(n_{ij})} d \ln n_{ij} \\ &= \sum_j \left(\frac{n_{ij} \bar{x}_{ij}}{w_i \bar{L}_i} \frac{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn}{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ij})} dn} \right) (d \ln \bar{f}_{ij} - d \ln \bar{L}_i) + \sum_j \frac{\bar{x}_{ij} n_{ij}}{w_i \bar{L}_i} (1 + \varrho_{ij}(n_{ij})) d \ln n_{ij} \\ &= \frac{1}{N_i} \sum_j \left(y_{ij} \frac{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn}{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ij})} dn} \right) (d \ln \bar{f}_{ij} - d \ln \bar{L}_i) + \frac{1}{N_i} \sum_j y_{ij} (1 + \varrho_{ij}(n_{ij})) d \ln n_{ij}, \end{aligned}$$

where the second equality uses $\bar{x}_{ij} = \frac{\rho_{ij}(n_{ij})}{\epsilon_{ij}(n_{ij})} \sigma \bar{f}_{ij} w_i$, the third equality uses (16) and (OA.3), and the fourth uses $y_{ij} \equiv N_i \bar{x}_{ij} n_{ij} / w_i \bar{L}_i$.

Thus,

$$d \ln N_i = -\pi_i d \ln \bar{F}_i / \bar{L}_i - \sum_j y_{ij} \pi_{ij}(n_{ij}) (d \ln \bar{f}_{ij} - d \ln \bar{L}_i) - \sum_j y_{ij} (1 + \varrho_{ij}(n_{ij})) d \ln n_{ij} \quad (\text{OA.17})$$

where

$$\pi_{ij}(n_{ij}) \equiv \frac{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn}{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ij})} dn} \quad (\text{OA.18})$$

and, by equation (OA.6),

$$\pi_i \equiv \sigma \frac{\bar{F}_i N_i}{\bar{L}_i} = 1 - \sum_j y_{ij} \pi_{ij}(n_{ij}). \quad (\text{OA.19})$$

Equation (OA.1) implies

$$d \ln w_i + d \ln \bar{L}_i = \sum_j y_{ij} (d \ln N_i + d \ln n_{ij} + d \ln \bar{x}_{ij}),$$

which combined with (OA.15) implies

$$-d \ln N_i + d \ln \bar{L}_i = \sum_j y_{ij} d \ln \bar{f}_{ij} + \sum_j y_{ij} (1 + \varrho_{ij}(n_{ij}) - \varepsilon_{ij}(n_{ij})) d \ln n_{ij}. \quad (\text{OA.20})$$

The combination of this equation and (OA.17) implies that

$$\sum_j y_{ij} \varepsilon_{ij}(n_{ij}) d \ln n_{ij} = -\pi_i d \ln \bar{F}_i + \sum_j y_{ij} (1 - \pi_{ij}(n_{ij})) d \ln \bar{f}_{ij}. \quad (\text{OA.21})$$

Finally, equation (OA.2) implies

$$(1 - \sigma) d \ln P_j = \sum_i x_{ij} (d \ln \bar{r}_{ij} - (\sigma - 1) d \ln w_i + (1 + \varrho_{ij}(n_{ij})) d \ln n_{ij} + d \ln N_i) \quad (\text{OA.22})$$

Equations (OA.14), (OA.17), (OA.21) and (OA.22) form a system that determines $\{d \ln n_{ij}, d \ln N_i, d \ln P_i, d \ln w_i\}_{i,j}$ for any arbitrary set of shocks. We now establish Part 1 of Proposition 2 by reducing this system to two sets of equations determining $\{d \ln P_i, d \ln w_i\}_i$ in terms of σ , $\{\theta_{ij}(n_{ij}), n_{ij}, X_{ij}\}_{i,j}$. To this end, note that the definition of $\theta_{ij}(n_{ij})$ in (17) implies that $\frac{1 + \varrho_{ij}(n_{ij})}{\varepsilon_{ij}(n_{ij})} = 1 + \frac{\theta_{ij}(n_{ij})}{1 - \sigma}$. Thus, equations (OA.22) and (OA.20) imply

$$\begin{aligned} (1 - \sigma) d \ln P_j &= \sum_i x_{ij} (d \ln \bar{r}_{ij} - (\sigma - 1) d \ln w_i + \varepsilon_{ij}(n_{ij}) d \ln n_{ij}) \\ &+ \sum_i x_{ij} \left[\left(\frac{\theta_{ij}(n_{ij})}{1 - \sigma} \right) \varepsilon_{ij}(n_{ij}) d \ln n_{ij} + d \ln N_i \right] \end{aligned} \quad (\text{OA.23})$$

$$d \ln N_i = d \ln \bar{L}_i - \sum_j y_{ij} d \ln \bar{f}_{ij} + \sum_j y_{ij} \left(\frac{\theta_{ij}(n_{ij})}{\sigma - 1} \right) \varepsilon_{ij}(n_{ij}) d \ln n_{ij}. \quad (\text{OA.24})$$

By substituting the second equation into the first, we get that

$$\begin{aligned}
(1 - \sigma)d \ln P_j &= \sum_i x_{ij} (d \ln \bar{r}_{ij} - (\sigma - 1)d \ln w_i + \varepsilon_{ij}(n_{ij})d \ln n_{ij}) \\
&- \sum_i x_{ij} \left[\left(\frac{\theta_{ij}(n_{ij})}{\sigma - 1} \right) \varepsilon_{ij}(n_{ij})d \ln n_{ij} - \sum_d y_{id} \left(\frac{\theta_{id}(n_{id})}{\sigma - 1} \right) \varepsilon_{id}(n_{id})d \ln n_{id} \right] \\
&+ \sum_i x_{ij} (d \ln \bar{L}_i - \sum_d y_{id} d \ln \bar{f}_{id})
\end{aligned}$$

By substituting (OA.14) into this expression,

$$\begin{aligned}
d \ln E_j - \sum_i x_{ij} d \ln w_i &= \sum_i x_{ij} (d \ln \bar{L}_i + d \ln \bar{f}_{ij} - \sum_d y_{id} d \ln \bar{f}_{id}) \\
&- \sum_i x_{ij} \left(\frac{\theta_{ij}(n_{ij})}{\sigma - 1} \right) (d \ln \bar{f}_{ij} / \bar{r}_{ij} + \sigma d \ln w_i - (\sigma - 1)d \ln P_j - d \ln E_j) \\
&+ \sum_i x_{ij} \sum_d y_{id} \left(\frac{\theta_{id}(n_{id})}{\sigma - 1} \right) (d \ln \bar{f}_{id} / \bar{r}_{id} + \sigma d \ln w_i - (\sigma - 1)d \ln P_d - d \ln E_d)
\end{aligned}$$

Substituting (OA.16) into this expression,

$$\begin{aligned}
\iota_j d \ln w_j - \sum_i x_{ij} d \ln w_i &= -\iota_j d \ln \bar{L}_j - (1 - \iota_j)d \ln \bar{T}_j + \sum_i x_{ij} (d \ln \bar{L}_i + d \ln \bar{f}_{ij} - \sum_d y_{id} d \ln \bar{f}_{id}) \\
&- \sum_i x_{ij} \frac{\theta_{ij}(n_{ij})}{\sigma - 1} (d \ln \bar{f}_{ij} / \bar{r}_{ij} + \sigma d \ln w_i - (\sigma - 1)d \ln P_j - \iota_j d \ln w_j) \\
&+ \sum_i x_{ij} \sum_d y_{id} \frac{\theta_{id}(n_{id})}{\sigma - 1} (d \ln \bar{f}_{id} / \bar{r}_{id} + \sigma d \ln w_i - (\sigma - 1)d \ln P_d - \iota_d d \ln w_d) \\
&+ \sum_i x_{ij} \left(\frac{\theta_{ij}(n_{ij})}{\sigma - 1} (\iota_j d \ln \bar{L}_j + (1 - \iota_j)d \ln \bar{T}_j) - \sum_d y_{id} \frac{\theta_{id}(n_{id})}{\sigma - 1} (\iota_d d \ln \bar{L}_d + (1 - \iota_d)d \ln \bar{T}_d) \right)
\end{aligned}$$

Thus, for market i ,

$$\sum_j v_{ij}^p d \ln P_j - \sum_j v_{ij}^w d \ln w_j = d \ln r_i^p - d \ln f_i^p + d \ln L_i^p + d \ln T_i^p \quad (\text{OA.25})$$

$$\begin{aligned}
v_{ij}^w &\equiv 1[i = j] \left(1 - \sum_j \frac{x_{ji} \theta_{ji}(n_{ji})}{\sigma - 1} \right) \iota_i \\
&+ \left[(\sum_o x_{oi} y_{oj} \theta_{oj}(n_{oj})) \left(\frac{\iota_j}{\sigma - 1} \right) - x_{ji} \left(1 - \frac{\sigma}{\sigma - 1} (\theta_{ji}(n_{ji}) - \sum_d y_{jd} \theta_{jd}(n_{jd})) \right) \right]
\end{aligned} \quad (\text{OA.26})$$

$$v_{ij}^p \equiv 1[i = j] \left(\sum_o x_{oi} \theta_{oi}(n_{oi}) \right) - \left(\sum_o x_{oi} y_{oj} \theta_{oj}(n_{oj}) \right) \quad (\text{OA.27})$$

$$d \ln r_i^p \equiv \sum_j x_{ji} \left(\frac{\theta_{ji}(n_{ji})}{1 - \sigma} d \ln \bar{r}_{ji} - \sum_d y_{jd} \frac{\theta_{jd}(n_{jd})}{1 - \sigma} d \ln \bar{r}_{jd} \right) \quad (\text{OA.28})$$

$$d \ln f_i^p \equiv \sum_j x_{ji} \left[\left(1 - \frac{\theta_{ji}(n_{ji})}{\sigma - 1} \right) d \ln \bar{f}_{ji} - \sum_d y_{jd} \left(1 - \frac{\theta_{jd}(n_{jd})}{\sigma - 1} \right) d \ln \bar{f}_{jd} \right] \quad (\text{OA.29})$$

$$d \ln L_i^p \equiv \left(1 - \sum_j x_{ji} \frac{\theta_{ji}(n_{ji})}{\sigma - 1} \right) \iota_i d \ln \bar{L}_i - \sum_j x_{ji} \left(d \ln \bar{L}_j - \sum_d y_{jd} \frac{\theta_{jd}(n_{jd})}{\sigma - 1} \iota_d d \ln \bar{L}_d \right) \quad (\text{OA.30})$$

$$d \ln T_i^p \equiv \left(1 - \sum_j x_{ji} \frac{\theta_{ji}(n_{ji})}{\sigma - 1} \right) (1 - \iota_i) d \ln \bar{T}_i + \sum_j x_{ji} \left(\sum_d y_{jd} \frac{\theta_{jd}(n_{jd})}{\sigma - 1} (1 - \iota_d) d \ln \bar{T}_d \right) \quad (\text{OA.31})$$

Equations (OA.21) and (OA.14) imply

$$\begin{aligned}
\sum_j y_{ij} (d \ln \bar{f}_{ij} - d \ln \bar{r}_{ij} + \sigma d \ln w_i - (\sigma - 1)d \ln P_j) &- \sum_j y_{ij} (\iota_j d \ln w_j + \iota_j d \ln \bar{L}_j + (1 - \iota_j)d \ln \bar{T}_j) \\
&= -\pi_i d \ln \bar{F}_i + \sum_j y_{ij} (1 - \pi_{ij}(n_{ij})) d \ln \bar{f}_{ij}
\end{aligned}$$

Thus,

$$\begin{aligned} \sigma d \ln w_i - \sum_j y_{ij} \iota_j d \ln w_j - \sum_j y_{ij} (\sigma - 1) d \ln P_j &= \sum_j y_{ij} d \ln \bar{r}_{ij} - \sum_j y_{ij} \pi_{ij} (n_{ij}) d \ln \bar{f}_{ij} \\ &\quad - \pi_i d \ln \bar{F}_i + \sum_j y_{ij} (1 - \iota_j) d \ln \bar{T}_j \\ &\quad + \sum_j y_{ij} \iota_j d \ln \bar{L}_j \end{aligned}$$

Thus,

$$\sum_j m_{ij}^w d \ln w_j - \sum_j m_{ij}^p d \ln P_j = d \ln r_i^w - d \ln f_i^w + d \ln F_i^w + d \ln T_i^w + d \ln L_i^w \quad (\text{OA.32})$$

$$m_{ij}^w \equiv 1[i = j] \sigma - y_{ij} \iota_j \quad (\text{OA.33})$$

$$m_{ij}^p \equiv y_{ij} (\sigma - 1) \quad (\text{OA.34})$$

$$d \ln r_i^w \equiv \sum_j y_{ij} d \ln \bar{r}_{ij}, \quad \text{and} \quad d \ln f_i^w \equiv \sum_j y_{ij} \pi_{ij} (n_{ij}) d \ln \bar{f}_{ij} \quad (\text{OA.35})$$

$$d \ln F_i^w \equiv -\pi_i d \ln \bar{F}_i, \quad d \ln T_i^w \equiv \sum_j y_{ij} (1 - \iota_j) d \ln \bar{T}_j, \quad d \ln L_i^w \equiv \sum_j y_{ij} \iota_j d \ln \bar{L}_j \quad (\text{OA.36})$$

Let us use bold letters to denote vectors, $\mathbf{v} = [v_i]_i$ and bold bar variables to denote matrices, $\bar{\mathbf{v}} = [v_{ij}]_{i,j}$. Thus, equations (OA.25)–(OA.32) imply

$$\begin{aligned} \bar{\mathbf{v}}^p d \ln \mathbf{P} - \bar{\mathbf{v}}^w d \ln \mathbf{w} &= d \ln \boldsymbol{\psi}^p \\ -\bar{\mathbf{m}}^p d \ln \mathbf{P} + \bar{\mathbf{m}}^w d \ln \mathbf{w} &= d \ln \boldsymbol{\psi}^w \end{aligned}$$

where

$$\begin{aligned} d \ln \psi_i^p &\equiv d \ln r_i^p - d \ln f_i^p + d \ln L_i^p + d \ln T_i^p \\ d \ln \psi_i^w &\equiv d \ln r_i^w - d \ln f_i^w + d \ln L_i^w + d \ln T_i^w + d \ln F_i^w \end{aligned}$$

We then use the first equation to solve for the price index change,

$$d \ln \mathbf{P} = (\bar{\mathbf{v}}^p)^{-1} (\bar{\mathbf{v}}^w d \ln \mathbf{w} + d \ln \boldsymbol{\psi}^p), \quad (\text{OA.37})$$

which we then substitute into the second equation to obtain,

$$\left[\bar{\mathbf{m}}^w - \bar{\mathbf{m}}^p (\bar{\mathbf{v}}^p)^{-1} \bar{\mathbf{v}}^w \right] d \ln \mathbf{w} = d \ln \boldsymbol{\psi}^w + \bar{\mathbf{m}}^p (\bar{\mathbf{v}}^p)^{-1} d \ln \boldsymbol{\psi}^p. \quad (\text{OA.38})$$

Notice that, because of the numeraire choice, solving (OA.38) requires dropping one row and one column by setting $d \ln w_n = 0$ for some arbitrary country n .

Recall that $\{X_{ij}\}_{i,j}$ immediately yields $\{\iota_j, x_{ij}, y_{ij}\}_{i,j}$. Thus, the system (OA.37)–(OA.38) determines $\{d \ln P_i, d \ln w_i\}_i$ as a function of shocks. Since $(\bar{\mathbf{m}}^w, \bar{\mathbf{m}}^p, \bar{\mathbf{v}}^w, \bar{\mathbf{v}}^p)$ and $(d \ln \mathbf{r}^p, d \ln \mathbf{r}^w)$ depend only on σ , $\{\theta_{ij}(n_{ij}), X_{ij}\}_{i,j}$, and $\{d \ln \bar{r}_{ij}\}_{i,j}$, $\frac{d \ln w_i}{d \ln \bar{r}_{od}}$ and $\frac{d \ln P_i}{d \ln \bar{r}_{od}}$ are functions of $(\sigma, \{\theta_{km}(n_{km})\}_{km}, \{X_{km}\}_{km})$. By (OA.16), $\frac{d \ln E_i}{d \ln \bar{r}_{od}}$ is also a function of $(\sigma, \{\theta_{km}(n_{km})\}_{km}, \{X_{km}\}_{km})$. For the other shocks, it is also necessary to know the share of country i 's labor force allocated to cover fixed costs of exporting to j , $\{\pi_{ij}(n_{ij}^0)\}_{i,j}$, which immediately yields π_i as defined in (OA.19).

To obtain changes in the number of entrants, we combine equations (OA.24) and (OA.14):

$$d \ln N_i = d \ln \bar{L}_i - \sum_j y_{ij} d \ln \bar{f}_{ij} + \sum_j y_{ij} \left(\frac{\theta_{ij}(n_{ij})}{\sigma - 1} \right) (d \ln \bar{f}_{ij} / \bar{r}_{ij} + \sigma d \ln w_i - (\sigma - 1) d \ln P_j - d \ln E_j). \quad (\text{OA.39})$$

This implies that $d \ln N_i$ is a function of $\{d \ln \bar{f}_{ij}, d \ln \bar{r}_{ij}\}_j$, σ , $\{\theta_{ij}(n_{ij})\}_j$, $\{d \ln P_j, d \ln w_j, d \ln E_j\}_j$, and $\{X_{ij}\}_{ij}$. Thus, given that $\frac{d \ln w_i}{d \ln \bar{r}_{od}}$, $\frac{d \ln P_i}{d \ln \bar{r}_{od}}$ and $\frac{d \ln E_i}{d \ln \bar{r}_{od}}$ are functions of $(\sigma, \{\theta_{km}(n_{km})\}_{km}, \{X_{km}\}_{km})$, $\frac{d \ln N_i}{d \ln \bar{r}_{od}}$ is a function of $(\sigma, \{\theta_{km}(n_{km})\}_{km}, \{X_{km}\}_{km})$.

Finally,

$$\begin{aligned} d \ln X_{ij} &= d \ln N_i + d \ln n_{ij} + d \ln \bar{x}_{ij} \\ &= d \ln N_i + d \ln w_i + (1 + \varrho_{ij}(n_{ij}) - \varepsilon_{ij}(n_{ij})) d \ln n_{ij} \\ &= d \ln N_i + d \ln \bar{f}_{ij} + d \ln w_i - \theta_{ij}(n_{ij}) \frac{\varepsilon_{ij}(n_{ij})}{\sigma - 1} d \ln n_{ij} \end{aligned}$$

where the first equality follows from $X_{ij} \equiv N_i n_{ij} \bar{x}_{ij}$, the second equality follows from (OA.15), and the third equality follows from the definition of $\theta_{ij}(n_{ij})$ in (17).

Using (OA.14),

$$d \ln X_{ij} = d \ln N_i + d \ln \bar{f}_{ij} + d \ln w_i - \left(\frac{\theta_{ij}(n_{ij})}{\sigma - 1} \right) (d \ln \bar{f}_{ij} / \bar{r}_{ij} + \sigma d \ln w_i - (\sigma - 1) d \ln P_j - d \ln E_j) \quad (\text{OA.40})$$

Thus, since $\{\frac{d \ln w_i}{d \ln \bar{r}_{od}}, \frac{d \ln P_i}{d \ln \bar{r}_{od}}, \frac{d \ln E_i}{d \ln \bar{r}_{od}}, \frac{d \ln N_i}{d \ln \bar{r}_{od}}\}_j$ are functions of $(\sigma, \{\theta_{km}(n_{km})\}_{km}, \{X_{km}\}_{km})$, $\frac{d \ln X_{ij}}{d \ln \bar{r}_{od}}$ is a function of $(\sigma, \{\theta_{km}(n_{km})\}_{km}, \{X_{km}\}_{km})$.

Part 2. Equation (OA.14) immediately implies that

$$d \ln n_{ij} = \frac{1}{\varepsilon_{ij}(n_{ij})} (d \ln \bar{f}_{ij} - d \ln \bar{r}_{ij} + \sigma d \ln w_i - (\sigma - 1) d \ln P_j - d \ln E_j)$$

Since $\{\frac{d \ln w_i}{d \ln \bar{r}_{od}}, \frac{d \ln P_i}{d \ln \bar{r}_{od}}, \frac{d \ln E_i}{d \ln \bar{r}_{od}}\}_j$ are functions of $(\sigma, \{\theta_{km}(n_{km})\}_{km}, \{X_{km}\}_{km})$, then $\frac{d \ln n_{ij}}{d \ln \bar{r}_{od}}$ is a function of $(\sigma, \{\theta_{km}(n_{km})\}_{km}, \{X_{km}\}_{km})$ and $\varepsilon_{ij}(n_{ij})$.

A.4 Proof of Proposition 3

Assume that we observe trade outcomes in the initial equilibrium, $\{n_{ij}^0, X_{ij}^0\}_{i,j}$, and changes in wages and trade outcomes between two equilibria, $\{\hat{w}_i^t, \hat{n}_{ij}^t, \hat{x}_{ij}^t, \hat{X}_{ij}^t\}_{i,j}$. We can immediately compute $y_{ij}^0 = X_{ij}^0 / \sum_d X_{id}^0$, $\iota_i^0 = (\sum_d X_{id}^0) / (\sum_o X_{oi}^0)$, $\hat{N}_i^t = \hat{X}_{ij}^t / \hat{x}_{ij}^t \hat{n}_{ij}^t$, $\hat{L}_i = \sum_j y_{ij}^0 \hat{X}_{ij}^t / \hat{w}_i^t$, $\hat{E}_i^t = \sum_i x_{ij}^0 \hat{X}_{ij}^t$, $\hat{\iota}_i = \hat{w}_i^t \hat{L}_i^t / \hat{E}_i^t$ and $\hat{T}_i^t = \hat{E}_i^t (1 - \iota_i^0) / (1 - \iota_i^0)$.

The first step of the proof is to establish the set of fundamental changes that rationalizes changes in bilateral trade outcomes and observed wages. In the main text we discuss the identification of $\hat{f}_{ij}$ and $\hat{r}_{ij}^t \equiv \hat{r}_{ij}^t / \hat{r}_{jj}^t$ – see equations (21) and (22). Given $\{n_{ij}^0, y_{ij}^0\}_j$ and $\{\hat{w}_i^t, \hat{n}_{ij}^t, \hat{x}_{ij}^t, \hat{X}_{ij}^t\}_j$, equation (OA.13) for origin i implies that

$$\hat{F}_i^t = \left[\frac{\hat{L}_i^t}{\hat{N}_i^t} - \sum_j y_{ij}^0 \frac{\hat{n}_{ij}^t \hat{x}_{ij}^t}{\hat{w}_i^t} \frac{\int_0^{n_{ij}^0 \hat{n}_{ij}^t} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn}{\int_0^{n_{ij}^0 \hat{n}_{ij}^t} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ij}^0 \hat{n}_{ij}^t)} dn} \right] \left(1 - \sum_j y_{ij}^0 \frac{\int_0^{n_{ij}^0} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn}{\int_0^{n_{ij}^0} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ij}^0)} dn} \right)^{-1} \quad (\text{OA.41})$$

The second step of the proof is to show that, given $\{n_{ij}^0, X_{ij}^0\}_{i,j}$ in the initial equilibrium, the observed

changes $\{\hat{w}_i^t, \hat{n}_{ij}^t, \hat{x}_{ij}^t, \hat{X}_{ij}^t\}_{i,j}$ are the equilibrium changes in our economy when

$$\hat{f}_{ij} = \hat{f}_{ij}^t \quad \forall i, j; \quad \hat{r}_{ij} = \hat{r}_{ij}^t \quad \forall i \neq j; \quad \hat{r}_{ii} = 1 \quad \forall i; \quad \hat{F}_i = \hat{F}_i^t \quad \forall i; \quad \hat{L}_i = \hat{L}_i^t; \quad \hat{T}_i = \hat{T}_i^t \quad \forall i. \quad (\text{OA.42})$$

To this end, we use (OA.8)–(OA.13) to write the conditions that determine the equilibrium vector $\{\hat{w}_i, \hat{n}_{ij}, \hat{x}_{ij}, \hat{N}_i, \hat{E}_j\}_{i,j}$ for the set of exogenous shocks in OA.42. By substituting $\hat{f}_{ij}^t$ in expression (21) into (OA.8) and (OA.9), $\hat{n}_{ij}$ and $\hat{x}_{ij}$ must satisfy

$$\frac{\epsilon_{ij}(n_{ij}^0 \hat{n}_{ij})}{\epsilon_{ij}(n_{ij}^0)} \frac{\hat{x}_{ij}}{\hat{w}_i} \frac{\rho_{ij}(n_{ij}^0)}{\rho_{ij}(n_{ij}^0 \hat{n}_{ij})} = \frac{\hat{x}_{ij}^t}{\hat{w}_i^t} \frac{\epsilon_{ij}(n_{ij}^0 \hat{n}_{ij}^t)}{\epsilon_{ij}(n_{ij}^0)} \frac{\rho_{ij}(n_{ij}^0)}{\rho_{ij}(n_{ij}^0 \hat{n}_{ij}^t)} \quad (\text{OA.43})$$

By substituting $\hat{r}_{ij} = \hat{r}_{ij}^t$ in expression (22) into the ratio of equation (OA.9) for pairs (i, j) and (j, j) , $\hat{x}_{ij}/\hat{x}_{jj}$ must satisfy

$$\frac{\hat{x}_{ij}}{\hat{x}_{jj}} \left(\frac{\hat{w}_i}{\hat{w}_j} \right)^{\sigma-1} \frac{\rho_{ij}(n_{ij}^0)}{\rho_{jj}(n_{jj}^0)} \frac{\rho_{jj}(n_{jj}^0 \hat{n}_{jj})}{\rho_{ij}(n_{ij}^0 \hat{n}_{ij})} = \frac{\hat{x}_{ij}^t}{\hat{x}_{jj}^t} \left(\frac{\hat{w}_i^t}{\hat{w}_j^t} \right)^{\sigma-1} \frac{\rho_{ij}(n_{ij}^0)}{\rho_{jj}(n_{jj}^0)} \frac{\rho_{jj}(n_{jj}^0 \hat{n}_{jj}^t)}{\rho_{ij}(n_{ij}^0 \hat{n}_{ij}^t)} \quad (\text{OA.44})$$

By substituting $\hat{r}_{ij} = \hat{r}_{ij}^t$ in expression (22) and $\hat{r}_{jj} = 1$ into equation (OA.9) for (j, j) , $\hat{x}_{jj}$ must satisfy

$$\hat{x}_{jj} = \hat{x}_{jj}^t \frac{\rho_{jj}(n_{jj}^0 \hat{n}_{jj}) \left[(\hat{w}_j)^{1-\sigma} \hat{E}_j \right]}{\sum_i x_{ij}^0 \left(\frac{\hat{x}_{ij}^t}{\hat{w}_j^t} \right)^{\sigma-1} \frac{\rho_{jj}(n_{jj}^0 \hat{n}_{jj}^t)}{\rho_{ij}(n_{ij}^0 \hat{n}_{ij}^t)} \rho_{ij}(n_{ij}^0 \hat{n}_{ij}) \hat{w}_i^{1-\sigma} \hat{n}_{ij} \hat{N}_i} \quad (\text{OA.45})$$

By substituting $\hat{F}_{ij}$ in expression (OA.41) into (OA.13), $\hat{N}_i$ must satisfy

$$\hat{N}_i = \left[\frac{1}{\hat{N}_i^t} - \sum_j y_{ij}^0 \frac{\hat{n}_{ij}^t \hat{x}_{ij}^t}{\hat{w}_i^t \hat{L}_i^t} \frac{\int_0^{n_{ij}^0 \hat{n}_{ij}^t} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn}{\int_0^{n_{ij}^0 \hat{n}_{ij}^t} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ij}^0 \hat{n}_{ij}^t)} dn} + \sum_j y_{ij}^0 \frac{\hat{n}_{ij}^t \hat{x}_{ij}^t}{\hat{w}_i^t \hat{L}_i^t} \frac{\int_0^{n_{ij}^0 \hat{n}_{ij}^t} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn}{\int_0^{n_{ij}^0 \hat{n}_{ij}^t} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ij}^0 \hat{n}_{ij}^t)} dn} \right]^{-1} \quad (\text{OA.46})$$

From (OA.10) and (OA.11), $\hat{E}_i$ and $\hat{w}_i$ must satisfy

$$\hat{E}_i = \iota_i^0 \left(\hat{w}_i \hat{L}_i^t \right) + (1 - \iota_i^0) \hat{T}_i^t, \quad (\text{OA.47})$$

$$\hat{w}_i \hat{L}_i = \sum_j y_{ij}^0 \left(\hat{N}_i \hat{n}_{ij} \hat{x}_{ij} \right). \quad (\text{OA.48})$$

We now verify that the system (OA.43)–(OA.48) is satisfied by $\{\hat{w}_i^t, \hat{n}_{ij}^t, \hat{x}_{ij}^t, \hat{N}_i^t, \hat{E}_j^t\}_{i,j}$ where $\hat{N}_i^t = \hat{X}_{ij}^t / \hat{x}_{ij}^t \hat{n}_{ij}^t$ and $\hat{E}_i^t = \sum_i x_{ij}^0 \hat{X}_{ij}^t$. It is straight forward to check that (OA.43), (OA.44) and (OA.46) are satisfied for $\{\hat{w}_i^t, \hat{n}_{ij}^t, \hat{x}_{ij}^t, \hat{N}_i^t, \hat{E}_j^t\}$. Since $\hat{E}_i^t = \sum_i x_{ij}^0 \hat{X}_{ij}^t$, equation (OA.45) holds:

$$\frac{\hat{x}_{jj}^t}{\sum_i x_{ij}^0 \left(\frac{\hat{x}_{ij}^t}{\hat{w}_j^t} \right)^{\sigma-1} \frac{\rho_{jj}(n_{jj}^0 \hat{n}_{jj}^t)}{\rho_{ij}(n_{ij}^0 \hat{n}_{ij}^t)} \rho_{ij}(n_{ij}^0 \hat{n}_{ij}^t) (\hat{w}_i^t)^{1-\sigma} \hat{n}_{ij}^t \hat{N}_i^t} \left[(\hat{w}_j^t)^{1-\sigma} \hat{E}_j^t \right] = \frac{\hat{x}_{jj}^t}{\sum_i x_{ij}^0 \left(\frac{\hat{x}_{ij}^t}{\hat{w}_j^t} \right)^{\sigma-1} \frac{\rho_{jj}(n_{jj}^0 \hat{n}_{jj}^t)}{\rho_{ij}(n_{ij}^0 \hat{n}_{ij}^t)} \rho_{ij}(n_{ij}^0 \hat{n}_{ij}^t) (\hat{w}_i^t)^{1-\sigma} \hat{n}_{ij}^t \hat{N}_i^t} \frac{\hat{E}_j^t}{\sum_i x_{ij}^0 \left(\frac{\hat{x}_{ij}^t}{\hat{w}_j^t} \right)^{\sigma-1} \frac{\rho_{jj}(n_{jj}^0 \hat{n}_{jj}^t)}{\rho_{ij}(n_{ij}^0 \hat{n}_{ij}^t)} \rho_{ij}(n_{ij}^0 \hat{n}_{ij}^t) (\hat{w}_i^t)^{1-\sigma} \hat{n}_{ij}^t \hat{N}_i^t} = \hat{x}_{jj}^t.$$

Using the definitions of $\hat{\bar{L}}_i^t$ and $\hat{\bar{T}}_i^t$, we can also show that equations (OA.47) and (OA.48) hold:

$$\begin{aligned} \iota_i^0 \hat{w}_i^t \hat{\bar{L}}_i^t + (1 - \iota_i^0) \hat{\bar{T}}_i^t &= \iota_i^0 \hat{w}_i^t \hat{\bar{L}}_i^t + (1 - \iota_i^0) \frac{\hat{E}_i^t}{1 - \iota_i^0} \left(1 - \iota_i^0 \frac{\hat{w}_i^t \hat{\bar{L}}_i^t}{\hat{E}_i^t} \right) = \hat{E}_i^t \\ \sum_j y_{ij}^0 \left(\hat{N}_i^t \hat{n}_{ij}^t \hat{x}_{ij}^t \right) &= \sum_j y_{ij}^0 \hat{X}_{ij}^t = \hat{w}_i^t \frac{\sum_j y_{ij}^0 \hat{X}_{ij}^t}{\hat{w}_i^t} = \hat{w}_i^t \hat{\bar{L}}_i^t. \end{aligned}$$

Notice that, by the definition of $\hat{N}_i^t$, $\hat{X}_{ij}^t = \hat{N}_i^t \hat{n}_{ij}^t \hat{x}_{ij}^t$. Thus, given $\{n_{ij}^0, X_{ij}^0\}_{i,j}$ in the initial equilibrium, $\{\hat{w}_i^t, \hat{n}_{ij}^t, \hat{x}_{ij}^t, \hat{X}_{ij}^t\}_{i,j}$ is an equilibrium vector of outcome changes implied by the set of exogenous shocks in OA.42.

Finally, we establish the second part of the proposition. Suppose that we also observe the change in the price index for country j , $\hat{P}_j^t$. We now derive the change in the domestic revenue shifter $\hat{r}_{jj}^t$ that generates $\hat{P}_j^t$ given observed trade outcomes in the initial equilibrium $\{n_{ij}^0, X_{ij}^0\}_{i,j}$ and observed changes in wages and trade outcomes between two equilibria, $\{\hat{w}_i^t, \hat{n}_{ij}^t, \hat{x}_{ij}^t, \hat{X}_{ij}^t\}_{i,j}$. Combining equation (OA.12) and $\hat{n}_{ij}^t \hat{N}_i^t = \hat{X}_{ij}^t / \hat{x}_{ij}^t$, we get that

$$\hat{r}_{jj}^t = \frac{\left(\hat{P}_j^t \right)^{1-\sigma}}{\sum_i x_{ij}^0 \left(\hat{r}_{ij}^t \frac{\rho_{ij}(n_{ij}^0, \hat{n}_{ij}^t)}{\rho_{ij}(n_{ij}^0, n_{ij}^0)} (\hat{w}_i^t)^{1-\sigma} \frac{\hat{X}_{ij}^t}{\hat{x}_{ij}^t} \right)} \quad (\text{OA.49})$$

where $\hat{r}_{jj}^t = 1$, $\hat{r}_{ij}^t \equiv \hat{r}_{ij}^t / \hat{r}_{jj}^t$ is given by (22) for $i \neq j$.

A.5 Proof of the expressions in Section 3.3

Equation (23). If $T_i = 0$, then $\hat{E}_j = \hat{w}_j$ and equation (11) for $i = j$ is

$$\frac{\epsilon_{ii}(n_{ii} \hat{n}_{ii})}{\epsilon_{ii}(n_{ii})} = \frac{\hat{f}_{ii}}{\hat{r}_{ii}} \left(\frac{w_i}{P_i} \right)^{\sigma-1},$$

which immediately yields the expression in (23).

Equation (24). For the case of balanced trade with $\iota_i = 1$, equation (11) implies that

$$\varepsilon_{ii}(n_{ii}) d \ln n_{ii} = d \ln \bar{f}_{ii} / \bar{r}_{ii} + (\sigma - 1) d \ln w_i / P_i.$$

Equation (14) implies that

$$d \ln \bar{x}_{ii} = d \ln \bar{r}_{ii} + \varrho_{ii}(n_{ii}) d \ln n_{ii} - (\sigma - 1) d \ln w_i / P_i + d \ln \hat{E}_i.$$

By summing these expressions, we get that

$$d \ln \bar{x}_{ii} = d \ln \bar{f}_{ii} + (\varrho_{ii}(n_{ii}) - \varepsilon_{ii}(n_{ii})) d \ln n_{ii} + d \ln \hat{E}_i$$

Thus,

$$\begin{aligned}
d \ln x_{ii} &= d \ln N_i + d \ln n_{ii} + d \ln \bar{x}_{ii} - d \ln E_i \\
&= d \ln \bar{f}_{ii} + d \ln N_i + (1 + \varrho_{ij}(n_{ij}) - \varepsilon_{ij}(n_{ij})) d \ln n_{ij} \\
&= d \ln \bar{f}_{ii} + d \ln N_i + \frac{(1 + \varrho_{ij}(n_{ij}) - \varepsilon_{ij}(n_{ij}))}{\varepsilon_{ij}(n_{ij})} \varepsilon_{ij}(n_{ij}) d \ln n_{ij}
\end{aligned}$$

Using the fact that $\varepsilon_{ii}(n_{ii}) d \ln n_{ii} = d \ln \bar{f}_{ii} / \bar{r}_{ii} + (\sigma - 1) d \ln w_i / P_i$,

$$d \ln x_{ii} / N_i \bar{f}_{ii} = - \left[(\sigma - 1) \left(1 - \frac{1 + \varrho_{ij}(n_{ij})}{\varepsilon_{ij}(n_{ij})} \right) \right] \left(d \ln w_i / P_i + \frac{1}{\sigma - 1} d \ln \bar{f}_{ii} / \bar{r}_{ii} \right)$$

By the definition of $\theta_{ii}(n_{ii})$ in (17),

$$d \ln x_{ii} / N_i \bar{f}_{ii} = -\theta_{ii}(n_{ii}) \left(d \ln w_i / P_i + \frac{1}{\sigma - 1} d \ln \bar{f}_{ii} / \bar{r}_{ii} \right),$$

which immediately yields the expression in (24).

The constant elasticity benchmark.

Assume that $\varepsilon_{ij}(n) = \varepsilon_i$ and $\varrho_{ij}(n) = \varrho_i$ for all n, i and j . By the definition of θ_{ij} , we immediately get that $\theta_{ij} \equiv \theta_i = (\sigma - 1) \left(1 - \frac{1 + \varrho_i}{\varepsilon_i} \right)$. By equation (16), $\rho_i^m(n) = (1 + \varrho_i(n)) \rho_i(n)$ and, therefore, $\rho_i^m(n) = (1 + \varrho_i) n^{\varrho_i}$. Consider the free entry condition in equation (OA.5):

$$\begin{aligned}
\sigma w_i \bar{F}_i &= \sum_j n_{ij} \bar{x}_{ij} \left(1 - \frac{\int_0^{n_{ij}} n^{\varrho_i - \varepsilon_i} dn}{n_{ij}^{\varepsilon_i} \int_0^{n_{ij}} n^{\varrho_i} dn} \right) \\
&= \sum_j n_{ij} \bar{x}_{ij} \left(1 - \frac{1 + \varrho_i}{1 + \varrho_i - \varepsilon_i} \frac{n_{ij}^{1 + \varrho_i - \varepsilon_i}}{n_{ij}^{1 + \varrho_i}} \right) = \left(\frac{-\varepsilon_i}{1 + \varrho_i - \varepsilon_i} \right) \sum_j n_{ij} \bar{x}_{ij}.
\end{aligned}$$

The market clearing condition in (OA.1) implies that $\sum_j n_{ij} \bar{x}_{ij} = w_i \bar{L}_i / N_i$ and, therefore, $N_i = \frac{\bar{L}_i}{\sigma \bar{F}_i} \left(\frac{-\varepsilon_i}{1 + \varrho_i - \varepsilon_i} \right)$.

A.6 Gains from Trade

We now compute the gains from trade in our model. We assume that $\hat{\tau}_{ij} \rightarrow \infty$ for all $i \neq j$, and that $\hat{a}_i = \hat{\bar{F}}_i = \hat{f}_{ij} = \hat{\tau}_{ii} = \hat{\bar{L}}_i = 1$ for all i and j .

Corollary 1. *Consider an economy moving from the trade equilibrium to the autarky equilibrium with $\hat{T}_i^A = 0$. The change in the real wage is given by (23) where $\hat{n}_{ii}^A$ and $\hat{N}_i^A$ solve*

$$\frac{\epsilon_{ii}(n_{ii} \hat{n}_{ii}^A)}{\epsilon_{ii}(n_{ii})} = \left(\frac{x_{ii}}{\iota_i} \right) \left(\hat{n}_{ii}^A \hat{N}_i^A \right) \frac{\rho_{ii}(n_{ii} \hat{n}_{ii}^A)}{\rho_{ii}(n_{ii})}, \quad (\text{OA.50})$$

$$\left(1 - \sum_j \frac{X_{ij}}{\sum_{j'} X_{ij'}} \frac{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn}{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ij})} dn} \right) \hat{N}_i^A = 1 - \frac{\int_0^{n_{ii} \hat{n}_{ii}^A} \frac{\rho_{ii}^m(n)}{\epsilon_{ii}(n)} dn}{\int_0^{n_{ii} \hat{n}_{ii}^A} \frac{\rho_{ii}^m(n)}{\epsilon_{ii}(n_{ii} \hat{n}_{ii}^A)} dn}. \quad (\text{OA.51})$$

In order to compute the gains from trade using (23), we need to compute changes in n_{ii} when the economy moves to autarky (i.e., $\hat{\tau}_{ij} \rightarrow \infty$ for all $i \neq j$). Equation (OA.50) captures the change in the profitability of the domestic market that determines the change in the domestic survival rate of firms (given $\hat{N}_i^A$). Equation (OA.51) is the free entry condition that determines the change in the number of entrants when the country

moves to autarky. The left hand size of (OA.51) is the profit/revenue ratio (inclusive of entry costs) that firms have in different markets in the initial equilibrium. The right hand size is the profit/revenue ratio that entrants have in the domestic market in the autarky equilibrium.

A.6.1 Proof of Corollary 1

To simplify the notation, we drop the superscript A and use “hat” variables to denote the change from the initial equilibrium to the autarky equilibrium. We assume that $\hat{\tau}_{ij} \rightarrow \infty$ for all $i \neq j$, and that $\hat{a}_i = \hat{F}_i = \hat{f}_{ij} = \hat{\tau}_{ii} = \hat{L}_i = 1$ for all i and j . We set the wage of i to be the numerarie, $w_i \equiv 1$, so that $\hat{w}_i = 1$. Equation (OA.12) implies that

$$\left(\hat{P}_i\right)^{1-\sigma} = x_{ii} \frac{\rho_{ii}(n_{ii}\hat{n}_{ii})}{\rho_{ii}(n_{ii})} \left(\hat{n}_{ii}\hat{N}_i\right) \quad (\text{OA.52})$$

From equation (OA.8), we get that, for all $i \neq j$, $\epsilon_{ij}(n_{ij}\hat{n}_{ij}) \rightarrow \infty$ and, therefore, $\hat{n}_{ij} = 0$. In addition, it implies that

$$\frac{\epsilon_{ii}(n_{ii}\hat{n}_{ii})}{\epsilon_{ii}(n_{ii})} = \frac{\left(\hat{P}_i\right)^{1-\sigma}}{\hat{E}_i} \quad (\text{OA.53})$$

Using the fact that $\hat{E}_i = \iota_i$, (OA.52) and (OA.53) imply that

$$\frac{\epsilon_{ii}(n_{ii}\hat{n}_{ii})}{\epsilon_{ii}(n_{ii})} = \frac{x_{ii}}{\iota_i} \frac{\rho_{ii}(n_{ii}\hat{n}_{ii})}{\rho_{ii}(n_{ii})} \left(\hat{n}_{ii}\hat{N}_i\right). \quad (\text{OA.54})$$

From expression (OA.13),

$$\begin{aligned} \hat{N}_i &= \left[\left(1 - \sum_j y_{ij} \frac{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn}{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ij})} dn} \right) + y_{ii} \hat{n}_{ii} \hat{x}_{ii} \frac{\int_0^{n_{ii}\hat{n}_{ii}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn}{\int_0^{n_{ii}\hat{n}_{ii}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ii}\hat{n}_{ii})} dn} \right]^{-1} \\ 1 &= \hat{N}_i \left(1 - \sum_j y_{ij} \frac{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn}{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ij})} dn} \right) + y_{ii} \hat{N}_i \hat{n}_{ii} \hat{x}_{ii} \frac{\int_0^{n_{ii}\hat{n}_{ii}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn}{\int_0^{n_{ii}\hat{n}_{ii}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ii}\hat{n}_{ii})} dn} \end{aligned}$$

Recall that $\hat{x}_{ii} = \frac{\hat{N}_i \hat{n}_{ii} \hat{x}_{ii}}{\hat{E}_i} = 1/x_{ii}$. Thus, $y_{ii} \hat{N}_i \hat{n}_{ii} \hat{x}_{ii} = y_{ii} \frac{\iota_i}{x_{ii}} = \frac{X_{ii}}{w_i L_i} \frac{E_i}{X_{ii}} \frac{w_i \bar{L}_i}{E_i} = 1$ and, therefore,

$$\left(1 - \sum_j y_{ij} \frac{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn}{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ij})} dn} \right) \hat{N}_i = 1 - \frac{\int_0^{n_{ii}\hat{n}_{ii}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn}{\int_0^{n_{ii}\hat{n}_{ii}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ii}\hat{n}_{ii})} dn}.$$

A.7 Derivation of Equation (31)

By plugging (28) into (11)–(14) we have that

$$\ln \epsilon_{ij}(n_{ij}) = z_{ij}/\kappa^\epsilon + \left(\tilde{\sigma} \eta_{ij}^\tau + \eta_{ij}^f \right) + \left[\ln \sigma w_i \left(\frac{\sigma}{\sigma-1} \frac{w_i}{\bar{a}_i} \right)^{\sigma-1} + \tilde{\sigma} \delta_i^\tau + \delta_i^f \right] - \left[\ln (P_j^{\sigma-1} E_j) - \tilde{\sigma} \zeta_j^\tau - \zeta_j^f \right]$$

$$\ln \bar{x}_{ij} - \ln \rho_{ij}(n_{ij}) = -\tilde{\sigma} \kappa^\tau z_{ij} - \tilde{\sigma} \eta_{ij}^\tau + \left[\ln \left(\frac{\sigma}{\sigma-1} \frac{w_i}{\bar{a}_i} \right)^{1-\sigma} - \tilde{\sigma} \delta_j^\tau \right] + [\ln (P_j^{\sigma-1} E_j) - \tilde{\sigma} \zeta_j^\tau]$$

where $\tilde{\sigma} \equiv \sigma - 1$ and $\kappa^\epsilon \equiv 1/(\tilde{\sigma} \kappa^\tau + \kappa^f)$.

This implies that

$$\begin{aligned} -\kappa^\epsilon \left(\tilde{\sigma} \eta_{ij}^\tau + \eta_{ij}^f \right) &= z_{ij} - \kappa^\epsilon \ln \epsilon_{ij}(n_{ij}) + \kappa^\epsilon \left[\ln \sigma w_i \left(\frac{\sigma}{\sigma-1} \frac{w_i}{\bar{a}_i} \right)^{\sigma-1} + \tilde{\sigma} \delta_i^\tau + \delta_i^f \right] \\ &\quad - \kappa^\epsilon \left[\ln (P_j^{\sigma-1} E_j) - \tilde{\sigma} \zeta_j^\tau - \zeta_j^f \right], \end{aligned}$$

$$-\tilde{\sigma} \eta_{ij}^\tau = \ln \bar{x}_{ij} + \tilde{\sigma} \kappa^\tau z_{ij} - \ln \rho_{ij}(n_{ij}) - \left[\ln \left(\frac{\sigma}{\sigma-1} \frac{w_i}{\bar{a}_i} \right)^{1-\sigma} - \tilde{\sigma} \delta_j^\tau \right] - [\ln (P_j^{\sigma-1} E_j) - \tilde{\sigma} \zeta_j^\tau].$$

We can then write

$$\begin{aligned} v_{ij}^\epsilon &= z_{ij} - \kappa^\epsilon \ln \epsilon_{ij}(n_{ij}) - \delta_i^\epsilon - \zeta_j^\epsilon \\ v_{ij}^\rho &= \ln \bar{x}_{ij} + \tilde{\sigma} \kappa^\tau z_{ij} - \ln \rho_{ij}(n_{ij}) - \delta_i^\rho - \zeta_j^\epsilon \end{aligned} \tag{OA.55}$$

where

$$\begin{aligned} v_{ij}^\epsilon &\equiv -\kappa^\epsilon \left(\tilde{\sigma} \eta_{ij}^\tau + \eta_{ij}^f \right) \quad \text{and} \quad v_{ij}^\rho \equiv -\tilde{\sigma} \eta_{ij}^\tau, \\ \delta_i^\epsilon &\equiv -\kappa^\epsilon \left[\ln \sigma w_i \left(\frac{\sigma}{\sigma-1} \frac{w_i}{\bar{a}_i} \right)^{\sigma-1} + \tilde{\sigma} \delta_i^\tau + \delta_i^f \right] \quad \text{and} \quad \zeta_j^\epsilon \equiv \kappa^\epsilon [\ln (P_j^{\sigma-1} E_j) - \tilde{\sigma} \zeta_j^\tau] - \kappa^\epsilon \zeta_j^f, \\ \delta_i^\rho &\equiv \ln \left(\frac{\sigma}{\sigma-1} \frac{w_i}{\bar{a}_i} \right)^{1-\sigma} - \tilde{\sigma} \delta_j^\tau \quad \text{and} \quad \zeta_j^\rho \equiv \ln (P_j^{\sigma-1} E_j) - \tilde{\sigma} \zeta_j^\tau. \end{aligned}$$

We obtain expression (31) by plugging the functional form assumptions in (29) into (OA.55). Finally, notice that the definitions of ζ_j^ϵ and ζ_j^ρ above immediately imply that

$$\kappa^\epsilon \zeta_j^f = \kappa^\epsilon \zeta_j^\rho - \zeta_j^\epsilon.$$

B Empirical Appendix

B.1 Sample Statistics in Table OA.1

B.2 Restricted Cubic Spline Implementation

We follow [Harrell Jr \(2001\)](#) in setting up our restricted cubic splines. Formally we use a restricted cubic spline with knot values u_k for $k = 1, \dots, K$:

$$\begin{aligned} f_1(\ln n) &= \ln n \\ f_{k+1}(\ln n) &= \frac{(\ln n - \ln u_k)_+^3 - \frac{(\ln n - \ln u_{k-1})_+^3 (\ln u_K - \ln u_{k+1}) - (\ln n - \ln u_k)_+^3 (\ln u_{k-1} - \ln u_k)}{(\ln u_k - \ln u_{k-1})}}{(\ln u_K - \ln u_1)^2}, \end{aligned}$$

with the auxiliary function $(n)_+ = n$ if $n > 0$ and zero otherwise.

Table OA.1: Data Availability

Country Country Name	$\{\bar{x}_{ij}, n_{ij}, z_{ij}\}$		$\{\tau_{ij}\}$		Developed
	Origin	Dest.	Origin	Dest.	
AUS	43	32	3	0	1
AUT	43	37	2	0	1
BEL	43	36	3	0	1
BGR	43	37	3	0	0
BRA	43	31	3	0	0
CAN	41	34	3	0	1
CHE	0	37	0	0	1
CHN	43	37	3	0	0
CYP	24	36	1	0	1
CZE	43	36	2	31	0
DEU	37	37	2	0	1
DNK	43	37	3	0	1
ESP	43	36	3	0	1
EST	43	36	3	0	0
FIN	37	36	3	0	1
FRA	43	37	3	0	1
GBR	43	36	3	0	1
GRC	43	36	3	0	1
HRV	43	29	3	0	0
HUN	37	36	3	0	0
IDN	0	31	0	0	0
IND	0	37	0	0	0
IRL	37	37	3	0	1
ITA	43	36	3	0	1
JPN	0	36	0	0	1
KOR	42	32	3	0	1
LTU	43	35	3	0	0
LUX	35	35	3	0	1
LVA	38	36	3	0	0
MEX	43	36	3	0	0
MLT	42	34	3	0	1
NLD	43	37	3	0	1
NOR	43	36	3	0	1
POL	43	36	2	0	0
PRT	43	36	3	0	1
ROU	43	36	3	0	0
RUS	0	37	0	0	0
SVK	42	36	2	36	0
SVN	42	35	3	0	1
SWE	40	37	3	0	1
TUR	43	36	3	0	0
TWN	0	29	0	0	1
USA	39	37	2	36	1
Count > 0	37	43	37	3	26
Observations	1522		103		

Table OA.2: Constant Elasticity Gravity of Firm Exports

<i>Dep. Var.:</i>	$\ln n_{ij}$	$\ln \bar{x}_{ij}$	$\ln X_{ij}$
	(1)	(2)	(3)
<i>Panel A: Constant elasticity gravity estimation</i>			
Log of to Distan	-1.148*** (0.0478)	-0.778*** (0.0420)	-1.927*** (0.0708)
R^2	0.913	0.691	0.870
<i>Panel B: Implied structural parameters</i>			
	$\kappa^\epsilon \times \epsilon$	ρ	
	-0.84	-0.56	

Note. Sample of 1,524 origin-destination pairs in 2012 – see Table OA.1 of Appendix B.1. All specifications include origin and destination fixed-effects. Implied structural parameters computed with $\tilde{\sigma} = 2.9$ and $\kappa^\tau = 0.36$ from column (3) of Table 1. Standard errors clustered by origin-destination pair. *** $p < 0.01$

In our main specification, we choose $K = 4$ knots. To determine the values u_i , we follow [Harrell Jr \(2001\)](#) and divide the data into the 5th, 35th, 65th, and 95th percentiles.

B.3 The Impact of Distance on the Extensive and Intensive Margins of Firm Exports

B.3.1 Constant-Elasticity Gravity Equations

Table OA.2 presents the estimates of (34). Column (1) indicates that the exporter firm share falls sharply with distance: a 1% higher bilateral distance leads to a 1.2% decline in exporter firm share. Column (2) indicates that average sales also decline with distance. As pointed out by [Fernandes et al. \(2019\)](#), this evidence is inconsistent with the lack of average revenue responses in the Melitz-Pareto model ([Chaney, 2008](#)). Finally, column (3) reports an elasticity of bilateral trade flows to distance of -2 , which is slightly lower than the typical estimates in the literature reviewed by [Head and Mayer \(2014\)](#).

In Panel B of Table OA.2, we use the expressions in (35) to recover $\kappa^\epsilon \epsilon$ and ρ . We use our baseline calibration of $\tilde{\sigma} \kappa^\tau = 1.04$ that sets $\tilde{\sigma} = \sigma - 1 = 2.9$ from [Hottman et al. \(2016\)](#) and $\kappa^\tau = 0.36$ from column (3) of Table 1. The negative extensive margin elasticity implies that $\kappa^\epsilon \epsilon < 0$. Thus, in line with our model, $\epsilon < 0$ whenever distance increases trade costs, $\kappa^\epsilon > 0$. In addition, the implied value of ρ indicates that the average revenue potential of all exporters falls by 0.2% when the exporter firm share increases by 1%. Hence, marginal exporters have a lower revenue potential than incumbent exporters in each market.

B.3.2 Piece-wise Linear Approximation of Baseline Estimates

In this section, we extend the specification in (34) by allowing the coefficients β^ϵ and β^ρ to differ across country pairs in different ranges of the support of n_{ij} . This specification sheds light on how the gravity elasticities of the extensive and intensive margins of firms exports vary with the exporter firm share. Accordingly, it illustrates the main features of the data behind our estimates of the non-linear elasticities reported in Figure 3.

Table OA.3: Constant Elasticity Gravity of Firm Exports

	(1)	(2)	(3)
	$\ln n_{ij}$	$\ln \bar{x}_{ij}$	$\ln X_{ij}$
Log of Distance $\times n_{ij} \in (P_0(n), P_5(n)]$	-0.992*** (0.0360)	-0.701*** (0.0469)	-1.693*** (0.0680)
Log of Distance $\times n_{ij} \in (P_5(n), P_{35}(n)]$	-0.864*** (0.0399)	-0.724*** (0.0479)	-1.588*** (0.0729)
[1em] Log of Distance $\times n_{ij} \in (P_{35}(n), P_{65}(n)]$	-0.778*** (0.0414)	-0.702*** (0.0515)	-1.480*** (0.0774)
Log of Distance $\times n_{ij} \in (P_{95}(n), P_{100}(n)]$	-0.692*** (0.0456)	-0.667*** (0.0575)	-1.358*** (0.0848)
R^2	0.912	0.696	0.886

Note. Sample of 1,524 origin-destination pairs in 2012 – see Table OA.1 of Appendix B.1. All specifications include origin and destination fixed-effects. The four quantiles of n_{ij} are defined by the knots used in the estimation of the semiparametric specification in Figure 3, with $P_c(n)$ denoting percentile c of the empirical distribution of n_{ij} . Standard errors clustered by origin-destination pair. *** $p < 0.01$

Table OA.3 reports estimates of the gravity elasticity over four ranges of the support of n_{ij} defined by the same knots used to in our semi-parametric estimates in Figure 3. Column (1) shows that the extensive margin elasticity is less sensitive to distance among country pairs with high n_{ij} . In contrast, column (2) indicates that the intensive margin elasticity is roughly constant across country pairs with different levels of n_{ij} . Finally, column (3) shows that, because of the declining extensive margin elasticity, bilateral trade flows become less sensitive to distance among countries with high values of n_{ij} .

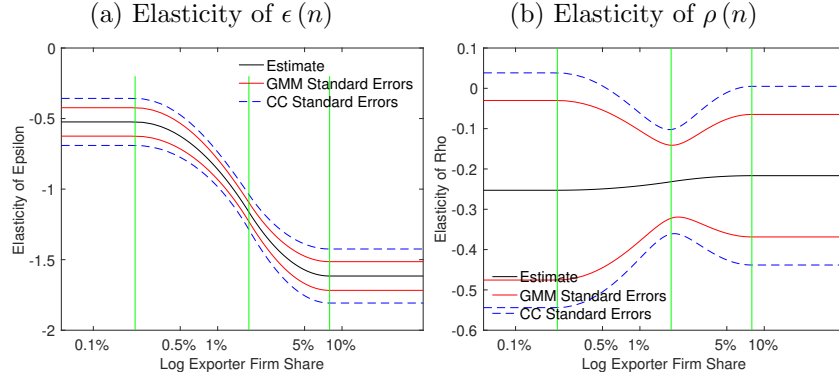
B.4 Robustness of Baseline Estimates in Section 5.4

In this section, we investigate the robustness of the baseline estimates of $\epsilon(n)$ and $\rho(n)$ presented in Section 5.4. First, we implement the inference procedure in [Chen and Christensen \(2018\)](#) for the estimation of NPIV sieve estimators. Second, we show that results are similar when we use data for different years that have a similar country coverage. Third, we show that results are similar when we ignore observations associated with domestic sales. Fourth, we investigate how our results depend on the assumptions used to compute the number of domestic entrants, N_i . Fifth, we report our estimates with alternative calibrations of the elasticity of substitution, σ .

B.4.1 Alternative Inference

In this section, we investigate the robustness of our standard estimates of $\epsilon(n)$ and $\rho(n)$. We estimate standard errors with a criterion value estimated using the bootstrapped procedure in [Chen and Christensen \(2018\)](#). This method accounts for the fact that the function basis used in estimation are intended to approximate the true nonparametric functions $\epsilon(n)$ and $\rho(n)$. Our results show that the confidence intervals implied by the method in [Chen and Christensen \(2018\)](#) are similar to those implied by the robust standard errors of the parameter estimates.

Figure OA.1: Semiparametric Gravity of Firm Exports – Alternative Inference Procedure

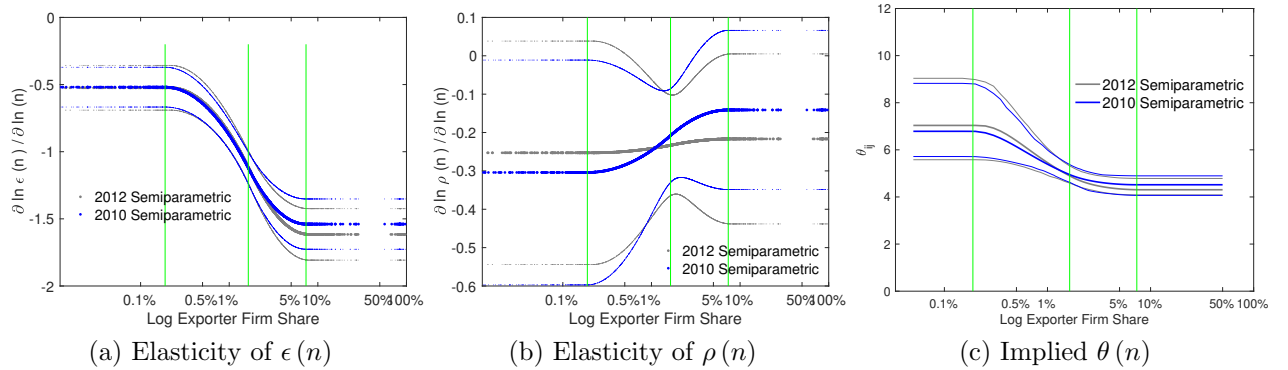


Note. Estimates obtained with GMM estimator in (33) in the 2012 sample of 1,522 origin-destination pairs described in Table OA.1 of Appendix B.1. Estimates obtained with a cubic spline over three intervals ($K = 3$) for a single group ($G = 1$). Calibration of $\tilde{\sigma} = 2.9$. Thick lines are the point estimates and thin lines are the 95% confidence intervals computed with robust standard errors for $\epsilon(n)$ and $\rho(n)$. CC Standard Errors obtained using the bootstrapped procedure in [Chen and Christensen \(2018\)](#).

B.4.2 Alternative Sample Years

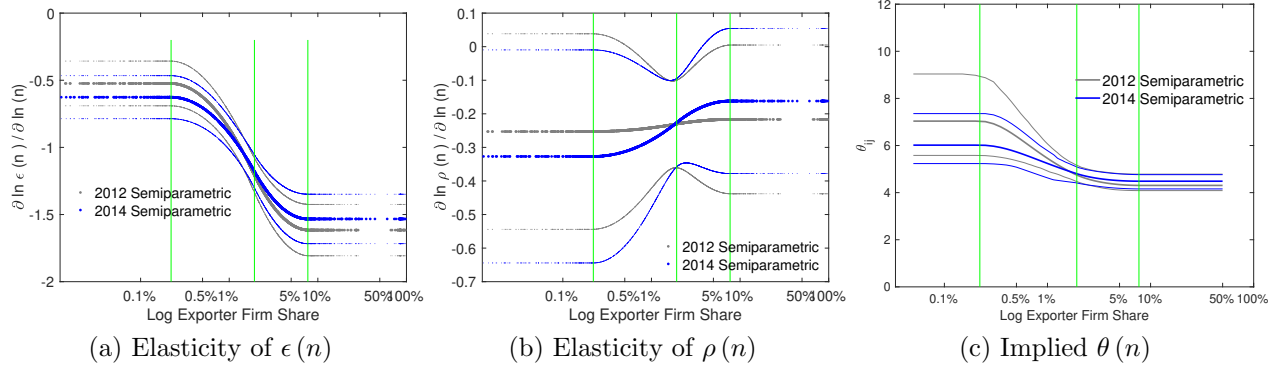
Our baseline estimates use the sample of country pairs for 2012. Our data has a similar country coverage for all years between 2010 and 2014. We thus estimate the model with alternative samples for 2010 and 2014. Figures OA.2 and OA.3 show that results are broadly consistent with the baseline estimates obtained from the sample for 2012.

Figure OA.2: Semiparametric Gravity of Firm Exports – 2010 Sample



Note. Estimates obtained with GMM estimator in (33). Estimates obtained with a cubic spline over three intervals ($K = 3$) for a single group ($G = 1$). Calibration of $\tilde{\sigma} = 2.9$. Thick lines are the point estimates and thin lines are the 95% confidence intervals computed with robust standard errors for $\epsilon(n)$ and $\rho(n)$ and 1,000 bootstrap draws for $\theta(n)$.

Figure OA.3: Semiparametric Gravity of Firm Exports – 2014 Sample

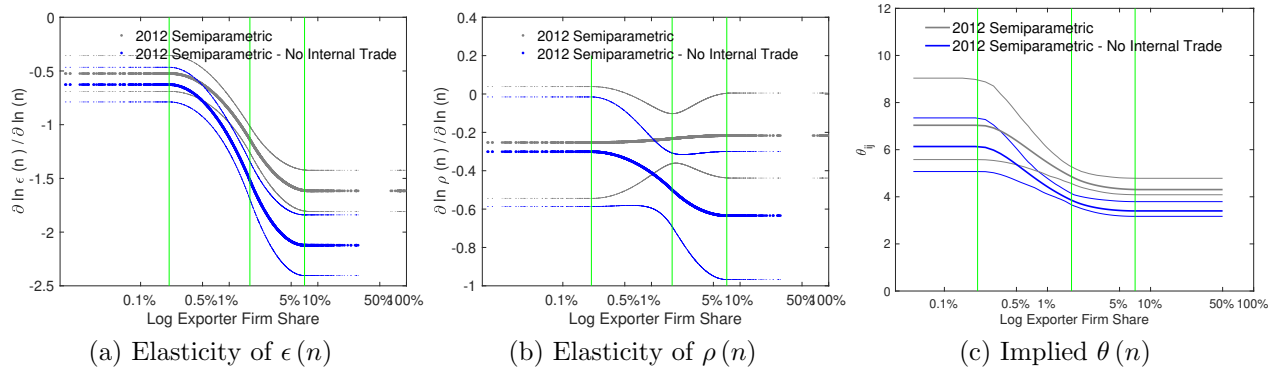


Note. Estimates obtained with GMM estimator in (33). Estimates obtained with a cubic spline over three intervals ($K = 3$) for a single group ($G = 1$). Calibration of $\bar{\sigma} = 2.9$. Thick lines are the point estimates and thin lines are the 95% confidence intervals computed with robust standard errors for $\epsilon(n)$ and $\rho(n)$ and 100 bootstrap draws for $\theta(n)$.

B.4.3 Baseline Sample Excluding Domestic Trade Observations

Our main estimation combines data on international trade with domestic sales. This requires not only n_{ij} and z_{ij} for $i \neq j$, but also n_{ii} and z_{ii} . Importantly, domestic sales are a high fraction of observations in the top knot where the trade elasticity is lower. To access whether these estimates depend on domestic sales, we re-run our estimation procedure in an alternative sample without domestic trade observations. Figure OA.4 shows that this has only a small impact on our baseline estimates of the trade elasticity function.

Figure OA.4: Semiparametric Gravity of Firm Exports – Sample excluding domestic trade observations



Note. Estimates obtained with GMM estimator in (33). Estimates obtained with a cubic spline over three intervals ($K = 3$) for a single group ($G = 1$). Calibration of $\bar{\sigma} = 2.9$. Thick lines are the point estimates and thin lines are the 95% confidence intervals computed with robust standard errors for $\epsilon(n)$ and $\rho(n)$ and 100 bootstrap draws for $\theta(n)$.

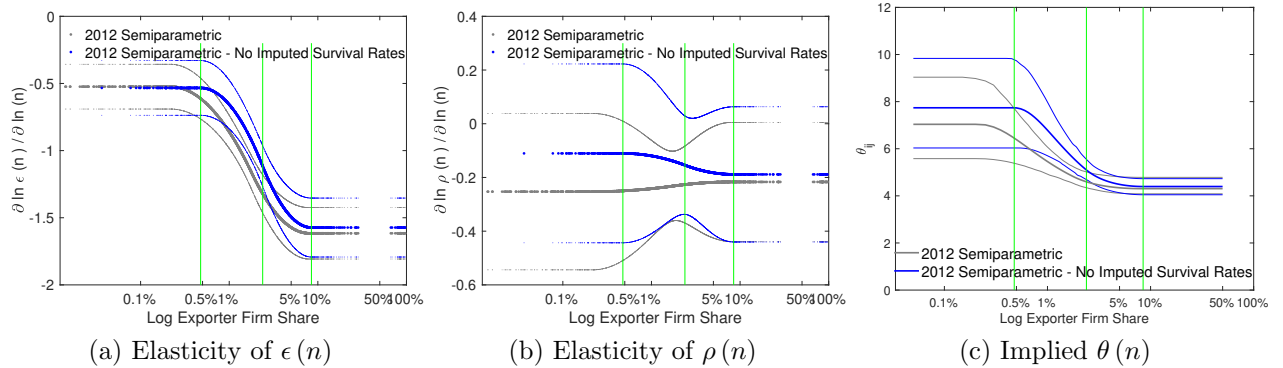
B.4.4 Alternative Measures of the Number of Entrants

In our data construction, we measure the share of successful entrants using one-year survival rates for manufacturing firms. This accounts for the fact that not all entrant firms are successful in entry and may leave the market (in the spirit of Melitz (2003)). We now conduct four robustness checks with respect to the construction of n_{ii} . We first exclude all countries with imputed values of n_{ii} from our baseline sample. We

also re-estimate the elasticity functions under the alternative assumptions that n_{ii} is either one (survival rate of 100%), the 2-year firm survival rate, or the 3-year firm survival rate.

Alternative sample excluding origin countries with imputed survival rate. In Figure OA.5, we replicate our baseline estimation in a sample that excludes all observations for origin countries with imputed values of the one-year survival rate. Results are roughly similar to our baseline estimates.

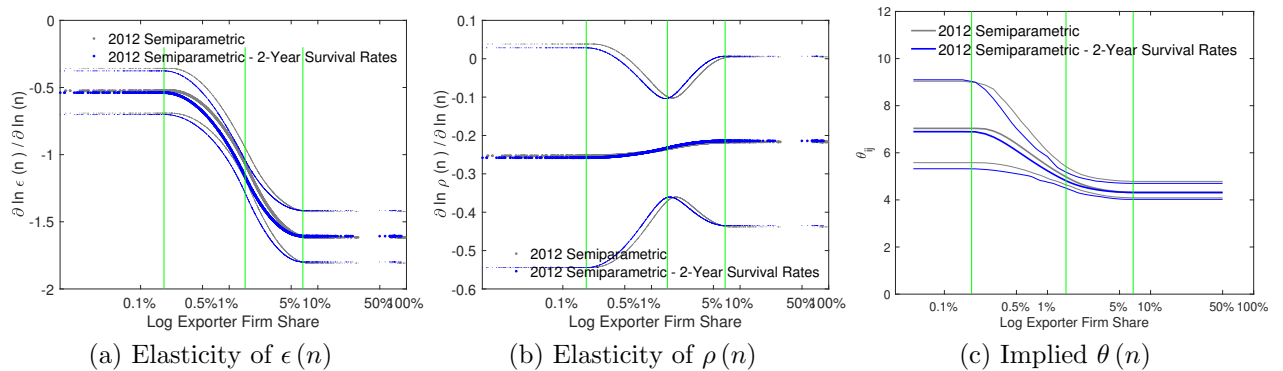
Figure OA.5: Semiparametric Gravity of Firm Exports – Sample excluding countries with imputed survival rates



Note. Estimates obtained with GMM estimator in (33). Estimates obtained with a cubic spline over three intervals ($K = 3$) for a single group ($G = 1$). Calibration of $\tilde{\sigma} = 2.9$. Thick lines are the point estimates and thin lines are the 95% confidence intervals computed with robust standard errors for $\epsilon(n)$ and $\rho(n)$ and 100 bootstrap draws for $\theta(n)$.

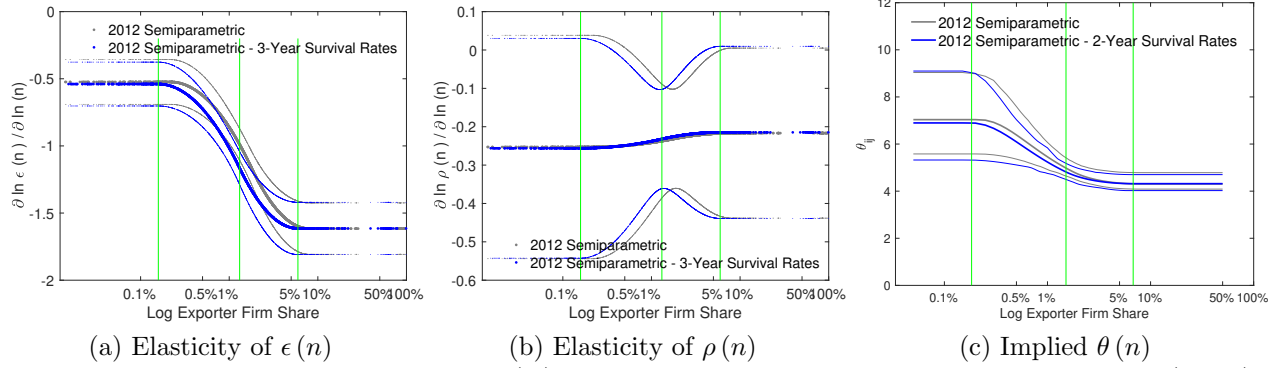
Alternative sample with n_{ii} measured with survival rates over different horizons. We now investigate how our baseline estimates change when we measure n_{ii} using 2-year and 3-year survival rates in manufacturing. Figures OA.6 and OA.7 show that the estimated elasticity functions are almost identical in both cases.

Figure OA.6: Semiparametric Gravity of Firm Exports – n_{ii} is two-year survival rate in manufacturing



Note. Estimates obtained with GMM estimator in (33). Estimates obtained with a cubic spline over three intervals ($K = 3$) for a single group ($G = 1$). Calibration of $\tilde{\sigma} = 2.9$. Thick lines are the point estimates and thin lines are the 95% confidence intervals computed with robust standard errors for $\epsilon(n)$ and $\rho(n)$ and 100 bootstrap draws for $\theta(n)$.

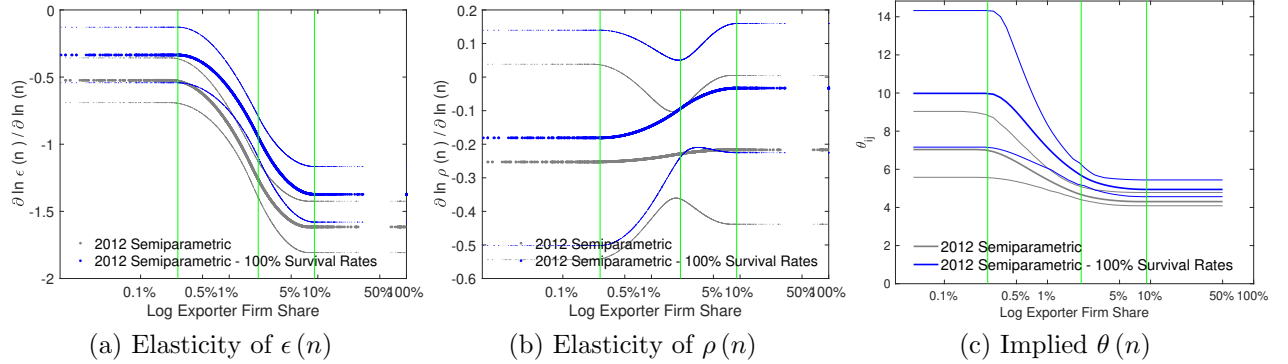
Figure OA.7: Semiparametric Gravity of Firm Exports – n_{ii} is three-year survival rate in manufacturing



Note. Estimates obtained with GMM estimator in (33). Estimates obtained with a cubic spline over three intervals ($K = 3$) for a single group ($G = 1$). Calibration of $\tilde{\sigma} = 2.9$. Thick lines are the point estimates and thin lines are the 95% confidence intervals computed with robust standard errors for $\epsilon(n)$ and $\rho(n)$ and 100 bootstrap draws for $\theta(n)$.

Alternative sample with $n_{ii} = 1$ (survival rate of 100%). In Figure OA.8, we replicate our baseline estimates under the assumption that all entrants produce for the domestic market (i.e., $\bar{f}_{ii} = 0$ and $n_{ii} = 1$). Panel (a) shows that the estimates of $\epsilon(n)$ are robust to imposing $n_{ii} = 1$. Panel (b) indicates that point estimates for the elasticity of $\rho(n)$ are slightly increasing, but are not statistically different from our baseline estimates. The lower elasticity of $\rho(n)$ yields estimates of the trade elasticity that are close to 10 in the bottom knot.

Figure OA.8: Semiparametric Gravity of Firm Exports – $n_{ii} = 1$

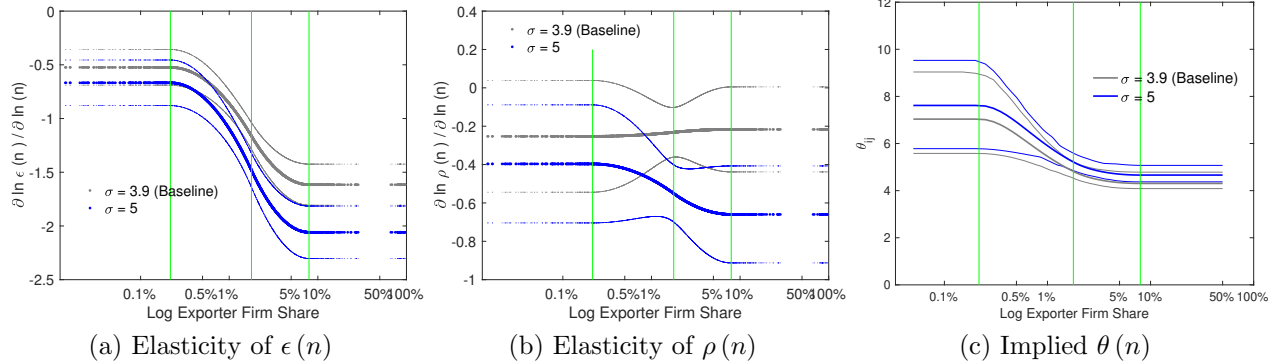


Note. Estimates obtained with GMM estimator in (33). Estimates obtained with a cubic spline over three intervals ($K = 3$) for a single group ($G = 1$). Calibration of $\tilde{\sigma} = 2.9$. Thick lines are the point estimates and thin lines are the 95% confidence intervals computed with robust standard errors for $\epsilon(n)$ and $\rho(n)$ and 100 bootstrap draws for $\theta(n)$.

B.4.5 Alternative Calibrations of the Elasticity of Substitution

In our baseline specification, we follow [Hottman et al. \(2016\)](#) to specify $\sigma = 3.9$. In Figure OA.9, we investigate how our estimates depend on the value of σ . In particular, we implement our estimator for a higher elasticity of $\sigma = 5$. Our estimates for $\epsilon(n)$ are broadly similar, with a small upward level shift. Our estimates for $\rho(n)$ are slightly lower, but still close to the baseline of -0.2. Panel (c) shows that the implied $\theta(n)$ are nearly identical, with similar point estimates and confidence intervals.

Figure OA.9: Semiparametric Gravity of Firm Exports – Alternative values of σ



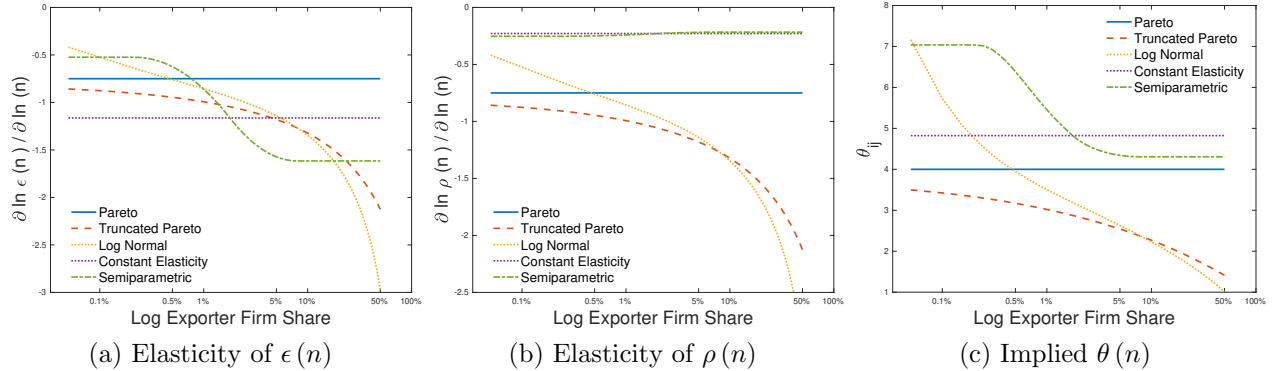
Note. Estimates obtained with GMM estimator in (33). Estimates obtained with a cubic spline over three intervals ($K = 3$) for a single group ($G = 1$). Baseline calibration of $\tilde{\sigma} = \sigma - 1 = 2.9$ and alternative calibration of $\tilde{\sigma} = \sigma - 1 = 4$. Thick lines are the point estimates and thin lines are the 95% confidence intervals computed with robust standard errors for $\epsilon(n)$ and $\rho(n)$ and 100 bootstrap draws for $\theta(n)$.

B.5 Counterfactual Analysis: Additional Results

B.5.1 Gains From Trade: Comparison to the Literature

In Figure OA.10, we compare the baseline estimates in Figure 3 to calibrated elasticity functions obtained from estimates in literature of parametric distributions of firm fundamentals.

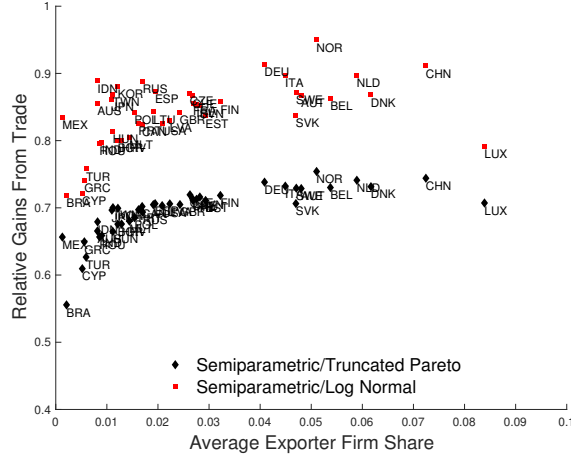
Figure OA.10: Baseline Estimates and Parametric Distributions in the Literature



Note. Pareto is the Melitz-Pareto model in [Chaney \(2008\)](#) with a trade elasticity of four. Truncated Pareto uses the productivity distribution in [Melitz and Redding \(2015\)](#). Log-normal uses the baseline estimate of the productivity distribution in [Head et al. \(2014\)](#). Constant elasticity and semiparametric specifications correspond to the baseline estimates in Section 5.4.

We compute the ratio between the gains from trade implied by our semiparametric gravity specification and the gains implied by specifications based on the assumption that firm productivity has either the Truncated Pareto distribution in [Melitz and Redding \(2015\)](#) or the Log-normal distribution in [Bas et al. \(2017\)](#). We use the parameter estimates reported on these papers. Figure OA.11 presents the cross-country relationship between these ratios and initial trade outcomes.

Figure OA.11: Importance of Functional Form Assumptions for the Gains from Trade



Note. Gains from Trade is the percentage change in the real wage implied by moving from autarky to the observed equilibrium in 2012. For each specification, gains from trade are computed with the formula in Section 3.3 for $\hat{n}_{ii}$ and $\hat{N}_i$ solving the system in Appendix A.6. Gains for semiparametric specification computed with the semiparametric estimates in Figure 3. Gains for Truncated Pareto specification computed with elasticity functions implied by the productivity distribution in Melitz and Redding (2015). Gains for Log-normal specification computed with elasticity functions implied by the productivity distribution in Head et al. (2014). Vertical axis is the ratio between the gains from trade implied by our baseline semiparametric estimates and those obtained with each parametric assumption.

The diamond-shaped dots in Figure OA.11 show that the Truncated Pareto specification leads to much higher gains from trade for all countries when compared to those implied by our baseline estimates. This is a direct consequence of the low trade elasticities implied by the parametrization in Melitz and Redding (2015) – see Figure OA.10. The square-shaped dots in Figure OA.11 show that the gains from trade are also lower for the Log-normal specification. Again, this follows from the average trade elasticity implied by the productivity distribution in Bas et al. (2017). Figure OA.10 shows that the implied trade elasticity in the log-normal case is slightly lower than our baseline estimate for all values of the exporter firm share.

B.5.2 The Welfare Impact of Observed Changes in Trade Costs: Additional Results

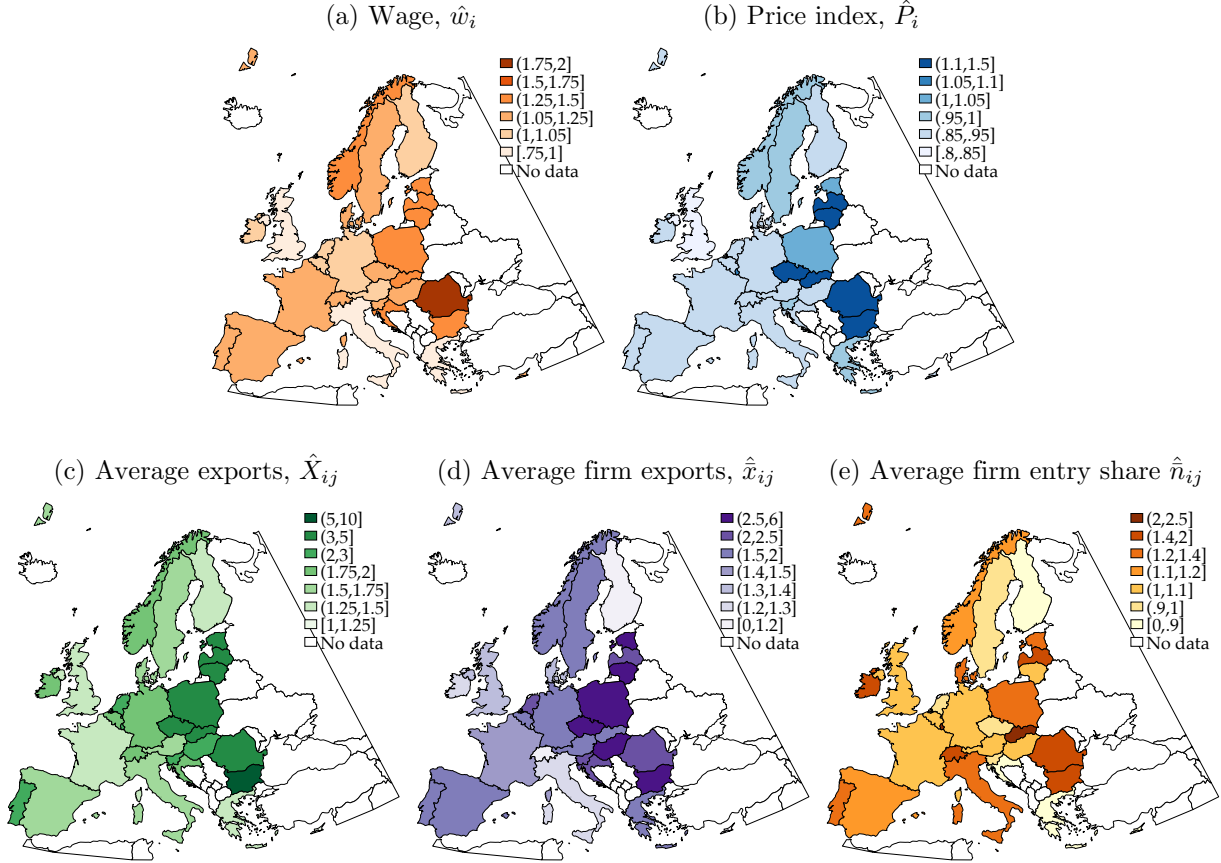
This section complements the quantitative results in Section 6.2 regarding the welfare impact of changes in trade costs among members of the European single market.

Summary statistics: changes in observed variables. Figure OA.12 depicts the change in the outcomes used in the inversion of changes in economic fundamentals for each origin country. Panel (a) shows that countries experienced vary different changes in their wage relative to the U.S. wage. We observe strong wage gains in Eastern Europe, which are 50% higher than U.S. wage gains for several countries. For western countries, wage gains are often small, with relative wages falling by 0.8% in Italy, 2.0% in the UK, and 10.2% in Greece. These wage patterns contribute to the weak growth in relative revenue shifters that we observe in West Europe. Panel (b) reports the price index changes used to measure the domestic revenue shifters, $\hat{r}_{jj}^t$. Relative to U.S. wage growth, the price index fell slightly in West Europe, but increased in East Europe.

Panels (c)–(e) illustrate the changes in export outcomes for each origin country. For most countries, we observe strong overall growth in total exports (panel (c)), firm average exports (panel (d)), and firm export shares (panel (e)). Again, export growth was substantially stronger in East Europe than in West Europe.

The different scales in panels (d) and (e) indicate that the growth in firm average exports was stronger than the growth in firm export shares. In our model, this fact is an important force contributing to the observed rises of both inverted fixed export costs and inverted revenue shifters – see the expressions in (36) and the companion discussion.

Figure OA.12: Change in Outcomes by Origin Country, 2003-2012



Note: Each panel reports the origin's outcome in 2012 divided by the origin's outcome in 2003. For bilateral variables in panels (c)-(e), we report the simple average of the ratio across all other destination countries j in the European single market (with $i \neq j$).

Comparison to the literature: alternative measures of trade cost shocks. We now compare the welfare impact of shocks in bilateral trade costs measured with different methodologies. Figure OA.15 reports the changes in real wages implied by three different approaches to measure trade cost shocks. In panel (a), we use the baseline changes in bilateral trade costs, $(\hat{\tau}_{ij})^{1-\sigma} = \hat{\tau}_{ij}/\hat{\tau}_{ii}$, implied by Proposition 3. In panel (b), we measure the changes in trade costs using observed tariff changes between countries – in our application, we consider tariff changes caused by the EU enlargement, as in [Caliendo et al. \(2017\)](#). Finally, in panel (c), we measure trade cost shocks using a non-parametric extension of the approach in [Head and Ries \(2001\)](#) that imposes symmetry of trade cost shocks. Specifically, by assuming that $(\hat{\tau}_{ij})^{1-\sigma} = (\hat{\tau}_{ji})^{1-\sigma}$, we set trade cost shocks to

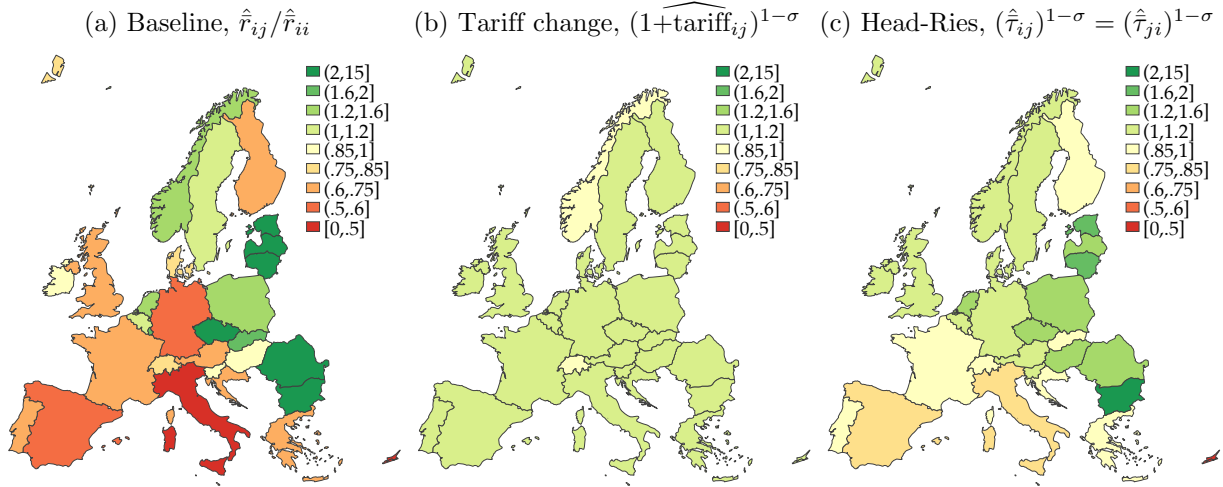
$$(\hat{\tau}_{ij})^{1-\sigma} = (\hat{\tau}_{ji})^{1-\sigma} = \left(\frac{\hat{x}_{ij} \hat{x}_{ji}}{\hat{x}_{ii} \hat{x}_{jj}} \left[\frac{\rho_{ij}(n_{ij})}{\rho_{ii}(n_{ii})} \frac{\rho_{ji}(n_{ji})}{\rho_{jj}(n_{jj})} \right] \left[\frac{\rho_{ij}(n_{ij} \hat{n}_{ij})}{\rho_{ii}(n_{ii} \hat{n}_{ii})} \frac{\rho_{ji}(n_{ji} \hat{n}_{ji})}{\rho_{jj}(n_{jj} \hat{n}_{jj})} \right]^{-1} \right)^{1/2}, \quad (\text{OA.56})$$

as implied by the semiparametric equation of firm average exports in (14).

Figures OA.13 and OA.14 compare the shocks measured with the different approaches. Panel (a) shows great cross-country dispersion in our baseline inverted trade shocks – the standard deviation across origins of the average log-change was 1.04. In Panel (b), tariff changes created much smaller changes in bilateral revenue shifters that were on average positive for all origin countries – the average log-change was 0.075, with a standard deviation of 0.11. Figure OA.14 indicates that tariff changes and our inverted shocks have a correlation close to zero. Finally, panel (c) shows the average revenue shifters implied by the symmetric shocks measured with (OA.56). Because it is “approximately” a bilateral average of our inverted trade shocks, expression (OA.56) yields smaller increases (declines) in bilateral revenue shifters for East (West) Europe. Figure OA.14 shows that, despite this attenuation bias, the two measures are positively correlated – their correlation is 0.56.

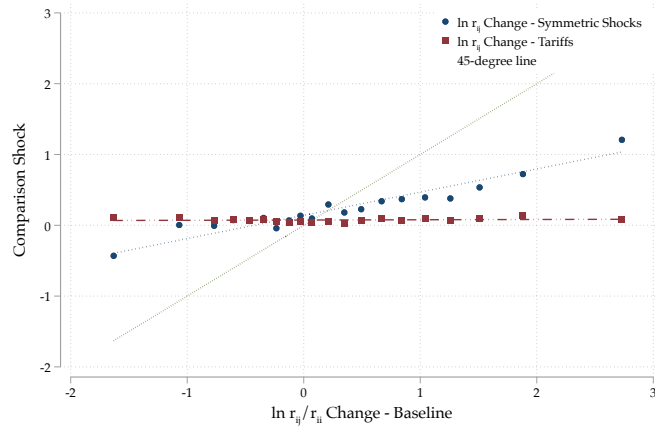
Figure OA.15 reports the welfare changes implied by the three alternative ways of measuring trade cost shocks. The comparison between panels (a) and (b) indicates that the welfare impact of the tariff changes is much smaller than that of our inverted trade cost shocks. The average effect is 0.7% in panel (a), but only 0.3% in panel (b). This follows from the fact that tariff changes are much smaller and more homogeneous than the inverted changes in trade costs. Lastly, when we use the symmetric trade cost shocks in panel (c), we get smaller welfare losses in West Europe and smaller welfare gains in East Europe. This attenuation of trade cost shocks is a natural implication by the symmetry assumptions (as discussed above).

Figure OA.13: Average Change in Bilateral Revenue Shifters in The European Single Market, 2003-2012 – Alternative Approaches to Measure Shocks



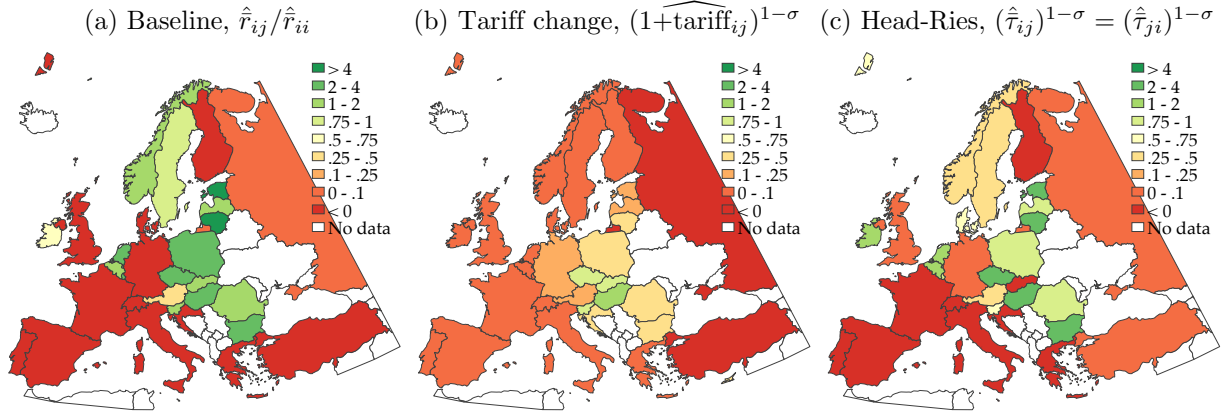
Note: Changes in variable trade costs recovered from changes in outcomes between 2003 and 2012. For each origin country i , we report the simple average of the change in bilateral revenue shifter across all other destination countries j in the European single market (with $i \neq j$). In Panel (a), we set $(\hat{\tau}_{ij})^{1-\sigma} = \hat{\tau}_{ij}/\hat{\tau}_{ii}$ for $i \neq j$ with $\hat{\tau}_{ij}$ computed using Proposition 3. In Panel (b), we set $(\hat{\tau}_{ij})^{1-\sigma} = (1+\widehat{\text{tariff}}_{ij})^{1-\sigma}$ to be the change in tariffs between EU members, as in [Caliendo et al. \(2017\)](#). In Panel (c), for $i \neq j$, we use $(\hat{\tau}_{ij})^{1-\sigma} = (\hat{\tau}_{ji})^{1-\sigma}$ given by the generalization of the approach in [Head and Ries \(2001\)](#) in expression (OA.56).

Figure OA.14: Bin Scatter Plot - Comparison of Changes in Bilateral Revenue Shifters Implied by Different Approaches



Note: Changes in variable trade costs recovered from changes in outcomes between 2003 and 2012. The horizontal axis is the baseline shock $(\hat{\tau}_{ij})^{1-\sigma} = \hat{r}_{ij}/\hat{r}_{ii}$ with $\hat{r}_{ij}$ computed using Proposition 3. The vertical axis is the change in bilateral revenue shifter implied by an alternative approach. For tariff change, we set $(\hat{\tau}_{ij})^{1-\sigma} = (1+\text{tariff}_{ij})^{1-\sigma}$ to be the change in tariffs between EU members, as in [Caliendo et al. \(2017\)](#). For symmetric shock, we use $(\hat{\tau}_{ij})^{1-\sigma} = (\hat{\tau}_{ji})^{1-\sigma}$ given by the generalization of the approach in [Head and Ries \(2001\)](#) in expression (OA.56).

Figure OA.15: The Welfare Impact of Changing Bilateral Variable Trade Costs in The European Single Market, 2003-2012 – Alternative Approaches to Measure Shocks



Note: Real wage changes caused by changes in bilateral trade shifters between 2003 and 2012 among 30 countries in the European single market. In Panel (a), we set $(\hat{\tau}_{ij})^{1-\sigma} = \hat{r}_{ij}/\hat{r}_{ii}$ for $i \neq j$ with $\hat{r}_{ij}$ computed using Proposition 3. In Panel (b), we set $\hat{\tau}_{ij}$ to be the change in tariffs between EU members, as in [Caliendo et al. \(2017\)](#). In Panel (c), for $i \neq j$, we use $(\hat{\tau}_{ij})^{1-\sigma} = (\hat{\tau}_{ji})^{1-\sigma}$ given by the generalization of the approach in [Head and Ries \(2001\)](#) in expression (OA.56). For each shock, we report minus the real wage change obtained with the baseline semiparametric estimates in Figure 3.